6. Find The Intersection Of The Line 7 And The Plane . 1:=(4,-1,4)+t(5,-2,3) : 2x+5y+z+2=0 4

6. Find The Intersection Of The Line 7 And The Plane 1: = (4, -1, 4) + t(5, -2, 3); 2x + 5y + z + 2 = 0

In this comprehensive guide, we will explore the process of finding the intersection point between a parametric line and a plane in three-dimensional space. The specific problem involves a line defined parametrically as *L(t) = (4, -1, 4) + t(5, -2, 3)* and a plane described by the equation *2x + 5y + z + 2 = 0*. Our objective is to determine the value of the parameter *t* at which this line intersects the plane, and then find the corresponding point of intersection.

Understanding the Components of the Problem

The Parametric Equation of the Line

The line is given in parametric form as:

- Point on the line: *(4, -1, 4)*
- Direction vector: *(5, -2, 3)*

The parametric equations for the line can be written as:

```
x(t) = 4 + 5t
y(t) = -1 - 2t
z(t) = 4 + 3t
```

The Equation of the Plane

The plane is defined by:

```
2x + 5y + z + 2 = 0
```

This is a standard form of a plane equation, where the coefficients of *x*, *y*, and *z* are the components of the normal vector to the plane.

Methodology for Finding the Intersection

Step 1: Substitute the parametric equations into the plane equation

To find the intersection point, substitute *x(t)*, *y(t)*, and *z(t)* into the plane equation:

```
2(4 + 5t) + 5(-1 - 2t) + (4 + 3t) + 2 = 0
```

Step 2: Simplify the resulting equation

Distribute and combine like terms:

```
(8 + 10t) + (-5 - 10t) + (4 + 3t) + 2 = 0
```

Breaking it down:

1. Combine constants: 8 - 5 + 4 + 2 = 9
2. Combine the terms with *t*: 10t - 10t + 3t = 3t

Thus, the simplified equation becomes:

```
9 + 3t = 0
```

Step 3: Solve for *t*

Isolate *t*:

```
3t = -9
```

```
t = -3
```

Finding the Coordinates of the Intersection Point

Step 4: Substitute *t = -3* back into the parametric equations

Calculate each coordinate:

- **x-coordinate:** $x(-3) = 4 + 5(-3) = 4 - 15 = -11$
- **y-coordinate:** $y(-3) = -1 - 2(-3) = -1 + 6 = 5$
- **z-coordinate:** $z(-3) = 4 + 3(-3) = 4 - 9 = -5$

Step 5: Write the intersection point

The point where the line intersects the plane is:

```
(-11, 5, -5)
```

Verification of the Intersection Point

Step 6: Verify by substituting back into the plane equation

Substitute $x = -11$, $y = 5$, and $z = -5$ into the plane equation to verify:

```
2(-11) + 5(5) + (-5) + 2 = ?
```

Calculate step-by-step:

- 2(-11) = -22
- 5(5) = 25
- Adding all: -22 + 25 - 5 + 2 = (-22 + 25) + (-5 + 2) = 3 - 3 = 0

Since the sum is zero, the point lies on the plane, confirming the correctness of our intersection point.

Summary

In conclusion, the parametric line intersects the plane at the point *(-11, 5, -5)*. The process involved substituting the parametric equations into the plane equation, solving for the parameter t, and then calculating the corresponding coordinates. The verification step confirmed the accuracy of our solution, demonstrating a systematic approach to solving line-plane intersection problems in three-dimensional space.

Additional Considerations

Parallel Lines and Planes

If, during the process, the resulting equation in t had no solution, it would indicate that the line and the plane are parallel and do not intersect. Conversely, if the equation simplifies to an identity (e.g., 0 = 0), the line lies entirely within the plane.

Other Applications

- Finding intersections in computer graphics
- Collision detection in physics simulations
- Geometric modeling and CAD applications

Conclusion

Finding the intersection point between a line and a plane is a fundamental problem in vector geometry and analytical geometry. By converting the line into parametric equations, substituting into the plane equation, solving for the parameter, and finally calculating the intersection point, we can efficiently determine the exact point of intersection. This method is applicable across various scientific and engineering disciplines, highlighting its importance in spatial analysis and geometric computations.

