2. Given That F(x) = 1/x-6' And G(x)=x-7(a) Find (fg)(x). (b) Find The Domain Of (fog)(x).

Understanding Function Composition and Domain: A Comprehensive Guide

2. Given That F(x) = 1/x-6' And G(x)=x-7(a) Find (fg)(x). (b) Find The Domain Of (fog)(x).

In the realm of mathematics, particularly in algebra and calculus, functions are fundamental building blocks. Understanding how functions interact through composition and how to determine their domains is essential for solving complex problems. This article provides an in-depth explanation of function composition, specifically focusing on two functions: F(x) and G(x). We will explore how to find the composite function (fg)(x) and analyze the domain of (fog)(x). By the end of this article, you'll have a clear understanding of these concepts, along with practical steps to approach similar problems.

Understanding the Given Functions

Before diving into the problem, it's crucial to comprehend the given functions:

- F(x) = 1/(x - 6)
- G(x) = x - 7

Notice that the original statement has some formatting ambiguities, but based on standard function notation and common algebraic forms, we interpret:

- F(x) as a rational function: F(x) = 1 divided by (x - 6).
- G(x) as a linear function: G(x) = x - 7.

The problem asks two things:

a) Find (fg)(x), which is the composition F(g(x)).

b) Find the domain of (fog)(x), which is G(F(x)).

Let's analyze each part step-by-step.

Part A: Finding (fg)(x) = F(g(x))

Step 1: Understand the Composition of Functions

Function composition involves applying one function to the result of another. For (fg)(x), which is often written as F(g(x)), the process is:

- First, evaluate G(x) to find g(x).
- Then, substitute g(x) into F(x) to find F(g(x)).

Mathematically:

$$
(fg)(x) = F(g(x))
$$

Given G(x) = x - 7, we have:

$$
F(g(x)) = F(x - 7)
$$

And since F(x) = 1 / (x - 6), substituting g(x) gives:

$$
F(x - 7) = \frac{1}{(x - 7) - 6}
$$

Simplify the denominator:

$$
(x - 7) - 6 = x - 13
$$

Therefore,

$$
(fg)(x) = \frac{1}{x - 13}
$$

Step 2: Final Expression for (fg)(x)

The composite function is:

$$
\boxed{
(fg)(x) = \frac{1}{x - 13}
}
$$

Part B: Finding the Domain of (fog)(x) = G(F(x))

Step 1: Understand the Composition G(F(x))

Similarly, G(F(x)) involves:

- Applying F(x) first.
- Then applying G to the result of F(x).

Given:

$$
F(x) = \frac{1}{x - 6}
$$
$$
G(x) = x - 7
$$

The composition G(F(x)) becomes:

$$
(G \circ F)(x) = G(F(x)) = F(x) - 7
$$

Because G(x) = x - 7, G(F(x)) is:

$$
G(F(x)) = \frac{1}{x - 6} - 7
$$

Step 2: Simplify G(F(x))

Express as a single fraction:

$$
G(F(x)) = \frac{1}{x - 6} - 7 = \frac{1}{x - 6} - \frac{7(x - 6)}{x - 6}
$$

Combine into one fraction:

$$
G(F(x)) = \frac{1 - 7(x - 6)}{x - 6}
$$

Simplify numerator:

$$
1 - 7x + 42 = (1 + 42) - 7x = 43 - 7x
$$

Thus,

$$
G(F(x)) = \frac{43 - 7x}{x - 6}
$$

Step 3: Find the Domain of G(F(x))

The domain of G(F(x)) depends on:

- The domain of F(x): where F(x) is defined.
- The domain restrictions imposed by G(F(x)).

Key points:

- F(x) is undefined when the denominator is zero:

$$
x - 6 \neq 0 \Rightarrow x \neq 6
$$

- G(F(x)) involves the denominator $x - 6$. To ensure G(F(x)) is defined, the original F(x) must be defined, and the denominator in G(F(x)) must not be zero. But since G(F(x)) simplifies to $\frac{43 - 7x}{x - 6}$, the denominator is $x - 6$, which cannot be zero.

