(1 Point) Find An Equation Of The Curve That Satisfies Dydx=63yx6 And Whose Y-intercept Is 5.

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If you're studying differential equations and want to find the equation of a specific curve given its derivative and a point, you're in the right place. In this article, we'll explore how to find the equation of a curve when the derivative $\frac{dy}{dx}$ is given as $63 y x^6$, and the curve passes through the point where $y=5$ when $x=0$. This problem is a classic example of solving first-order differential equations using separation of variables and applying initial conditions to find the particular solution.

Understanding the Differential Equation $\frac{dy}{dx} = 63 y x^6$

Before jumping into solving the differential equation, it's important to understand what it represents.

What does the derivative tell us?

- The derivative $\frac{dy}{dx} = 63 y x^6$ indicates how y changes with respect to x.
- Notice that the rate of change of y is proportional to both y itself and x^6.

Type of differential equation

- This is a separable differential equation because it can be written in the form $dy/y = \text{function of } x \, dx$.

Step-by-Step Solution Process

To find the equation of the curve, follow these steps:

1. Rewrite the differential equation in separable form

$$
\frac{dy}{dx} = 63 y x^6
$$

Dividing both sides by y (assuming $y \neq 0$):

$$
\frac{1}{y} \frac{dy}{dx} = 63 x^6
$$

which can be written as:

$$
\frac{dy}{y} = 63 x^6 dx
$$

2. Integrate both sides

Integrate with respect to their variables:

$$
\int \frac{1}{y} dy = \int 63 x^6 dx
$$

Calculating each integral:

- Left side:

$$
\int \frac{1}{y} dy = \ln|y| + C_1
$$

- Right side:

$$
\int 63 x^6 dx = 63 \times \frac{x^7}{7} + C_2 = 9 x^7 + C
$$

where $C = C_2 - C_1$, a combined constant of integration.

3. Write the general solution

$$
\ln|y| = 9 x^7 + C
$$

Exponentiating both sides:

$$
|y| = e^{9 x^7 + C} = e^{C} e^{9 x^7}
$$

Let $A = e^{C}$, which is a positive constant. Therefore:

$$
y = \pm A e^{9 x^7}
$$

Since the problem states the initial condition y-intercept is 5 (meaning $y(0) = 5$), and the exponential function is always positive, we take the positive branch:

$$
\boxed{
y = A e^{9 x^7}
}
$$

Applying the Initial Condition

The initial condition states:

$$
y(0) = 5
$$

Substitute $x=0$:

$$
5 = A e^{9 \times 0^7} = A e^{0} = A \times 1
$$

So,

$$
A = 5
$$

Thus, the particular solution is:

$$
\boxed{
y = 5 e^{9 x^7}
}
$$

Final Equation of the Curve

The equation of the curve that satisfies $\frac{dy}{dx} = 63 y x^6$ and passes through the point $(0, 5)$ is:

$$
\boxed{
\boxed{
y = 5 e^{9 x^7}
}
}
$$

This exponential function describes the behavior of y in relation to x, with rapid growth as x increases due to the $e^{9 x^7}$ term.

Key Takeaways for Solving Similar Differential Equations

Separable Differential Equations

- These equations can be written as $\frac{dy}{dx} = g(y) h(x)$.
- The solution involves separating variables y and x and integrating both sides.

Applying Initial Conditions

- Always substitute the given point into the general solution to find the constant of integration.
- Initial conditions help convert the general solution into a particular solution.

Handling Exponential Functions

- When integrating, constants of integration transform into multiplicative constants after exponentiation.
- Always consider the absolute value when dealing with $\ln|y|$, but choose the sign based on initial conditions.

Conclusion

Finding the equation of a curve from a differential equation involves understanding the type of equation, separating variables, integrating, and then applying initial conditions to find the specific solution. In this case, with $\frac{dy}{dx} = 63 y x^6$ and the y-intercept at 5, the solution is:

$$
\boxed{
y = 5 e^{9 x^7}
}
$$

This process highlights the power of separation of variables and the importance of initial conditions in differential equations. Whether you're tackling homework, preparing for exams, or working on real-world modeling, mastering these techniques is essential for solving first-order differential equations effectively.

Frequently Asked Questions

What is the differential equation given in the problem?

The differential equation is dy/dx = 63 y x^6.

What is the initial condition provided for the curve?

The y-intercept is 5, which means when x = 0, y = 5.

How do we separate variables to solve the differential equation dy/dx = 63 y x^6?

We rewrite it as dy/y = 63 x^6 dx to separate variables.

What is the integral form needed to find the solution for y?

Integrate both sides: ∫ (1/y) dy = ∫ 63 x^6 dx.

What is the result of integrating ∫ (1/y) dy?

The integral is ln|y| + C1.

What is the integral of 63 x^6 dx?

The integral is 63 (x^7/7) + C2, which simplifies to 9 x^7 + C.

What is the general solution of the differential equation?

ln|y| = 9 x^7 + C, which implies y = A e^{9 x^7} where A = e^C.

How do we determine the constant A using the y-intercept condition?

Substitute $x=0$ and $y=5$ into $y = A e^{9 x^7}$ to get $5 = A e^0$, so $A = 5$.

What is the final equation of the curve satisfying the given conditions?

The equation is $y = 5 e^{9 x^7}$.

