Write The Equation Of The Line That Is Parallel To Y=4x+12 Through The Point (2,3).

Write The Equation Of The Line That Is Parallel To Y=4x+12 Through The Point (2,3)

When working with linear equations, a common problem involves finding the equation of a line that satisfies specific conditions—such as passing through a particular point and being parallel to another given line. In this case, we are asked to write the equation of the line that is parallel to the line y = 4x + 12 and passes through the point (2, 3). This process involves understanding the properties of parallel lines, the concept of slope, and the point-slope form of a line's equation. This comprehensive guide will walk you through each step with clarity, providing insights into the mathematics involved to help you grasp the concept thoroughly.

Understanding the Problem

Before diving into calculations, it's essential to understand what the question asks:

- Line to be parallel to: y = 4x + 12
- Point through which the new line passes: (2, 3)
- Goal: Find the equation of the line satisfying both conditions.

This task involves two key concepts:

1. Parallel Lines: Lines that have the same slope but different y-intercepts.
2. Point-Slope Form: A way to write the equation of a line when you know a point on the line and its slope.

Let's explore each concept in detail.

Key Concepts in Finding the Equation of a Parallel Line

1. Slope of the Given Line

The given line y = 4x + 12 is in slope-intercept form, y = mx + b, where:

- m is the slope
- b is the y-intercept

From the equation, the slope (m) is 4. Since parallel lines have identical slopes, the new line will also have a slope of 4.

2. Parallel Lines and Their Slopes

Parallel lines are lines in the same plane that never intersect. The defining property of parallel lines in Euclidean geometry is that they have:

- Equal slopes: $m1 = m2$

In this problem:

- The original line's slope: 4
- The new line's slope: also 4

3. Using the Point-Slope Form

The point-slope form of a line's equation is:

$y - y1 = m(x - x1)$

where:

- $(x1, y1)$ is a point on the line
- m is the slope

Given the point (2, 3) and the slope $m = 4$, the equation becomes:

$y - 3 = 4(x - 2)$

This form makes it straightforward to write the equation of the required line.

Step-by-Step Solution

Let's now go through each step to find the equation of the line.

Step 1: Identify the known values

- Point: (2, 3)
- Slope: 4

Step 2: Write the point-slope form

Using the point-slope formula:

$y - y1 = m(x - x1)$

Substitute:

- $y1 = 3$
- $x1 = 2$
- $m = 4$

Resulting in:

$y - 3 = 4(x - 2)$

Step 3: Simplify to slope-intercept form

Distribute the slope:

$y - 3 = 4x - 8$

Add 3 to both sides:

$y = 4x - 8 + 3$

Simplify:

$y = 4x - 5$

This is the slope-intercept form of the line.

Final Equation of the Line

The equation of the line that is parallel to $y = 4x + 12$ and passes through the point (2, 3) is:

$y = 4x - 5$

This line has the same slope as the original line and passes through the specified point.

Additional Information and Variations

Understanding different forms of a line's equation can be helpful in various contexts.

1. Slope-Intercept Form

- The form $y = mx + b$ directly gives the slope (m) and y-intercept (b).

- For the solution line, $y = 4x - 5$, the slope is 4 and y-intercept is -5.

2. Standard Form

- The standard form of a line is $Ax + By = C$.
- Convert $y = 4x - 5$ to standard form:

1. Subtract 4x from both sides:

$y - 4x = -5$

2. Rearrange:

$-4x + y = -5$

3. Multiply through by -1 to make A positive:

$4x - y = 5$

- The standard form is $4x - y = 5$.

3. Graphical Representation

Plotting these lines can help visualize their parallelism:

- The original line $y = 4x + 12$ passes through points such as (0, 12) and (1, 16).
- The new line $y = 4x - 5$ passes through points such as (0, -5) and (1, -1).

Both have the same slope (4), confirming their parallelism.