Therefore, the domain of G(F(x)) is:

$$
\boxed{
\text{Domain of } G(F(x)) = \{ x \in \mathbb{R} \mid x \neq 6 \}
}
$$

In words:

- All real numbers except $x = 6$, because at $x=6$, F(x) is undefined, which in turn makes G(F(x)) undefined.

Summary of Results

Part	Function	Expression	Domain Conditions
a	$(fg)(x) = F(g(x))$	$\frac{1}{x - 13}$	$x \neq 13$ (since denominator zero at $x=13$)

b	$(fog)(x) = G(F(x))$	$\frac{43 - 7x}{x - 6}$	$x \neq 6$

Understanding the Significance of Domain Restrictions

In function composition problems, domain restrictions are vital because they indicate where the functions are defined and meaningful. Ignoring these restrictions can lead to incorrect conclusions or undefined expressions.

- For (fg)(x): The denominator $x - 13$ must not be zero, so $x \neq 13$.
- For (fog)(x): The original function F(x) is undefined at $x=6$, thus the composition G(F(x)) is also undefined at this point.

Practical Tips for Solving Similar Problems

When working with function compositions and domains, consider the following steps:

1. Identify the functions involved: Write down F(x) and G(x) explicitly.
2. Determine the composite function:
- For (f g)(x): substitute G(x) into F.
- For (g f)(x): substitute F(x) into G.
3. Simplify the resulting expression: Combine fractions, cancel common factors if possible.
4. Analyze domain restrictions:
- Find where each original function is undefined.
- Examine the composite expression for points where the denominator becomes zero.
- Exclude these points from the domain.
5. Express the domain in set notation: Clearly specify the real numbers that are valid inputs.

Conclusion: Mastering Function Composition and Domains

Understanding how to compose functions and determine their domains is a critical skill in algebra and calculus. The problem involving F(x) = 1/(x - 6) and G(x) = x - 7 exemplifies these concepts well. By carefully substituting, simplifying, and analyzing restrictions, you can accurately find composite functions and their domains. Remember, paying attention to domain restrictions not only ensures mathematical correctness but also deepens your understanding of how functions behave and interact.

Whether tackling rational functions, linear functions, or more complex compositions, these foundational principles serve as essential tools for success in mathematics. Practice with various functions and compositions to build confidence and proficiency in these vital areas.

Frequently Asked Questions

What is the expression for (fg)(x) if F(x) = 1/(x-6) and G(x) = x - 7?

To find (fg)(x), we substitute G(x) into F(x): (fg)(x) = F(G(x)) = 1 / (G(x) - 6) = 1 / ((x - 7) - 6) = 1 / (x - 13).

How do you find the domain of (fg)(x) given F(x) = 1/(x-6) and G(x) = x - 7?

The domain of (fg)(x) depends on the domain of G(x) and the restrictions from F. Since F(x) is undefined at x=6, and (fg)(x) = 1/(x-13), it is undefined at x=13. G(x) is defined for all real x, so the domain of (fg)(x) is all real numbers except x=13.

What is the composite function (fg)(x) based on the given functions?

The composite function is (fg)(x) = 1 / (x - 13).

What are the key considerations when determining the domain of a composite function like (fg)(x)?

You need to ensure that G(x) is within the domain of F, and also that the resulting expression from substitution doesn't cause division by zero or other undefined operations. Specifically, exclude values where the denominator becomes zero.

Are there any restrictions on the domain of G(x) = x - 7 in this problem?

No, G(x) = x - 7 is defined for all real numbers, so it has no restrictions on its domain.

Why is x=13 excluded from the domain of (fg)(x)?

Because at x=13, the denominator in (fg)(x) = 1 / (x - 13) becomes zero, making the function undefined.

How does understanding the composition of functions help in real-world applications?

Understanding function composition helps in modeling complex relationships where the output of one process serves as the input for another, such as in engineering, economics, and data analysis, ensuring all parts of the model are valid and well-defined.

Additional Resources

Understanding Composition of Functions: A Comprehensive Guide to (f∘g)(x) and Its Domain

When working with functions in algebra, especially in the context of calculus or advanced mathematics, understanding how to compose functions is essential. The composition of functions — denoted as (f∘g)(x) or "f composed with g" — involves applying one function to the result of another. This concept is fundamental in various applications, from solving equations to modeling real-world phenomena. In this guide, we'll explore a specific example involving the functions F(x) and G(x), and walk through the process of finding their composition, as well as determining the domain of the resulting composite function.