Why is the exponential function used in the solution?

Because the differential equation is separable and linear in y, leading to an exponential solution after integration.

Additional Resources

Find an equation of the curve that satisfies $\frac{dy}{dx} = 63 y x^6$ and whose y-intercept is 5 is a classic problem in differential equations, illustrating how calculus links the rate of change of a function to its original form. This problem requires us to find the explicit equation of a curve based on its derivative and a specific initial condition. Such problems are central in many fields, including physics, engineering, and mathematics, where understanding the relationship between variables is crucial. In this article, we will walk through a detailed, step-by-step process to solve this differential equation, interpret the conditions, and derive the explicit equation of the curve.

Understanding the Problem

At its core, the problem asks us to:

- Find the explicit function $y(x)$ that satisfies the differential equation:

$$
\frac{dy}{dx} = 63 y x^6
$$

- Ensure that this function passes through the point where y-intercept is 5, which means:

$$
y(0) = 5
$$

The key points to note are:

- The differential equation involves the product of y and x^6, hinting at the possibility of a separable differential equation.
- The initial condition $y(0) = 5$ allows us to solve for the constant of integration after finding the general solution.

Step 1: Recognizing the Type of Differential Equation

The given differential equation:

$$
\frac{dy}{dx} = 63 y x^6
$$

is a separable differential equation because it can be written as:

$$
\frac{1}{y} dy = 63 x^6 dx
$$

Separable equations are straightforward to solve as they involve functions of y and x that can be integrated separately.

Step 2: Separating the Variables

Rearranging the differential equation:

$$
\frac{dy}{dx} = 63 y x^6
$$

we isolate the variables:

$$
\frac{1}{y} dy = 63 x^6 dx
$$

This separation allows us to integrate both sides independently:

- Left side with respect to y
- Right side with respect to x

Step 3: Integrating Both Sides

Integrate the Left Side:

$$
\int \frac{1}{y} dy = \int 63 x^6 dx
$$

The integral of $\frac{1}{y}$ with respect to y is:

$$
\ln |y| + C_1
$$

where C_1 is the constant of integration (though we will consolidate constants later).

Integrate the Right Side:

$$
\int 63 x^6 dx = 63 \int x^6 dx
$$

The integral of x^6 is:

$$
\frac{x^{7}}{7}
$$

So, the right side becomes:

$$
63 \times \frac{x^{7}}{7} = 9 x^{7}
$$

Step 4: Write the General Solution

Putting both integrals together:

$$
\ln |y| = 9 x^{7} + C
$$

where C is the combined constant of integration.

To express y, exponentiate both sides:

$$
|y| = e^{9 x^{7} + C} = e^{C} e^{9 x^{7}}
$$

Let $A = e^{C}$, which is an arbitrary positive constant:

$$
y = A e^{9 x^{7}}
$$

Note: Since y can be positive or negative, but the initial condition will determine the sign.

Step 5: Applying the Initial Condition

Given that the y-intercept is 5, meaning:

$$
y(0) = 5
$$

Plug in $x = 0$:

$$
y(0) = A e^{9 \times 0^{7}} = A e^{0} = A \times 1 = A
$$

Therefore:

$$
A = 5
$$

The explicit solution is:

$$
y(x) = 5 e^{9 x^{7}}
$$

Final Equation of the Curve

The curve satisfying the differential equation $\frac{dy}{dx} = 63 y x^6$ with the y-intercept of 5 is:

$$
\boxed{
\quad y(x) = 5 e^{9 x^{7}}
}
$$

Additional Insights and Applications

Why Is This Solution Important?

This solution exemplifies how differential equations model exponential growth or decay processes where the rate depends on the current state. Here, the dependence on both y and x^6 creates a scenario where the growth rate accelerates rapidly as x increases, especially given the exponential nature of the result.

Real-World Contexts

- Population Dynamics: In some models, the growth rate of a population could depend on both the current population size (y) and some environmental variable modeled as x^6.
- Radioactive Decay or Growth: Exponential functions are common in nuclear physics, where the rate of decay or growth depends proportionally on the current quantity.

Summary of Steps

To recap, solving for the equation of the curve involved:

1. Recognizing the differential equation as separable.
2. Separating variables: $\frac{1}{y} dy = 63 x^6 dx$.
3. Integrating both sides: $\ln |y| = 9 x^7 + C$.
4. Exponentiating to solve for y: $y = A e^{9 x^7}$.
5. Applying the initial condition $y(0) = 5$ to find $A = 5$.
6. Presenting the final explicit function: $y(x) = 5 e^{9 x^7}$.

Final Thoughts

The process of solving differential equations like $\frac{dy}{dx} = 63 y x^6$ underscores the elegance of calculus in uncovering the underlying relationships between variables. Recognizing the type of differential equation, applying appropriate integration techniques, and using initial conditions to find specific solutions are essential skills in mathematical problem-solving. Whether modeling natural phenomena or designing engineering systems, these methods form the backbone of analytical problem solving in science and mathematics.

If you encounter similar problems, remember to:

- Look for separability or other recognizable forms.
- Carefully perform integrations, paying attention to constants.
- Use initial or boundary conditions to determine constants.
- Interpret the solution in the context of the problem.

By mastering these steps, you'll be well-equipped to tackle a wide range of differential equations with confidence.

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