Which Equation Represents The Line That Passes Through Points (1, 5) And (3, 17)?

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Understanding how to find the equation of a line passing through two given points is a fundamental skill in algebra and coordinate geometry. When provided with two specific points, such as (1, 5) and (3, 17), the goal is to determine a linear equation in the form y = mx + b that accurately represents the line passing through these points. This process involves calculating the slope of the line and then using one of the points to solve for the y-intercept.

In this article, we will explore the step-by-step method to derive the equation of the line, starting from understanding the slope formula, working through the calculations, and finally writing the equation in slope-intercept form. We will also examine alternative forms and verify the correctness of our result.

Understanding the Components of a Line Equation

The Slope (m)

The slope of a line indicates its steepness and direction. It is calculated as the ratio of the change in y-values to the change in x-values between two points:

- $m = (y_2 - y_1) / (x_2 - x_1)$

This ratio tells us how much y increases or decreases as x increases by one unit.

The Y-Intercept (b)

The y-intercept is the point where the line crosses the y-axis, i.e., when x = 0. Once the slope is known, the y-intercept can be found by substituting x and y from one of the known points into the equation y = mx + b and solving for b.

Calculating the Slope of the Line

Given Points

The points provided are:

1. Point 1: (1, 5)
2. Point 2: (3, 17)

Applying the Slope Formula

Using the formula:

- $m = (y_2 - y_1) / (x_2 - x_1)$

Plugging in the values:

- m = (17 - 5) / (3 - 1) = 12 / 2 = 6

This means the line has a slope of 6, indicating it rises 6 units vertically for every 1 unit it moves horizontally.

Deriving the Equation of the Line

Using the Slope-Intercept Form

The slope-intercept form is:

- y = mx + b

Since we now know m = 6, we can substitute one of the points into the equation to find b.

Calculating the Y-Intercept (b)

Let's use point (1, 5):

- 5 = 6(1) + b

- 5 = 6 + b

- b = 5 - 6 = -1

Therefore, the equation of the line is:

- y = 6x - 1

Verification of the Equation

To ensure the correctness of our derived equation, verify it with the second point (3, 17):

- Substitute x = 3:

- y = 6(3) - 1 = 18 - 1 = 17

Since the calculated y matches the given y-coordinate, the equation y = 6x - 1 passes through both points, confirming its validity.

Alternative Forms of the Line Equation

While the slope-intercept form is most common, the line can also be expressed in other forms:

Point-Slope Form

Using point (1, 5):

- $y - y_1 = m(x - x_1)$

- y - 5 = 6(x - 1)

Simplify:

- y - 5 = 6x - 6

- y = 6x - 6 + 5

- y = 6x - 1

This confirms our previous result.

Standard Form

Rearranged into standard form:

- y - 6x = -1

or

- 6x - y = 1

Both are equivalent representations.

Summary and Final Equation

- The line passing through points (1, 5) and (3, 17) has a slope of 6.

- Using either point to solve for the y-intercept, we find b = -1.

- The final equation in slope-intercept form is:

y = 6x - 1

This equation accurately models the line passing through the given points.

Key Takeaways

- The slope is key to understanding the line's inclination and is calculated from two points.
- Substituting known points into the slope-intercept form allows solving for the y-intercept.
- Verification with both points ensures the correctness of the equation.
- Alternative forms such as point-slope and standard form are useful depending on context.

Understanding how to derive a line's equation from two points is fundamental in algebra. It not only aids in graphing and analysis but also forms the basis for more complex topics such as linear systems and inequalities. The method outlined here can be applied universally to any two points to determine the equation of the line passing through them.

Frequently Asked Questions

How do you find the equation of a line passing through the points (1, 5) and (3, 17)?

First, calculate the slope using $m = (17 - 5) / (3 - 1) = 12 / 2 = 6$. Then, use point-slope form: $y - 5 = 6(x - 1)$, which simplifies to $y = 6x - 1$.

What is the slope of the line passing through (1, 5) and (3, 17)?

The slope is 6, calculated as $(17 - 5) / (3 - 1) = 12 / 2 = 6$.

What is the equation of the line passing through (1, 5) and (3, 17)?

The equation is $y = 6x - 1$.

How can I verify that the line y = 6x - 1 passes through points (1, 5) and (3, 17)?

Plug in $x=1$: $y=6(1)-1=5$, matches (1, 5). Plug in $x=3$: $y=6(3)-1=18-1=17$, matches (3, 17).

What is the slope-intercept form of the line through (1, 5) and (3, 17)?

It is $y = 6x - 1$.

Can the line passing through (1, 5) and (3, 17) be written in point-slope form?

Yes, it can be written as $y - 5 = 6(x - 1)$.

What steps are involved in deriving the equation of a line from two points?

Calculate the slope, then use either point-slope form or slope-intercept form to write the equation.

Is the equation y = 6x - 1 unique for the points (1, 5) and (3, 17)?

Yes, since the slope is uniquely determined and the line passing through both points is unique, the equation y = 6x - 1 is the only line passing through them.

Additional Resources

Which Equation Represents The Line That Passes Through Points (1, 5) And (3, 17)?