Frequently Asked Questions

How do you find the intersection point of the line 1:=(4, -1, 4)+ t(5, -2, 3) with the plane 2x + 5y + z + 2 = 0?

To find the intersection, substitute the parametric equations of the line into the plane equation and solve for t. Then, plug t back into the line equations to get the point of intersection.

What is the parametric form of the line given in the problem?

The line is given as x=4+5t, y=-1-2t, z=4+3t, where t is a parameter.

How do you substitute the line equations into the plane equation?

Replace x, y, and z in the plane equation with their expressions from the line: 2(4+5t) + 5(-1-2t) + (4+3t) + 2 = 0.

What is the step to solve for t after substitution?

Simplify the resulting equation in t and solve for t to find the specific value where the line intersects the plane.

Can you demonstrate the calculation to find t?

Yes. Substituting gives: 2(4+5t) + 5(-1-2t) + (4+3t) + 2 = 0. Simplify to: 8 + 10t - 5 - 10t + 4 + 3t + 2 = 0. Combine like terms: (8 - 5 + 4 + 2) + (10t - 10t + 3t) = 0, which simplifies to 9 + 3t = 0. Solving for t gives t = -3.

What is the intersection point after finding t?

Plug t = -3 into the parametric equations: x=4+5(-3)=4-15=-11, y=-1-2(-3)=-1+6=5, z=4+3(-3)=4-9=-5. So, the intersection point is (-11, 5, -5).

Why is it important to verify the solution of t in the context of the intersection?

Verifying ensures that the computed t indeed satisfies both the line and plane equations and confirms the point of intersection is accurate.

What are common mistakes to avoid when finding the intersection of a line and a plane?

Common mistakes include incorrect substitution, algebraic errors when simplifying, forgetting to solve for t before plugging back in, or mixing up the signs during calculations.

Additional Resources

Find The Intersection Of The Line 7 And The Plane 1:=(4,-1,4)+t(5,-2,3) : 2x+5y+z+2=0 4 is a classic problem in analytic geometry that combines understanding of lines and planes in three-dimensional space. This problem is not only fundamental for students learning the principles of spatial relationships but also highly applicable in fields such as engineering, computer graphics, and physics. The core challenge involves determining the specific point where a given line intersects a plane, which requires a solid grasp of parametric equations, plane equations, and algebraic manipulation. This article provides a comprehensive overview of the steps involved, the concepts underpinning the problem, and the significance of mastering such intersection problems.

Understanding the Components of the Problem

Before delving into the solution, it's essential to understand the individual components involved: the line, the plane, and the concept of intersection.

The Line

The line is given in parametric form:

$$ \mathbf{r}(t) = (4, -1, 4) + t(5, -2, 3) $$

This means that every point on the line can be described as:

- $x = 4 + 5t$
- $y = -1 - 2t$
- $z = 4 + 3t$

Here, t is a real parameter that can take any value from $-\infty$ to ∞. The point $(4, -1, 4)$ is known as the point through which the line passes, and $(5, -2, 3)$ is the direction vector indicating the line’s orientation.

Features:
- Parametric form makes it straightforward to generate any point on the line.
- The direction vector indicates the slope or orientation in space.

Pros:
- Easy to substitute into plane equations.
- Useful for visualization and calculations involving intersection.

Cons:
- Requires understanding of parametric equations.
- Can be abstract if not familiar with vector notation.

The Plane

The plane is given by the equation:

$$ 2x + 5y + z + 2 = 0 $$

This is a standard form of a plane equation, where the coefficients $(2, 5, 1)$ serve as the normal vector to the plane, indicating its orientation in space.

Features:
- The normal vector $\mathbf{n} = (2, 5, 1)$ is perpendicular to the plane.
- The constant term $+2$ shifts the plane relative to the origin.

Pros:
- Straightforward to test whether a point lies on the plane by substitution.
- The normal vector provides geometric intuition about the plane’s orientation.

