# A Coin Is Tossed 12 Times. What Is The Probability Of Obtaining Five Heads And Seven Tails?

**A Coin Is Tossed 12 Times. What Is The Probability Of Obtaining Five Heads And Seven Tails?**

Coin tossing is a fundamental concept in probability theory and statistics, often used as an introductory example to explain randomness, probability calculations, and combinatorial analysis. When a fair coin is tossed multiple times, each outcome is independent, and the probability of specific arrangements can be determined using combinatorial formulas. In this article, we explore the specific scenario: tossing a coin 12 times and calculating the probability of getting exactly five heads and seven tails. We will break down the problem, introduce relevant concepts, and demonstrate step-by-step calculations to provide a comprehensive understanding.

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## Understanding the Basics of Coin Tossing and Probability

### Probability of a Single Coin Toss

- When tossing a fair coin, there are two equally likely outcomes:

  - Heads (H)
  - Tails (T)

- The probability of getting a head in a single toss: **P(H) = 1/2**
- The probability of getting a tail in a single toss: **P(T) = 1/2**

### Multiple Tosses and Independent Events

- Each coin toss is independent; the outcome of one toss does not affect the others.
- The total number of possible outcomes after multiple tosses is exponential:
**2^n**, where *n* is the number of tosses.

### Outcome Distribution in Multiple Tosses

- The number of ways to get a specific number of heads (or tails) follows the binomial distribution.
- For example, the probability of getting exactly *k* heads in *n* tosses is given by the binomial

probability formula:

$$
P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}
$$

where:
- $\binom{n}{k}$ is the binomial coefficient (number of ways to choose *k* successes out of *n* trials).
- $p$ is the probability of success in a single trial (here, success = heads).

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# Calculating the Probability of Exactly Five Heads and Seven Tails in 12 Tosses

## Applying the Binomial Distribution

Given:
- Number of tosses, $n = 12$
- Desired number of heads, $k = 5$
- Probability of getting heads in a single toss, $p = 1/2$

The probability of getting exactly 5 heads (and consequently 7 tails) is:

$$
P(\text{5 heads}) = \binom{12}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{7}
$$

Since the probabilities are identical and independent, this simplifies to:

$$
P(\text{5 heads}) = \binom{12}{5} \left(\frac{1}{2}\right)^{12}
$$

which can be written as:

$$
P(\text{5 heads}) = \binom{12}{5} \times \frac{1}{2^{12}}
$$

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## Calculating the Binomial Coefficient $\binom{12}{5}$

The binomial coefficient, $\binom{n}{k}$, is calculated as:

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!}
$$

where:
- $n!$ represents factorial of $n$,
- $k!$ is factorial of $k$,
- $(n-k)!$ is factorial of $n - k$.

Applying this to $\binom{12}{5}$:

$$
\binom{12}{5} = \frac{12!}{5!(12-5)!} = \frac{12!}{5! \times 7!}
$$

Calculating factorials:
- $12! = 479001600$
- $5! = 120$
- $7! = 5040$

So,

$$
\binom{12}{5} = \frac{479001600}{120 \times 5040} = \frac{479001600}{604800} = 792
$$

Thus,

$$
P(\text{5 heads}) = 792 \times \frac{1}{2^{12}}
$$

Since $2^{12} = 4096$,

$$
P(\text{5 heads}) = \frac{792}{4096}
$$

which simplifies to:

$$
P(\text{5 heads}) \approx 0.1934
$$

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# Interpreting the Result and Additional Considerations

## Probability of Exactly Five Heads and Seven Tails

- The probability of obtaining exactly five heads (and thus seven tails) in 12 independent tosses of a fair coin is approximately 19.34%.
- This indicates that while possible, this specific outcome is less common than some others (e.g., getting six heads and six tails).

## Symmetry and Other Outcomes

- Due to the symmetry of the binomial distribution with $p=0.5$, the probability of getting exactly 7 heads and 5 tails is identical.
- The total probability of getting any combination of outcomes with 5 or 7 heads can be computed by summing probabilities for different values.

## General Formula Recap

- For any number of coins, the probability of obtaining exactly $k$ heads in $n$ tosses with a fair coin is:

$$
P = \binom{n}{k} \times \left(\frac{1}{2}\right)^n
$$

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# Expanding the Analysis: Probability of At Least 5 Heads

While the initial question focuses on exactly 5 heads, sometimes it's useful to understand the probability of at least 5 heads, reflecting broader outcomes.

