

8. The Sum Of The Squares Of Two Consecutive Positive Even Numbers Is 52. Find The Numbers.

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Understanding mathematical problems involving consecutive numbers and their properties is essential in developing problem-solving skills. One such intriguing problem asks: If the sum of the squares of two consecutive positive even numbers equals 52, what are these numbers? This question not only tests your algebraic abilities but also enhances your understanding of number patterns and equations. In this comprehensive guide, we will explore the step-by-step process to find these numbers, delve into related concepts, and provide tips to solve similar problems efficiently.

Introduction to Consecutive Even Numbers

Before diving into the problem-solving process, it's crucial to understand what consecutive even numbers are and how they behave.

What Are Consecutive Even Numbers?

- Definition: Consecutive even numbers are even integers that follow each other in sequence, with a difference of 2.
- Examples:
 - 2 and 4
 - 14 and 16
 - 100 and 102

Key Characteristics:

- They are symmetric around an average number.
- The difference between two consecutive even numbers is always 2.
- If the first even number is x , then the next consecutive even number is $x + 2$.

Understanding the Problem Statement

The problem states:

"The sum of the squares of two consecutive positive even numbers is 52. Find the numbers."

Let's break it down:

- The numbers are positive and even.
- The two numbers are consecutive, meaning if one is x , the other is $x + 2$.
- The sum of their squares is 52.

Our goal is to find the specific numbers that satisfy these conditions.

Setting Up the Mathematical Equation

To find the numbers, we need to translate the problem into an algebraic equation.

Step 1: Assign Variables

Let:

- The first even number be x (where $x > 0$ and x is even).
- The next consecutive even number be $x + 2$.

Step 2: Write the Equation Based on the Problem

The sum of the squares:

$$\begin{aligned} &\backslash[\\ &x^2 + (x + 2)^2 = 52 \\ &\backslash] \end{aligned}$$

This equation captures the essence of the problem.

Solving the Equation Step-by-Step

Now, we will proceed to solve the quadratic equation to find the value(s) of x .

Step 1: Expand the Equation

$$x^2 + (x^2 + 4x + 4) = 52$$

Combine like terms:

$$x^2 + x^2 + 4x + 4 = 52$$

$$2x^2 + 4x + 4 = 52$$

Step 2: Simplify the Equation

Subtract 52 from both sides:

$$2x^2 + 4x + 4 - 52 = 0$$

$$2x^2 + 4x - 48 = 0$$

Divide the entire equation by 2 to simplify:

$$x^2 + 2x - 24 = 0$$

Step 3: Solve the Quadratic Equation

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Here, $(a = 1)$, $(b = 2)$, $(c = -24)$:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-24)}}{2(1)}$$

Calculate discriminant:

$$\sqrt{4 + 96} = \sqrt{100} = 10$$

Find the roots:

$$x = \frac{-2 \pm 10}{2}$$

- For the positive root:

$$x = \frac{-2 + 10}{2} = \frac{8}{2} = 4$$

- For the negative root:

$$x = \frac{-2 - 10}{2} = \frac{-12}{2} = -6$$

Since we're looking for positive even numbers, we discard $x = -6$.

Identifying the Numbers

With the value of $x = 4$, find the second number:

$$x + 2 = 4 + 2 = 6$$

Therefore, the two consecutive positive even numbers are 4 and 6.

Verification of the Solution

Let's verify whether these numbers satisfy the original problem:

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\[
4^2 + 6^2 = 16 + 36 = 52
\]
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Yes, the sum of their squares is indeed 52, confirming our solution.

Additional Insights and Related Problems

Understanding the solution process opens the door to solving similar problems involving sequences, algebraic equations, and number properties.

Common Variations of Similar Problems

- Find two consecutive odd numbers where the sum of their squares equals a specific number.
- Determine two numbers with a certain difference whose squares sum to a given value.
- Solve for numbers in geometric sequences where the sums of powers meet specific criteria.

Tips for Solving Similar Problems

- Always assign variables carefully, considering the constraints (positive, even/odd, consecutive).
- Translate words into algebraic equations precisely.
- Simplify equations step-by-step, checking for common factors.
- Use the quadratic formula when equations are quadratic in form.
- Verify your solutions by substituting back into the original problem.

Summary and Conclusion

In this article, we've explored how to approach and solve the problem:

"The sum of the squares of two consecutive positive even numbers is 52. Find

the numbers."

By assigning variables, setting up the algebraic equation, and solving systematically, we identified the numbers as 4 and 6. This example highlights the importance of translating word problems into algebraic expressions and solving them methodically.