Real-World Applications of Parallel Line Equations

Understanding how to find the equation of a parallel line has practical applications across various fields:

- Engineering: Designing parallel components or structures.
- Architecture: Planning parallel walls or features.
- Navigation: Charting routes that maintain a consistent bearing.
- Data Analysis: Fitting parallel trend lines to datasets to compare patterns.

Practice Problems to Reinforce Learning

Engaging with similar problems can solidify your understanding:

1. Find the equation of a line parallel to y = -2x + 7 passing through the point (4, 5).
2. Determine the equation of a line parallel to y = (1/3)x - 4 that passes through the point (-1, 2).
3. Write the equation of a line parallel to y = 5x + 3 and passing through (0, 0).

Each problem involves similar steps: identifying the slope, applying the point-slope form, and converting to the desired form.

Summary and Key Takeaways

- The slope of the original line y = 4x + 12 is 4.
- Parallel lines share the same slope.
- Using the point-slope form with the given point and the known slope, we derived the equation y = 4x - 5.
- This equation can be expressed in various forms (slope-intercept, standard) depending on the context.

In conclusion, understanding how to find the equation of a parallel line involves recognizing the importance of slope, applying the point-slope formula, and being comfortable with algebraic manipulations. Mastering this process enhances your ability to solve a broad range of problems involving linear equations, which are foundational in mathematics and its applications.

If you need further clarification or additional examples, feel free to ask!

Frequently Asked Questions

How do you find the equation of a line parallel to y = 4x + 12 passing through a specific point?

To find a parallel line, use the same slope as the original line (which is 4). Then, substitute the point (2, 3) into y = mx + b to solve for b: 3 = 4(2) + b, so b = 3 - 8 = -5. The equation is y = 4x - 5.

What is the slope of the line parallel to y = 4x + 12 that passes through (2, 3)?

The slope is the same as the original line, which is 4.

Can you provide the step-by-step process to write the equation of a parallel line through (2, 3)?

Yes. First, identify the slope from the original line (4). Then, use the point-slope form: $y - y_1 = m(x - x_1)$. Substitute $m=4$, $x_1=2$, $y_1=3$: $y - 3 = 4(x - 2)$. Simplify to get $y = 4x - 5$.

What is the final equation of the line parallel to y=4x+12 passing through (2,3)?

The equation is $y = 4x - 5$.

Why are lines parallel if they have the same slope, and how does that relate to this problem?

Lines with the same slope are parallel because they never intersect. In this problem, since the original line has slope 4, the parallel line must also have slope 4, ensuring they are parallel.

Additional Resources

Write The Equation Of The Line That Is Parallel To Y=4x+12 Through The Point (2,3)

Introduction

Mathematics, specifically coordinate geometry, is fundamental in understanding the spatial relationships between lines and points. One common problem involves finding the equation of a line that satisfies specific conditions, such as being parallel to a given line and passing through a particular point. This article delves into the problem: Write the equation of the line that is parallel to $y=4x+12$ through the point (2,3), exploring the principles, methods, and underlying concepts involved.

Understanding the Problem

The problem asks for the equation of a line with two key attributes:
1. The line must be parallel to the given line: $y = 4x + 12$.
2. The line must pass through the point (2, 3).

To solve this, we need to understand the fundamental properties of lines in the coordinate plane, especially regarding their slopes and equations.

Fundamental Concepts in Line Equations

1. Slope-Intercept Form

The most common form of a linear equation is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line.
- b is the y-intercept — the point where the line crosses the y-axis.

In the given line y=4x+12, the slope $m = 4$.

2. Parallel Lines and Equal Slopes

Two lines are parallel if and only if they have the same slope but different y-intercepts. This means that if we know the slope of one line, the parallel line must have the same slope.

Given:
- Original line: $y = 4x + 12$
- Slope of original line: $m = 4$

Therefore, the parallel line must also have a slope $m = 4$.

Step-by-Step Solution Approach

Step 1: Identify the slope of the parallel line

Since the original line's slope is 4, the parallel line must also have:

$$ m_{parallel} = 4 $$

Step 2: Use the point-slope form to find the new line

The point-slope form of a line's equation is:

$$ y - y_1 = m (x - x_1) $$

where:
- (x_1, y_1) is a point on the line.
- m is the slope.