## Calculating the Probability of 5 or More Heads

- Sum the probabilities of getting 5, 6, 7, ..., 12 heads:

$$
P(\text{at least 5 heads}) = \sum_{k=5}^{12} \binom{12}{k} \left(\frac{1}{2}\right)^{12}
$$

- Using binomial coefficients, this sum can be computed directly or via statistical software.

## Complement Rule for Efficiency

- Alternatively, use the complement rule:

$$
P(\text{at least 5 heads}) = 1 - P(\text{fewer than 5 heads}) = 1 - \sum_{k=0}^{4} \binom{12}{k} \left(\frac{1}{2}\right)^{12}
$$

Calculations of these probabilities involve similar binomial coefficient calculations for $k=0, 1, 2, 3, 4$.

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# Practical Applications and Real-World Implications

## Statistical Significance and Coin Toss Experiments

- Understanding these probabilities helps in designing fair tests of coin fairness.
- If an observed outcome deviates significantly from expected probabilities, it could suggest bias in the coin.

## Games of Chance and Gambling

- Probabilities like these underpin strategies in games involving coin tosses or similar binary outcomes.
- Knowing the likelihood of specific outcomes informs risk assessment.

## Educational Tools and Teaching Probability

- Coin toss experiments serve as excellent pedagogical tools to teach concepts such as probability distributions, independence, and combinatorics.

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# Conclusion

Determining the probability of obtaining exactly five heads and seven tails in 12 coin tosses is a straightforward application of the binomial distribution. By calculating the binomial coefficient $\binom{12}{5}$ and applying the probability formula, we find that this specific outcome has roughly a 19.34% chance of occurring with a fair coin. This calculation highlights the usefulness of combinatorial mathematics in understanding randomness and probability in simple, real-world scenarios. Whether for academic purposes, game theory, or statistical analysis, mastering these fundamental concepts provides a solid foundation for exploring more complex probabilistic systems.

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## Additional Resources for Further Learning

- Books: "Introduction to Probability" by Joseph K. Blitzstein and Jessica Hwang
- Online Tools: Binomial calculator tools available on many educational websites
- Software: Statistical software like R, Python (with SciPy), or Excel can perform binomial probability calculations efficiently

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Meta Description: Discover how to calculate the probability of getting exactly five heads and seven tails when tossing a coin 12 times. Learn the binomial distribution, step-by-step calculations, and real-world applications in this comprehensive guide.

## Frequently Asked Questions

### What is the probability of getting exactly 5 heads and 7 tails when tossing a coin 12 times?

The probability is given by the binomial formula: $C(12, 5) (0.5)^5 (0.5)^7 = C(12, 5) (0.5)^{12} = 792 (0.5)^{12} \approx 0.193$.

### How do you calculate the probability of obtaining a specific number of heads in multiple coin tosses?

Use the binomial probability formula: $P = C(n, k) p^k (1-p)^{n-k}$, where n is the total tosses, k is the number of heads, and p is the probability of heads (0.5 for a fair coin).

### What is the significance of the binomial coefficient C(12, 5) in this probability calculation?

C(12, 5) represents the number of ways to choose 5 positions for heads out of 12 tosses, accounting for all arrangements that result in exactly five heads.

### Is the probability of getting 5 heads and 7 tails higher or lower than getting 6 heads and 6 tails in 12 tosses?

They are equal because the binomial coefficients C(12, 5) and C(12, 6) are symmetric, but C(12, 6) is larger, so the probability of 6 heads and 6 tails is higher.

### How does the probability change if the coin is biased instead

## of fair?

If the coin is biased with probability p of heads, the binomial formula adjusts to P = C(12, 5) p^5 (1-p)^7, which can increase or decrease the probability depending on p.

## Can the probability of obtaining exactly 5 heads and 7 tails be considered a rare event?

No, with a probability of approximately 19.3%, it is a relatively common outcome in 12 coin tosses.

## What is the total number of possible outcomes when tossing a coin 12 times?

There are 2^{12} = 4096 possible outcomes, since each toss has 2 possible results and there are 12 tosses.

# Additional Resources

A Coin Is Tossed 12 Times. What Is The Probability Of Obtaining Five Heads And Seven Tails? This question touches on fundamental concepts in probability theory and combinatorics, making it a fascinating problem for students, educators, and enthusiasts alike. The scenario involves a simple, fair coin being tossed multiple times, and calculating the likelihood of a particular outcome—specifically, getting five heads and seven tails in 12 tosses. Understanding this problem requires grasping the principles of binomial probability, basic combinatorics, and the underlying assumptions about the fairness of the coin. In this article, we will explore the problem in detail, dissect the mathematical principles involved, and discuss its broader implications in probability theory.