Key Takeaways:

- Consecutive even numbers differ by 2.
- Translating word problems into equations is crucial.
- Quadratic equations often arise in such problems and can be solved using the quadratic formula.
- Always verify your solutions to confirm correctness.

Practice Questions to Enhance Your Skills

1. Find two consecutive odd numbers such that the sum of their squares is 130.
2. The sum of the squares of two positive integers is 85. If the larger number exceeds the smaller by 4, find the numbers.
3. Two numbers are consecutive even numbers. Their squares add up to 100. Find the numbers.

Solving these problems will strengthen your understanding of algebra and number properties.

Final Thoughts

Mathematics is a subject of patterns, logical reasoning, and problem-solving. Problems like the one discussed serve as excellent exercises to develop critical thinking skills. Remember, the key to solving such problems is patience, systematic approach, and verification.

By mastering these techniques, you'll be well-equipped to tackle a wide array of algebraic problems involving sequences, powers, and number properties. Keep practicing, and you'll find these problems becoming intuitive over time!

Frequently Asked Questions

What is the main problem asking us to find in the given statement?

It asks us to find two consecutive positive even numbers whose squares sum up to 52.

How should we represent the two consecutive positive even numbers mathematically?

Let the first even number be x ; then the next consecutive even number is $x + 2$.

What is the key equation derived from the problem statement?

The sum of their squares is 52, so the equation is $x^2 + (x + 2)^2 = 52$.

How do we simplify the equation $x^2 + (x + 2)^2 = 52$?

Expand it to $x^2 + x^2 + 4x + 4 = 52$, which simplifies to $2x^2 + 4x + 4 = 52$.

What steps are involved in solving the quadratic equation for x ?

Subtract 52 from both sides to get $2x^2 + 4x - 48 = 0$, then divide through by 2 to simplify to $x^2 + 2x - 24 = 0$, and solve using factoring or quadratic formula.

What are the solutions for x after solving the quadratic equation?

Factoring $x^2 + 2x - 24 = 0$ gives $(x + 6)(x - 4) = 0$, so $x = -6$ or $x = 4$.

Which solution for x is valid based on the problem's conditions?

Since we're looking for positive even numbers, $x = 4$ is valid; $x = -6$ is invalid because it is negative.

What are the two consecutive positive even numbers found from $x = 4$?

The numbers are 4 and 6.

How do we verify that these numbers satisfy the original problem?

Calculate $4^2 + 6^2 = 16 + 36 = 52$; since this matches the given sum, the numbers are correct.

Additional Resources

Mathematics Problem-Solving

Introduction: Unraveling the Puzzle of Consecutive Even Numbers

Mathematics, often regarded as the language of the universe, presents us with intriguing problems that challenge our reasoning and analytical skills. One such problem involves discovering two consecutive positive even numbers based on a given condition about their squares. The problem statement reads:

“The sum of the squares of two consecutive positive even numbers is 52. Find the numbers.”

This seemingly straightforward question invites us into a deeper exploration of algebraic expressions, equations, and number properties. Whether you are a student brushing up on algebra or a math enthusiast keen on sharpening your problem-solving skills, this problem offers a rich opportunity to understand the mechanics behind number relationships and algebraic modeling.

In this article, we delve into the problem comprehensively, explaining each step with clarity and providing insights into the underlying concepts. As we proceed, we'll explore how to model the problem mathematically, solve the resulting equations, and interpret the solution within the context of the problem.

Understanding the Problem: Key Elements and Terminology

Before jumping into calculations, it's essential to comprehend the problem's core components:

- Consecutive positive even numbers: These are numbers that follow one after the other in the sequence of even integers (2, 4, 6, 8, ...). The term consecutive indicates they are directly next to each other, with no gaps.
- Sum of their squares: The operation involves squaring each number and then adding these squares together, resulting in a total of 52.
- Find the numbers: The goal is to identify these two specific even numbers satisfying the above condition.

This problem combines number properties with algebraic modeling, making it a classic example of how real-world problems can be translated into mathematical equations.

Modeling the Problem with Algebra

The first step in solving such a problem involves translating the words into algebraic expressions:

Defining Variables

Let's denote the smaller of the two numbers as x . Since the numbers are consecutive even numbers, the next number will be $x + 2$ because:

- Even numbers differ by 2 (e.g., 2 and 4, 4 and 6, etc.)
- The sequence proceeds by adding 2 to the previous term.