Given:
- Point $(2, 3)$
- Slope $m = 4$

Substituting into the point-slope form:

$$ y - 3 = 4 (x - 2) $$

Step 3: Simplify to slope-intercept form

Expanding:

$$ y - 3 = 4x - 8 $$

Adding 3 to both sides:

$$ y = 4x - 8 + 3 $$
$$ y = 4x - 5 $$

This is the equation of the line that is parallel to $y=4x+12$ and passes through $(2,3)$.

Final Equation and Validation

Answer:

$$ y = 4x - 5 $$

This line has:
- The same slope as the original line (parallel condition).
- Passes through $(2, 3)$, verified by substituting $x=2$:

$$ y = 4(2) - 5 = 8 - 5 = 3 $$

which matches the y-coordinate of the point, confirming the correctness.

Deeper Exploration: Why Does Slope Determine Parallelism?

The Geometric Significance of Slope

In the coordinate plane, the slope of a line indicates its steepness and direction. Two lines with identical slopes are always parallel because they never intersect (unless they are coincident, which would mean they are the same line).

The slope is a measure of the ratio of vertical change to horizontal change:

$$ m = \frac{\Delta y}{\Delta x} $$

For the original line $y=4x+12$, every unit increase in x results in a 4-unit increase in y. This consistent ratio ensures the line's direction. To be parallel, another line must have this same ratio, i.e., the same slope.

Variations and Related Problems

1. Finding a Perpendicular Line

Instead of parallel, what if the problem asked for the perpendicular line passing through the same point?

Since perpendicular lines have slopes that are negative reciprocals:

$$ m_{perp} = -\frac{1}{m_{original}} = -\frac{1}{4} $$

Using the point $(2, 3)$, the line's equation:

$$ y - 3 = -\frac{1}{4} (x - 2) $$
$$ y - 3 = -\frac{1}{4}x + \frac{1}{2} $$
$$ y = -\frac{1}{4}x + \frac{1}{2} + 3 $$
$$ y = -\frac{1}{4}x + \frac{7}{2} $$

Practical Applications in Real-World Contexts

While this problem appears purely mathematical, the principles of parallel lines and slope are vital in various fields:

- Engineering: Designing parallel beams, roads, or rails.
- Architecture: Ensuring walls or structural components are parallel.
- Navigation: Calculating paths with consistent slopes for safety or efficiency.
- Data Analysis: Understanding trends that run parallel in scatter plots or regression lines.

Summary and Key Takeaways

- To write the equation of a line parallel to a given line, match the slope.
- Use the point-slope form when given a point (x_1, y_1) and slope m.
- Convert to slope-intercept form for simplicity and clarity.
- Verify by substituting the point into the equation.

Final Result:

$$
\boxed{
\text{The line parallel to } y=4x+12 \text{ passing through } (2,3) \text{ is } y=4x-5
}
$$

Conclusion

The process of deriving the line equation demonstrates fundamental principles in coordinate geometry — chiefly, the significance of slope and point positioning. Understanding how to manipulate these equations equips students and professionals to solve complex spatial problems efficiently. The problem also exemplifies how straightforward algebraic techniques can be applied to interpret real-world scenarios involving parallelism and linear relationships.

By grasping these concepts, learners can confidently approach similar problems, whether in academic settings or practical applications, reinforcing the importance of a solid foundation in the geometry of lines.

Write The Equation Of The Line That Is Parallel To Y 4x 12 Through The Point 2 3

Write The Equation Of The Line That Is Parallel To Y 4x 12 Through The Point 2 3

Related Articles

- The Insertion Tendon Of The______ Group Contains A Large Sesamoid Bone, The Patella
- One Of The Most Distinctive Cultural Features Of Culture In Central Asia Includes
- The Issue Of The Constitutional Protections Afforded Flag Burning Was Addressed In.

Back to Home