# Understanding the Problem

At its core, the problem asks: When a fair coin is tossed 12 times, what is the probability that exactly 5 of these tosses will result in heads, and the remaining 7 will be tails? To approach this, it’s essential to clarify a few assumptions and concepts:

- Fair Coin Assumption: The coin has an equal chance of landing on heads or tails, with each outcome having a probability of 0.5.
- Independence of Tosses: Each toss is independent; the outcome of one toss does not influence the next.
- Discrete Outcomes: Each toss results in either heads or tails, with no other possibilities.

Given these assumptions, the problem aligns with a binomial probability framework, which models the number of successes (in this case, heads) in a fixed number of independent Bernoulli trials.

# Mathematical Foundations

## Binomial Distribution Basics

The binomial distribution describes the probability of having exactly k successes in n independent Bernoulli trials, each with success probability p. Its probability mass function (PMF) is:

$$ P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k} $$

where:

- $\binom{n}{k}$ is the binomial coefficient, representing the number of ways to choose k successes from n trials.
- $p$ is the probability of success on a single trial.
- $k$ is the number of successes desired.

In this scenario:

- $n = 12$ (number of tosses)
- $k = 5$ (desired number of heads)
- $p = 0.5$ (probability of heads)

## Calculating the Probability

Applying the binomial formula:

$$ P(\text{5 heads in 12 tosses}) = \binom{12}{5} \times (0.5)^5 \times (0.5)^{7} $$

Since $(0.5)^5 \times (0.5)^7 = (0.5)^{12}$, the expression simplifies to:

$$ P = \binom{12}{5} \times (0.5)^{12} $$

Calculating $\binom{12}{5}$:

$$
\binom{12}{5} = \frac{12!}{5! \times 7!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = \frac{95,040}{120} = 792
$$

Therefore:

$$
P = 792 \times (0.5)^{12}
$$

And since $(0.5)^{12} = \frac{1}{2^{12}} = \frac{1}{4096}$:

$$
P = \frac{792}{4096} \approx 0.1934
$$

This means there is approximately a 19.34% chance of getting exactly five heads and seven tails in 12 tosses.

# Broader Perspectives and Variations

While the calculation above provides a clear and precise answer, it also opens the door to exploring related questions and variations that deepen our understanding of probability.

## What if the Coin Is Biased?

If the coin is biased, with a probability $p \neq 0.5$ of landing heads, the calculation changes accordingly. For example, if $p = 0.6$:

$$
P = \binom{12}{5} \times (0.6)^5 \times (0.4)^7
$$

This calculation would yield a different probability, reflecting the bias.

Pros:

- Allows modeling real-world scenarios where outcomes are not equally likely.
- Demonstrates the flexibility of the binomial distribution.

Cons:

- Requires knowledge of the actual bias, which may not always be available.

## Calculating Probabilities for Other Outcomes

Beyond exactly five heads, one might be interested in the probability of getting at least five heads, at most five heads, or exactly a different number of heads.

For example, the probability of getting at least 5 heads in 12 tosses:

$$
P(\geq 5) = \sum_{k=5}^{12} \binom{12}{k} (0.5)^k (0.5)^{12 - k}
$$

Calculating this sum involves summing binomial probabilities for $k=5$ through $k=12$, which can be efficiently computed using statistical software or binomial tables.

## Applications and Real-World Implications

Understanding the probability of specific outcomes in coin tosses has broader implications beyond simple games. It forms the foundation for various fields including:

- Statistics and Data Analysis: Modeling binary data and outcomes.
- Computer Science: Designing algorithms involving randomness.
- Physics: Modeling particle decay events.
- Economics: Analyzing binary decision outcomes.

In educational settings, problems like this serve as excellent exercises to develop intuition about probability, combinatorics, and the binomial distribution.

## Limitations and Considerations

While the calculations are straightforward under ideal assumptions, real-world scenarios may involve complexities:

- Biased Coins: Not all coins are perfectly fair.
- Dependent Tosses: Physical factors could introduce dependencies.
- Small Sample Sizes: Variance can be high with small numbers of trials.

Understanding these limitations is crucial for applying probability models accurately in practical situations.

## Conclusion

The probability of obtaining exactly five heads and seven tails in 12 tosses of a fair coin is approximately 19.34%. This calculation hinges on the principles of the binomial distribution, combinatorics, and the assumptions of fairness and independence. The problem exemplifies how simple probabilistic models can be used to analyze seemingly straightforward scenarios, providing insights into randomness and likelihood. Whether used for educational purposes, simulations, or real-world decision-making, understanding such probability problems enhances our grasp of uncertainty and statistical reasoning. As with all models, recognizing their assumptions and limitations is key to applying them effectively across various domains.

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