Formulating the Equation

The problem states that the sum of the squares of these numbers is 52. Therefore, the algebraic equation becomes:

$$x^2 + (x + 2)^2 = 52$$

This equation captures the entire problem: the sum of the squares of the two consecutive even numbers equals 52.

Expanding and Simplifying the Equation

To solve the equation, we need to expand and simplify it:

$$\begin{aligned} & \backslash[\\ & x^2 + (x + 2)^2 = 52 \\ & \backslash] \end{aligned}$$

Expanding $\backslash((x + 2)^2 \backslash)$:

$$\begin{aligned} & \backslash[\\ & x^2 + (x^2 + 4x + 4) = 52 \\ & \backslash] \end{aligned}$$

Combine like terms:

$$\begin{aligned} & \backslash[\\ & x^2 + x^2 + 4x + 4 = 52 \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & 2x^2 + 4x + 4 = 52 \\ & \backslash] \end{aligned}$$

Subtract 52 from both sides to bring all terms to one side:

$$\begin{aligned} & \backslash[\\ & 2x^2 + 4x + 4 - 52 = 0 \\ & \backslash] \end{aligned}$$

Simplify:

$$\begin{aligned} & \backslash[\\ & 2x^2 + 4x - 48 = 0 \\ & \backslash] \end{aligned}$$

Divide the entire equation by 2 to simplify:

$$\begin{aligned} & \backslash[\\ & x^2 + 2x - 24 = 0 \\ & \backslash] \end{aligned}$$

Now, we have a quadratic equation:

$$\begin{aligned} & \backslash[\\ & x^2 + 2x - 24 = 0 \\ & \backslash] \end{aligned}$$

Solving the Quadratic Equation

The quadratic equation $\backslash(x^2 + 2x - 24 = 0 \backslash)$ can be solved using various

methods: factoring, completing the square, or the quadratic formula. Here, the quadratic formula is most straightforward:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 1$, $b = 2$, and $c = -24$.

Calculating the Discriminant:

$$\Delta = b^2 - 4ac = (2)^2 - 4(1)(-24) = 4 + 96 = 100$$

Finding the Roots:

$$x = \frac{-2 \pm \sqrt{100}}{2} = \frac{-2 \pm 10}{2}$$

This yields two solutions:

- $x = \frac{-2 + 10}{2} = \frac{8}{2} = 4$
- $x = \frac{-2 - 10}{2} = \frac{-12}{2} = -6$

Since the problem specifies positive even numbers, we discard $x = -6$ and accept $x = 4$.

Verifying the Solution and Finding the Numbers

Having determined $x = 4$, the two numbers are:

- First number: 4
- Second number: $4 + 2 = 6$

Checking the solution:

Calculate the sum of their squares:

$$4^2 + 6^2 = 16 + 36 = 52$$

This confirms that the numbers satisfy the original condition.

Conclusion: The Two Consecutive Even Numbers

Based on the calculations and verification, the two positive consecutive even numbers are:

4 and 6

The problem, though simple at first glance, showcases the power of algebra in solving number-based puzzles. It emphasizes the importance of translating words into equations, simplifying, and applying the quadratic formula to find solutions.

Insights and Broader Applications

This problem is representative of a class of algebraic puzzles that involve:

- Modeling real-world or abstract problems with variables
- Recognizing number properties (e.g., evenness, consecutive numbers)
- Applying algebraic techniques (expansion, simplification, quadratic formula)
- Validating solutions within context (positive numbers, specific constraints)

Understanding this process enhances problem-solving skills across various domains, including engineering, computer science, and finance.

Additional Variations and Practice Problems

To deepen your understanding, consider exploring similar problems:

- Find two consecutive positive odd numbers such that the sum of their squares is 130.
- The sum of the squares of two consecutive positive integers (not necessarily even) is 85. Find the numbers.
- If the sum of the squares of two positive numbers is 50, and the larger number is 3 more than twice the smaller, find the numbers.

These variations challenge you to adapt the modeling process and reinforce the core algebraic skills.

Final Thoughts

Mathematics problems like this serve as excellent exercises in logical reasoning and algebraic manipulation. By carefully defining variables, translating words into equations, and systematically solving, we gain not only the solution but also a deeper appreciation for the structure and beauty of mathematics.

In summary, the two positive even numbers whose squares sum to 52 are 4 and 6, exemplifying how algebra transforms abstract words into concrete solutions—a fundamental skill for anyone seeking mastery over mathematical problem-solving.

Happy problem-solving!

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