4) A Line Has A Slope Of -5 And Passes Through The Point (2, 2). An Equation For This Line Is

4) A Line Has A Slope Of -5 And Passes Through The Point (2, 2). An Equation For This Line Is is a fundamental concept in coordinate geometry that involves understanding the relationship between a line's slope, a given point, and how to derive the line's equation. Whether you are a student learning about linear equations for the first time or someone brushing up on algebraic concepts, grasping how to find the equation of a line given specific information is essential. In this article, we will explore the process step-by-step, discuss related concepts, and provide practical examples to deepen your understanding of line equations.

Understanding the Slope-Point Form of a Line

Before deriving the equation, it's important to understand the core components involved.

What Is the Slope of a Line?

The slope of a line, often denoted by the letter m, measures the steepness or incline of the line. It is calculated as the ratio of the change in y-coordinates to the change in x-coordinates between two points on the line:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

A negative slope, such as -5, indicates that as x increases, y decreases.

What Does Passing Through a Specific Point Mean?

When a line passes through a particular point, such as (2, 2), it means that the coordinates (2, 2) satisfy the line's equation. Essentially, plugging in x = 2 and y = 2 into the equation should make it true.

Deriving the Equation of the Line

Given the slope and a point, the most straightforward way to find the line's equation is to use the point-slope form.

Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

where (x_1, y_1) is a point on the line, and m is the slope.

Applying the Given Data

For our specific case:
- Slope $m = -5$
- Point $(x_1, y_1) = (2, 2)$

Plugging these into the point-slope form:

$$
y - 2 = -5(x - 2)
$$

Converting to Slope-Intercept Form

To express the line in the more familiar slope-intercept form $y = mx + b$:

$$
y - 2 = -5x + 10
$$
$$
y = -5x + 10 + 2
$$
$$
y = -5x + 12
$$

Thus, the equation of the line passing through (2, 2) with a slope of -5 is:

$$
\boxed{y = -5x + 12}
$$

This is the explicit form that makes it easy to graph and analyze the line.

Key Concepts and Variations in Line Equations

Understanding the different forms of a line's equation enriches your comprehension of linear relationships.

Slope-Intercept Form

As shown above, $y = mx + b$ directly indicates the slope m and the y-intercept b. This form is especially useful for graphing.

Standard Form

Another common form is:

$$
Ax + By = C
$$

To convert our equation $y = -5x + 12$ into standard form:

$$
5x + y = 12
$$

This form is often useful in solving systems of equations.

Point-Slope Form

Reiterating, the point-slope form is:

$$
y - y_1 = m(x - x_1)
$$

which can be used to find the line’s equation given any point and slope.

Practical Applications of Line Equations

Understanding how to derive and interpret line equations has numerous real-world applications:

- **Physics:** Describing motion with constant velocity, where the position changes linearly over time.
- **Economics:** Modeling cost functions or revenue with linear relationships.
- **Engineering:** Analyzing structural slopes or material properties.
- **Data Analysis:** Fitting a linear regression line to data points for predictions.

In all these applications, knowing how to formulate the line equation from given data points and slopes is invaluable.

Practice Problems for Mastery

To solidify your understanding, here are some practice problems:

1. Find the equation of a line with a slope of 3 passing through the point (4, -1).
2. Given the line $y = 2x + 5$, find a point on the line where $x = -3$.
3. Convert the equation $4x - 3y = 6$ into slope-intercept form.
4. Determine the equation of a line passing through (1, 4) with a slope of -2.
5. Explain how the sign of the slope affects the graph of the line.

Solutions:

1. Using point-slope form: $y - (-1) = 3(x - 4) \Rightarrow y + 1 = 3x - 12 \Rightarrow y = 3x - 13$.

2. Plug in $x = -3$: $y = 2(-3) + 5 = -6 + 5 = -1$. The point is $(-3, -1)$.

3. Rearranged: $4x - 3y = 6 \Rightarrow -3y = -4x + 6 \Rightarrow y = \frac{4}{3}x - 2$.

4. Using point-slope form: $y - 4 = -2(x - 1) \Rightarrow y - 4 = -2x + 2 \Rightarrow y = -2x + 6$.

5. A negative slope (like -5) indicates the line falls as x increases, whereas a positive slope indicates an incline.

Conclusion

The equation of a line can be efficiently determined when given a point and a slope, using the point-slope form, then converting to the slope-intercept or standard forms as needed. In our case, with a slope of -5 passing through (2, 2), the equation $y = -5x + 12$ succinctly captures the line's behavior. Mastering these techniques allows you to analyze and graph linear relationships confidently, which can be applied in numerous academic and practical contexts.

Remember, the key steps involve identifying the knowns, applying the correct form, and simplifying to the desired equation. With practice, deriving line equations becomes an intuitive and valuable skill in your mathematical toolkit.

Frequently Asked Questions

What is the equation of a line with a slope of -5 passing through the point (2, 2)?

Using point-slope form: $y - 2 = -5(x - 2)$. Simplifying, the equation is $y = -5x + 12$.

How do you find the equation of a line given a slope and a point?

Use the point-slope form $y - y_1 = m(x - x_1)$, substituting the slope m and the point (x_1, y_1).

What is the slope-intercept form of the line passing through (2, 2) with slope -5?

The slope-intercept form is $y = -5x + 12$.

If a line has a slope of -5 and passes through (2, 2), what is its y-intercept?

The y-intercept is 12, as seen in the equation $y = -5x + 12$.

How can I verify that the point (2, 2) lies on the line with equation y = -5x + 12?