In the realm of analytical geometry, understanding the equation of a line passing through two distinct points is a fundamental skill that underpins numerous applications, from engineering to physics, and even data analysis. This investigation aims to thoroughly explore the process of determining the specific linear equation that passes through the points (1, 5) and (3, 17), including derivations, methodology, and common pitfalls. By dissecting this problem, we hope to elucidate the principles behind line equations and provide clarity for students, educators, and professionals alike.

Understanding the Basics: The Equation of a Line

Before delving into calculations, it is essential to revisit the foundational concepts that describe lines in a two-dimensional Cartesian coordinate system.

The Slope-Intercept Form

The most familiar form of a line's equation is the slope-intercept form:

$$ y = m x + b $$

where:
- m is the slope of the line,
- b is the y-intercept (the point where the line crosses the y-axis).

This form is particularly useful when the slope and y-intercept are known, but it requires knowledge of these parameters.

The Point-Slope Form

When a point on the line and the slope are known, the point-slope form is advantageous:

$$ y - y_1 = m (x - x_1) $$

where:
- (x_1, y_1) is a point on the line,
- m is the slope.

The Two-Point Form

Given two points, the most direct approach involves first calculating the slope, then employing the point-slope form.

Determining the Slope: The Heart of the Solution

The core step in identifying the line's equation is calculating the slope m, which quantifies the steepness and direction of the line.

Calculating the Slope from Two Points

Given points (x_1, y_1) and (x_2, y_2), the slope m is:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

Applying this to our points:

$$ (x_1, y_1) = (1, 5) $$
$$ (x_2, y_2) = (3, 17) $$

we find:

$$ m = \frac{17 - 5}{3 - 1} = \frac{12}{2} = 6 $$

Thus, the slope m of the line passing through these points is 6.

Formulating the Equation: Step-by-Step Derivation

With the slope known, the next step involves constructing the explicit equation of the line.

Using Point-Slope Form

Choose either of the two points; here, we'll use $(1, 5)$:

$$y - 5 = 6 (x - 1)$$

Expanding:

$$y - 5 = 6x - 6$$

Adding 5 to both sides:

$$y = 6x - 6 + 5$$

$$y = 6x - 1$$

This is the slope-intercept form.

Verifying with the Second Point

To ensure accuracy, substitute $x = 3$:

$$y = 6(3) - 1 = 18 - 1 = 17$$

which matches the y-coordinate of the second point, confirming the correctness of the equation.

Alternative Forms and Their Significance

While the slope-intercept form $y = 6x - 1$ is concise and convenient, other forms can be employed depending on context.

Standard Form

Rearranged from the slope-intercept form:

$$ y = 6x - 1 \Rightarrow 6x - y = 1 $$

or

$$ 6x - y - 1 = 0 $$

This form is often preferred in algebraic manipulations, geometric proofs, and certain computational methods.

Two-Point Form Equation

Alternatively, the two-point form directly relates the points without explicitly calculating the slope:

$$ \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} $$

which, in our case:

$$ \frac{y - 5}{x - 1} = 6 $$

Cross-multiplied:

$$ y - 5 = 6 (x - 1) $$

which leads back to the same line equation.

Common Errors and Misconceptions

Despite the straightforward nature of the problem, several pitfalls can obscure the correct solution.

1. Confusing the Order of Points

Using the points in the wrong order can lead to a negative slope if the differences are reversed. Consistent ordering is vital.

2. Arithmetic Mistakes in Slope Calculation

Incorrect subtraction or division can lead to wrong slope values.

3. Overlooking the Validity of the Points

Ensure the points are distinct; identical points would not define a unique line or would imply infinitely many lines.

4. Sign Errors in Equation Rearrangement

When converting between forms, be cautious with signs and coefficients.

Applications and Implications of the Derived Equation

Understanding the precise equation of a line passing through two points is more than an academic exercise; it has practical implications.

Data Modeling and Regression Analysis

Linear equations form the basis of regression models, especially in predictive analytics.

Engineering and Design

Knowing the line's equation helps in designing components, paths, or trajectories.

Physics and Motion

The equation describes constant velocity motion in kinematics.

Graphical Representation

Plotting the line accurately depends on the derived equation, facilitating visual analysis.

Summary and Final Equation

To synthesize, the process of determining the equation of a line passing through points (1, 5) and (3, 17) involves:

1. Calculating the slope:

$$ m = \frac{17 - 5}{3 - 1} = 6 $$

2. Applying the point-slope form with either point:

$$ y - 5 = 6(x - 1) $$

3. Simplifying to slope-intercept form:

$$ y = 6x - 1 $$

4. Verifying with the second point confirms the equation's correctness.

Final Equation:

$$ \boxed{y = 6x - 1} $$

This linear equation fully characterizes the unique line passing through the given points.

Conclusion

Determining the equation of a line through two points is a cornerstone of analytical geometry that combines algebraic manipulation with geometric intuition. By methodically calculating the slope and employing the appropriate form, one can accurately and efficiently derive the line's equation. The specific case of points (1, 5) and (3, 17) illustrates this process clearly, resulting in the elegant and precise line equation $y = 6x - 1$.

This exercise underscores the importance of fundamental concepts and careful calculation, serving as a model approach applicable to more complex scenarios involving lines and curves in mathematics and applied sciences.

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