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Understanding how to find tangent and normal lines to a function at a specific point is a fundamental concept in calculus. These lines provide valuable insights into the behavior of functions, such as the slope and the rate of change at a given point. In this article, we will explore the process of finding the equation of both the tangent line and the normal line for the function $G(x) = \ln(x e^{4x})$ at $x=1$. This involves calculating derivatives, simplifying expressions, and applying the point-slope form of a line.

Understanding the Function $G(x) = \ln(x e^{4x})$

Before diving into derivatives and line equations, it's crucial to understand the structure of the given function.

Breaking Down the Function

The function $G(x)$ is a natural logarithm of the product x and e^{4x}. Recall that the natural logarithm of a product can be expressed as the sum of the logarithms:

$$
G(x) = \ln(x e^{4x}) = \ln x + \ln e^{4x}
$$

Since $\ln e^{4x} = 4x$, the function simplifies to:

$$
G(x) = \ln x + 4x
$$

This simplified form makes it easier to compute derivatives and evaluate the function at specific points.

Calculating the Derivative $G'(x)$

The next step involves finding the derivative of $G(x)$, which is essential for determining the slope of the tangent line at $x=1$.

Derivative of $G(x) = \ln x + 4x$

Using basic derivative rules:

- The derivative of $\ln x$ is $\frac{1}{x}$
- The derivative of $4x$ is 4

Thus,

$$
G'(x) = \frac{1}{x} + 4
$$

This derivative function gives the slope of the tangent line at any point x.

Evaluate $G'(x)$ at $x=1$

Substitute $x=1$:

$$
G'(1) = \frac{1}{1} + 4 = 1 + 4 = 5
$$

So, the slope of the tangent line at $x=1$ is 5.

Finding the Point on the Graph at $x=1$

To write the equations of the tangent and normal lines, we need a specific point on the graph at $x=1$.

Calculate $G(1)$

Using the simplified form:

$$
G(1) = \ln 1 + 4 \times 1
$$

Since $\ln 1 = 0$,

$$
G(1) = 0 + 4 = 4
$$

Therefore, the point of tangency (and normal) is $(1, 4)$.

Equation of the Tangent Line at $x=1$

The tangent line at a point (x_0, y_0) with slope m is given by the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

Applying the Point-Slope Form

Using $(x_0, y_0) = (1, 4)$ and $m = 5$:

$$
y - 4 = 5 (x - 1)
$$

Simplify to slope-intercept form:

$$
y = 5x - 5 + 4
$$
$$
y = 5x - 1
$$

The equation of the tangent line at $x=1$ is:

$$
\boxed{ y = 5x - 1 }
$$

Equation of the Normal Line at $x=1$

The normal line to a curve at a point is perpendicular to the tangent line at that point. Therefore, its slope is the negative reciprocal of the tangent slope.

Calculating the Normal Line Slope

Since the tangent slope is 5, the normal line slope is:

$$
m_{\text{normal}} = -\frac{1}{5}
$$

Applying the Point-Slope Form for the Normal Line

Using $(1, 4)$:

$$
y - 4 = -\frac{1}{5}(x - 1)
$$

Expand and simplify:

$$
y - 4 = -\frac{1}{5}x + \frac{1}{5}
$$

$$
y = -\frac{1}{5}x + \frac{1}{5} + 4
$$

Convert 4 to a fraction with denominator 5:

$$
4 = \frac{20}{5}
$$

Add:

$$
y = -\frac{1}{5}x + \frac{1}{5} + \frac{20}{5} = -\frac{1}{5}x + \frac{21}{5}
$$

The equation of the normal line at $x=1$ is:

$$
\boxed{ y = -\frac{1}{5}x + \frac{21}{5} }
$$

Summary of Results

To summarize:

- The point of tangency at $x=1$ is $(1, 4)$.
- The slope of the tangent line at this point is 5.
- The tangent line equation: $y = 5x - 1$.
- The slope of the normal line: $-\frac{1}{5}$.
- The normal line equation: $y = -\frac{1}{5}x + \frac{21}{5}$.