Substitute $x = 2$ into the equation: $y = -5(2) + 12 = -10 + 12 = 2$. Since y equals 2, the point (2, 2) lies on the line.

Additional Resources

Understanding the Equation of a Line: Slope of -5 Passing Through (2, 2)

When exploring the fascinating world of coordinate geometry, one of the fundamental concepts that often captures the curiosity of students and enthusiasts alike is the equation of a straight line. It’s not just about plotting points; it’s about understanding the relationship between the slope, a point on the line, and how these elements come together to define an infinite set of points along a path. In this article, we delve deep into the problem: "A line has a slope of -5 and passes through the point (2, 2). An equation for this line is..." offering an expert’s perspective, detailed explanations, and practical insights.

What Is a Line’s Slope and Why Is It Important?

Before jumping into the specific problem, it’s crucial to understand what the slope of a line signifies and why it is a core component in the equation of a line.

Defining Slope

The slope of a line, often denoted by m, measures its steepness and the direction it inclines or declines as it moves along the x-axis. It is mathematically expressed as:

$$
m = \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x}
$$

This ratio indicates how much y (the vertical component) changes for a unit change in x (the horizontal component). For example:

- A slope of 0 indicates a horizontal line.
- A positive slope indicates the line rises as x increases.
- A negative slope indicates the line falls as x increases.

In our specific case, the slope is -5, which suggests a steep descent as x increases.

The Significance of the Slope

Understanding the slope allows us to grasp the line’s behavior visually and mathematically. It helps answer questions such as:

- How steep is the line?
- In which direction does it go?
- How does the line relate to specific points in the plane?

Moreover, the slope is integral to constructing the equation of the line, especially in the slope-intercept form discussed next.

The Slope-Intercept Form of a Line

The most common and intuitive form to express a line’s equation when its slope and a point on the line are known is the slope-intercept form:

$$
y = mx + b

\]

Where:

- m: the slope of the line
- b: the y-intercept, i.e., the point where the line crosses the y-axis

In our problem, we know m = -5 and a point on the line: (2, 2). Our task is to determine b and write the complete equation.

Deriving the Equation Step-by-Step

The process of finding the equation involves substituting the known point into the slope-intercept form and solving for b.

Step 1: Write the general form

\[
y = -5x + b
\]

Step 2: Plug in the known point (2, 2)

Since the point passes through (2, 2), substitute x = 2 and y = 2 into the equation:

\[
2 = -5(2) + b
\]

Simplify:

\[
2 = -10 + b
\]

Step 3: Solve for \(b\)

Add 10 to both sides:

\[
b = 2 + 10 = 12

\]

Step 4: Write the final equation

Substitute b = 12 back into the slope-intercept form:

\[
\boxed{
\textbf{y = -5x + 12}
}
\]

This is the equation of the line with a slope of -5 passing through (2, 2).

Alternative Forms and Interpretations

While the slope-intercept form is the most straightforward, understanding other forms and their advantages provides a more comprehensive grasp.

Point-Slope Form

Given a point \((x_1, y_1)\) and slope \(m\), the point-slope form is:

\[
y - y_1 = m(x - x_1)
\]

Applying our point (2, 2) and slope -5:

\[
y - 2 = -5(x - 2)
\]

This form is especially useful for quickly writing the line’s equation when a point and slope are known, without immediately solving for b.

Standard Form

Rearranging the slope-intercept form into standard form:

\[

$$
Ax + By = C
$$

Starting from $y = -5x + 12$:

$$
5x + y = 12
$$

This form is beneficial for certain algebraic operations and for understanding the geometric relationship between x and y.

Visualizing the Line

Understanding the equation geometrically enhances comprehension. Here’s what the line $y = -5x + 12$ looks like:

- Slope (-5): For every increase of 1 in x, y decreases by 5.
- Y-intercept (12): The line crosses the y-axis at (0, 12).

Plotting a couple of points:

- At $x = 0$, $y = 12$ (y-intercept)
- At $x = 2$, $y = 2$ (given point)
- At $x = 4$, $y = -8$ (since $y = -5(4) + 12 = -20 + 12 = -8$)

Connecting these points yields a straight line slanting downward steeply.

Practical Applications and Significance

Understanding how to derive the equation of a line with a known slope and passing through a given point isn’t just an academic exercise; it’s a foundational skill with practical applications:

- Engineering and Physics: Calculating trajectories, slopes of roads, or rates of change.
- Economics: Modeling supply and demand curves.
- Computer Graphics: Rendering lines and shapes based on equations.
- Data Analysis: Fitting lines to data points (regression).

This process exemplifies how theoretical knowledge translates into real-world problem-solving.

Summary and Key Takeaways

- The slope of a line indicates its steepness and direction; here, -5 denotes a steep decline.
- The point (2, 2) serves as a specific reference, anchoring the line’s position.
- The derivation involves substituting the known point into the slope-intercept form and solving for the y-intercept \(b\).
- The final equation: y = -5x + 12.
- Alternative forms (point-slope and standard) offer flexibility in different contexts.
- Visualizing the line enhances understanding and aids in practical application.

Conclusion

In conclusion, the equation of a line with a slope of -5 passing through the point (2, 2) is y = -5x + 12. This derivation exemplifies the fundamental approach in coordinate geometry: leveraging known data points and slopes to articulate the precise mathematical description of a line. Mastery of this process is essential for students and professionals alike, forming a critical building block in the broader landscape of mathematics, science, and engineering.

Whether you're plotting a graph, analyzing data, or solving a geometric problem, understanding how to derive and interpret linear equations ensures clarity, precision, and confidence in your mathematical endeavors.

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