Introduction to the Problem

Given the functions:

- F(x) = 1 / (x - 6)
- G(x) = x - 7

we are asked to:

1. Find (f∘g)(x), which means applying F to G(x), i.e., F(G(x)).
2. Determine the domain of (f∘g)(x).

Before diving into calculations, it's important to understand what these functions represent and how their composition works.

Understanding Basic Functions

The Functions at a Glance

- F(x) = 1 / (x - 6): This is a rational function with a vertical asymptote at x = 6, where the denominator becomes zero, and the function is undefined.
- G(x) = x - 7: This is a linear function, with no restrictions other than the domain of all real numbers.

Key Concepts

- Function Composition: The process of plugging G(x) into F(x). It results in a new function, often written as (f∘g)(x) = F(G(x)).
- Domain Considerations: The domain of the composite function depends on the domains of F and G, as well as where the composition is defined (e.g., where the denominator isn't zero).

Step-by-Step Guide to Finding (f∘g)(x)

Step 1: Write the Composition

The composition $(f \circ g)(x)$ is:

$(f \circ g)(x) = F(G(x))$

Substituting $G(x)$ into $F(x)$:

$(f \circ g)(x) = F(x - 7) = 1 / [(x - 7) - 6]$

Step 2: Simplify the Expression

Simplify the denominator:

$(x - 7) - 6 = x - 7 - 6 = x - 13$

Therefore:

$(f \circ g)(x) = 1 / (x - 13)$

This is the explicit form of the composite function.

Domain of the Composite Function $(f \circ g)(x)$

Step 1: Find Restrictions from $G(x)$

$G(x) = x - 7$ is defined for all real numbers, so no restrictions arise here individually.

Step 2: Find Restrictions from $F(G(x))$

Recall that $F(x) = 1 / (x - 6)$ is undefined when its denominator is zero. When composing, the argument of F is $G(x)$:

- The argument $G(x)$ must not make the denominator zero:

$G(x) \neq 6$

- Since $G(x) = x - 7$, set:

$x - 7 \neq 6$

Solve for x:

$x \neq 6 + 7$

$x \neq 13$

Conclusion: The composite function $(f \circ g)(x) = 1 / (x - 13)$ is undefined at $x = 13$ because it makes the denominator zero.

Final Results

a) The composite function (f∘g)(x):

$$
\boxed{
(f \circ g)(x) = \frac{1}{x - 13}
}
$$

b) The domain of (f∘g)(x):

$$
\boxed{
\text{Domain} = \{ x \in \mathbb{R} \mid x \neq 13 \}
}
$$

In words, the domain includes all real numbers except $x = 13$.

Additional Insights and Tips

1. Always Check for Domain Restrictions

When composing functions, pay close attention to the internal functions' restrictions. The points where the inner function G(x) hits values that make the outer function undefined must be excluded from the domain.

2. Simplify Carefully

Start by substituting G(x) into F(x), then simplify the expression to identify potential points of discontinuity or undefined behavior.

3. Visualize the Graphs

Plotting both functions can help understand how their composition behaves and where the function is undefined.

4. Practice with Variations

Try other pairs of functions, such as polynomial, rational, exponential, or logarithmic, to strengthen your understanding of composition and domain restrictions.

Summary

- The composition of functions involves substituting one function into another.
- For $F(x) = 1 / (x - 6)$ and $G(x) = x - 7$, the composition (f∘g)(x) simplifies to $1 / (x - 13)$.

- The domain of the composite function excludes any x-value that causes the denominator to be zero, which in this case is $x = 13$.
- Always analyze the inner and outer functions for restrictions to accurately determine the domain of the composite.

Mastering the art of function composition and domain analysis is vital for tackling more complex mathematical problems. Keep practicing these steps, and you'll develop a stronger intuition for how functions interact and where potential issues arise.

2 Given That F X 1 X 6 And G X X 7 A Find Fg X B Find The Domain Of Fog X

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