Additional Insights and Applications

Understanding how to find tangent and normal lines is not just a theoretical exercise; it has practical applications in various fields such as physics, engineering, and economics. For example:

- Physics: The tangent line indicates the instantaneous rate of change, such as velocity at a specific point.
- Engineering: Normal lines are used in designing components that require perpendicularity.
- Economics: Marginal analysis often involves tangent lines to curves representing cost or revenue functions.

Moreover, mastering these techniques enhances problem-solving skills and prepares students for more advanced topics like optimization and differential equations.

Conclusion

Calculating the equations of tangent and normal lines at a specific point involves a systematic process: simplifying the function, differentiating to find the slope, evaluating at the point of interest, and then applying the point-slope form. In the case of $G(x) = \ln(x e^{4x})$ at $x=1$, we determined the tangent line is $y = 5x - 1$, and the normal line is $y = -\frac{1}{5}x + \frac{21}{5}$. Mastery of these concepts is essential for understanding the geometric interpretation of derivatives and the behavior of functions.

Frequently Asked Questions

What is the function G(x) given in the problem?

$G(x)$ is given as $G(x) = \ln(xe^{4x})$.

How do we simplify the function G(x) = ln(xe^{4x})?

Using logarithm properties, G(x) = ln(x) + ln(e^{4x}) = ln(x) + 4x.

What is the first step to find the equation of the tangent line at x=1?

Calculate G(1) and find G'(x), then evaluate G'(1) for the slope.

How do you find the derivative G'(x) of G(x) = ln(x) + 4x?

G'(x) = (1/x) + 4.

What is the value of G(1)?

G(1) = ln(1) + 4(1) = 0 + 4 = 4.

What is the slope of the tangent line at x=1?

G'(1) = 1/1 + 4 = 1 + 4 = 5.

What is the equation of the tangent line at x=1?

Using point-slope form: y - 4 = 5(x - 1), so y = 5x - 1.

How do we find the equation of the normal line at x=1?

The normal line slope is the negative reciprocal of the tangent slope: -1/5. Using point (1,4): y - 4 = -1/5 (x - 1).

What is the equation of the normal line at x=1?

Simplifying: y - 4 = -1/5 x + 1/5, so y = -1/5 x + 21/5.

Why is understanding the derivative important for finding tangent and normal lines?

Because the derivative provides the slope of the tangent line at a point, which is essential for writing the equations of both the tangent and the normal lines.

Additional Resources

Given G(x) = ln(xe^{4x}) Find The Equation Of The Tangent Line And The Normal Line At x=1

Introduction

Mathematics often offers a rich landscape for exploration, especially when it comes to understanding the behavior of functions through their derivatives. One common problem involves finding the equations of tangent and normal lines to a given curve at a specific point. This task not only reinforces fundamental calculus concepts but also deepens our understanding of the function’s local behavior. In this detailed analysis, we will examine the function:

$$ G(x) = \ln(x e^{4x}) $$

and determine the equation of the tangent and normal lines at $x = 1$.

Understanding the Function $G(x) = \ln(x e^{4x})$

Before delving into derivatives and line equations, it is crucial to simplify and analyze the function:

$$
G(x) = \ln(x e^{4x})
$$

Using properties of logarithms, specifically the product rule:

$$
\ln(ab) = \ln a + \ln b
$$

we can rewrite $G(x)$ as:

$$
G(x) = \ln x + \ln e^{4x}
$$

Recall that:

$$
\ln e^{k} = k
$$

Therefore:

$$
G(x) = \ln x + 4x
$$

This simplified form makes differentiation more straightforward.

Step 1: Find $G'(x)$ — the derivative of $G(x)$

The derivative of $G(x)$ with respect to x involves differentiating each term:

- Derivative of $\ln x$ is $\frac{1}{x}$.
- Derivative of $4x$ is 4.