Cons:
- The equation doesn’t directly give the plane’s points; need to solve for specific points if necessary.
- Understanding the geometric significance of the coefficients may require familiarity with vectors.

Step-by-Step Solution Approach

The goal is to find the point $\mathbf{P} = (x, y, z)$ where the line intersects the plane. The general approach involves substituting the parametric coordinates of the line into the plane equation and solving for the parameter t.

Step 1: Write the parametric equations of the line

From the given line:

$$
\begin{cases}
x = 4 + 5t \\
y = -1 - 2t \\
z = 4 + 3t
\end{cases}
$$

Step 2: Substitute into the plane equation

Plugging into the plane equation $2x + 5y + z + 2 = 0$:

$$
2(4 + 5t) + 5(-1 - 2t) + (4 + 3t) + 2 = 0
$$

Expanding:

\[
8 + 10t - 5 - 10t + 4 + 3t + 2 = 0
\]

Combine like terms:

\[
(8 - 5 + 4 + 2) + (10t - 10t + 3t) = 0
\]

Simplify:

\[
9 + 3t = 0
\]

Step 3: Solve for \(t \)

\[
3t = -9
\]
\[
t = -3
\]

This value of \(t \) indicates the specific point on the line where it intersects the plane.

Step 4: Find the intersection point

Substitute \(t = -3 \) back into the parametric equations:

\[
x = 4 + 5(-3) = 4 - 15 = -11
\]
\[
y = -1 - 2(-3) = -1 + 6 = 5
\]
\[
z = 4 + 3(-3) = 4 - 9 = -5
\]

Result:

\[
\boxed{

\text{Intersection Point} = (-11, 5, -5)
}
\]

Geometric Interpretation and Significance

The computed point \((-11, 5, -5) \) is the unique point where the given line intersects the plane. Geometrically, this intersection signifies that the line passes through the plane at precisely this point, confirming the spatial relationship between the two geometric entities.

Visualizing the solution:

- The point lies on both the parametric line and the plane.
- The intersection can be viewed as the common solution to the system of equations defining the line and the plane.

Applications:

- In engineering, such calculations are fundamental when determining points of contact or intersection between components.
- In computer graphics, intersections are vital for rendering, ray tracing, and collision detection.
- In physics, they can represent points where particles or objects meet or cross paths.

Further Considerations and Variations

While this problem involves a straightforward substitution, several variations and more complex scenarios can arise.

Multiple Intersections

- For lines and planes, typically, there is either:
- One intersection point (as here),
- No intersection (parallel and distinct),
- Or infinitely many (if the line lies entirely within the plane).

Line Lying Entirely in the Plane

- When the parametric equations of the line satisfy the plane equation for all \(t \), the line is coplanar, meaning every point lies on both.

Skew Lines and Planes

- When lines are skewed or not intersecting, similar methods are used to determine the absence of intersection or to find the closest points.

Alternative Methods

- Vector Approach: Using dot products and cross products to analyze the orientation.
- Distance Formula: To verify whether a line and a plane are close or intersecting.
- Matrix Methods: Solving systems of equations via matrices for more complex geometries.

Pros and Cons of the Methodology

Pros:

- Straightforward substitution makes the problem accessible.
- Clear geometric interpretation aids understanding.
- Applicable to a wide range of problems involving lines and planes.

Cons:

- Assumes familiarity with parametric equations and vector algebra.
- Not always suitable for more complex or higher-dimensional problems.
- Can be algebraically intensive if equations are complicated.

Conclusion

Finding the intersection point of a line and a plane in three-dimensional space is a foundational skill in analytic geometry. The process involves expressing the line parametrically, substituting into the plane's equation, solving for the parameter, and then deriving the intersection point. In this particular problem, the line defined by \((4, -1, 4) + t(5, -2, 3) \) intersects the plane \(2x + 5y + z + 2 = 0 \) at \((-11, 5, -5) \). Mastering this technique enhances spatial reasoning and provides a basis for tackling more advanced geometric problems across various scientific and engineering disciplines. Whether for designing mechanical components, creating computer graphics, or studying physical phenomena, understanding line-plane intersections is an invaluable mathematical tool.

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