Thus,

$$
G'(x) = \frac{1}{x} + 4
$$

Step 2: Evaluate $G(x)$ and $G'(x)$ at $x=1$

Calculating $G(1)$:

$$
G(1) = \ln 1 + 4 \times 1 = 0 + 4 = 4
$$

Calculating $G'(1)$:

$$
G'(1) = \frac{1}{1} + 4 = 1 + 4 = 5
$$

The value $G(1) = 4$ gives the point $(1, 4)$ on the curve. The slope of the tangent line at $x=1$ is 5.

Step 3: Equation of the Tangent Line at $x=1$

The general equation of a line with point-slope form:

$$
y - y_0 = m (x - x_0)
$$

where:

- (x_0, y_0) is the point of tangency,
- m is the slope.

Plugging in our values:

$$
x_0 = 1, \quad y_0 = 4, \quad m = 5
$$

we obtain:

$$
\boxed{
\text{Tangent Line:} \quad y - 4 = 5(x - 1)
}
$$

which simplifies to:

$$
y = 5x - 1
$$

Step 4: Equation of the Normal Line at $x=1$

The normal line is perpendicular to the tangent line. Its slope m_{normal} is the negative reciprocal of the tangent slope:

$$
m_{\text{normal}} = - \frac{1}{m} = - \frac{1}{5}
$$

Using point-slope form again with the same point:

$$
x_0 = 1, \quad y_0 = 4
$$

we have:

$$
\boxed{
\text{Normal Line:} \quad y - 4 = - \frac{1}{5}(x - 1)
}
$$

which simplifies to:

$$
y = - \frac{1}{5}x + \frac{1}{5} + 4
$$
$$
y = - \frac{1}{5}x + \frac{1}{5} + \frac{20}{5} = - \frac{1}{5}x + \frac{21}{5}
$$

Summary of Results

| Line Type | Equation | Slope | Point of Tangency (at $x=1$) |

Tangent Line	$y = 5x - 1$	5	(1, 4)
Normal Line	$y = - \frac{1}{5}x + \frac{21}{5}$	-1/5	(1, 4)

Deep Dive: Additional Insights and Graphical Interpretation

Analyzing the Behavior of $G(x)$

The function $G(x) = \ln x + 4x$ combines a logarithmic component and a linear component. Its key features include:

- Domain: $x > 0$
- Increasing nature: Since $G'(x) = \frac{1}{x} + 4$, which is positive for $x > 0$, the function is monotonically increasing.
- Concavity: The second derivative:

$$
G''(x) = - \frac{1}{x^2}
$$

which is negative for all $x > 0$. This indicates the function is concave downward everywhere on its domain.

Geometric Interpretation at $x=1$

At $x=1$, the tangent line $y = 5x - 1$ touches the curve at $(1, 4)$. The slope of 5 indicates a steep increase at that point. The normal line, with slope $-1/5$, is relatively flat and intersects the curve perpendicularly at the same point.

Practical Applications and Context

Understanding the tangent and normal lines provides insights into the local behavior of the function:

- Tangent Line: Indicates the instantaneous rate of change at $x=1$. Useful in applications like physics (velocity at a point in time) or economics (marginal analysis).
- Normal Line: Offers a perpendicular reference, helpful in geometric constructions and optimization problems.

In advanced contexts, such analysis supports the study of curve concavity, inflection points, and optimization.

Concluding Remarks

The process of deriving tangent and normal lines to a function like $G(x) = \ln(x e^{4x})$ demonstrates the power of calculus in analyzing the local properties of functions. Simplifying the function was a critical step, transforming a complex logarithmic expression into a more manageable form. The derivative provided the slope of the tangent line, which, combined with the point of contact, led to the line equations. The normal line, being perpendicular to the tangent, shares the same point but has a reciprocal slope.

This comprehensive approach is fundamental in mathematical analysis, with broad applications across sciences and engineering. Mastery of these techniques enables a deeper understanding of the behavior of functions, their rates of change, and their geometric properties.

In summary:

- Tangent line at \(x=1 \): \(y = 5x - 1 \)
- Normal line at \(x=1 \): \(y = - \frac{1}{5}x + \frac{21}{5} \)

These lines encapsulate the local linear approximation of \(G(x) \) at \(x=1 \), offering valuable insights into the function’s immediate behavior.

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