A Line Has A Slope Of -10 And Includes The Points (4, W) And (3,0). What Isthe Value Of W?

A Line Has A Slope Of -10 And Includes The Points (4, W) And (3,0). What Is The Value Of W?

Understanding the relationship between points on a line and its slope is fundamental in coordinate geometry. When given specific points and the slope of a line, one can determine unknown coordinates using the slope formula. In this article, we will explore how to find the value of W in the points (4, W) and (3, 0) given that the slope of the line passing through these points is -10. We will walk through the concepts step-by-step, providing clear explanations and examples to enhance your understanding of calculating missing coordinate values in a line with a known slope.

Understanding the Slope of a Line

Definition of Slope

The slope of a line measures its steepness or inclination. It is calculated as the ratio of the vertical change to the horizontal change between two points on the line. Mathematically, the slope (m) between two points (x_1, y_1) and (x_2, y_2) is given by:

- m = $\frac{y_2 - y_1}{x_2 - x_1}$

This ratio indicates how much y increases or decreases as x increases. A negative slope means the line descends from left to right, while a positive slope indicates an ascent.

Relevance to Finding Unknown Coordinates

When the slope and some points on a line are known, you can find unknown coordinates by substituting the known values into the slope formula and solving for the unknown. This process is essential in many geometry problems, such as the one involving points (4, W) and (3, 0) with a slope of -10.

Applying the Slope Formula to Find W

Given Data Recap

- The slope of the line, m = -10
- Point 1: (4, W)
- Point 2: (3, 0)

Our goal is to find the value of W.

Step-by-Step Calculation

1. Write the slope formula using the known points:

  - -10 = \(\frac{0 - W}{3 - 4}\)

2. Simplify the denominator:

  - 3 - 4 = -1

3. Substitute back into the equation:

  - -10 = \(\frac{0 - W}{-1}\)

4. Recognize that dividing by -1 reverses the sign:

  - -10 = -(0 - W)\)
  - -10 = - (0 - W) = -0 + W = W

5. Therefore, W = -10.

This simple calculation reveals that the value of W is -10.

Interpreting the Result

Meaning of W = -10

The point (4, W) becomes (4, -10). This indicates that at x = 4, the y-coordinate on the line is -10. The line passing through the points (3, 0) and (4, -10) has a consistent slope of -10, confirming the calculations.

Verifying the Calculation

To ensure accuracy, we can verify the slope using the two points:

- Calculate $\frac{W - 0}{4 - 3}$ = $\frac{-10 - 0}{1}$ = -10

Since this matches the given slope, our value of W = -10 is correct.

Additional Concepts Related to Slope and Coordinates

Equation of the Line

Knowing one point and the slope allows you to write the line’s equation in point-slope form:

- $y - y_1 = m(x - x_1)$

Using point (3, 0):

- y - 0 = -10(x - 3)
- y = -10x + 30

Check the point (4, -10):

- Plug x = 4: y = -10(4) + 30 = -40 + 30 = -10

Matches the point (4, -10), confirming the line’s equation.

Real-World Applications

Understanding how to determine the unknown coordinate W based on the slope and other points has practical applications in various fields:
- Engineering: designing inclined planes or slopes

- Physics: analyzing motion along a line with a specific gradient
- Economics: modeling linear relationships between variables
- Computer Graphics: rendering lines with precise slopes

Summary and Key Takeaways

- The slope of a line indicates its steepness and is calculated using the change in y over the change in x between two points.
- Given the slope and two points, you can find unknown coordinates by manipulating the slope formula.
- In the problem involving points (4, W) and (3, 0) with a slope of -10, W equals -10.
- Verifying calculations through substitution ensures accuracy.
- Knowing how to derive the line’s equation from points and slope helps in understanding the line’s behavior and in solving related problems.

Final Thoughts

Mastering the process of calculating unknown coordinates using the slope formula is a foundational skill in algebra and geometry. Whether you're solving academic problems or applying these concepts in real-world scenarios, understanding the relationship between points, slope, and lines is crucial. Remember, always verify your calculations to ensure accuracy, especially when working with real data or complex problems.

If you find yourself working with lines and slopes frequently, consider practicing different problems involving various points and slopes. This will improve your confidence and proficiency in coordinate geometry, making you more adept at tackling both academic and practical challenges related to lines and their equations.

Frequently Asked Questions

Given a line with slope -10 passing through points (4, W) and (3, 0), how can you find the value of W?

Use the slope formula: m = (y2 - y1) / (x2 - x1). Plug in the known points and solve for W: -10 = (0 - W) / (3 - 4). Simplify to find W.

What is the value of W if a line with slope -10 passes

through points (4, W) and (3, 0)?

W equals 10. Calculated by setting -10 = (0 - W) / (3 - 4) and solving for W.

How do you determine W in the points (4, W) and (3, 0) given the slope of -10?

Apply the slope formula: -10 = (0 - W) / (3 - 4). Since 3 - 4 = -1, W = -10 (-1) = 10.

Why is W equal to 10 in the line passing through (4, W) and (3, 0) with slope -10?

Because substituting into the slope formula yields W = 10, ensuring the slope between the points is -10.

Can you verify the value of W for the line with slope -10 passing through (4, W) and (3, 0)?

Yes. Using the slope formula: W = 10, as the calculation confirms the points align with a slope of -10.

Additional Resources

Understanding the Relationship Between Slope and Coordinates in Line Equations

In the realm of analytical geometry, the study of lines and their properties forms a foundational pillar for understanding the spatial relationships between points in a plane. One of the key characteristics that define a line is its slope, a measure of its inclination or steepness. When given a line with a specific slope and certain points that lie on it, mathematicians can determine unknown quantities, such as the coordinate values of missing points. This article delves into an intriguing problem: a line with a slope of -10 passing through points (4, W) and (3, 0), asking us to find the value of W. Through comprehensive exploration, we will analyze the principles of slope calculation, the derivation of unknown coordinates, and the broader implications of such problems in mathematical reasoning.

Foundations of Line Equations and Slope Concept

Defining the Slope of a Line

The slope of a line, often denoted as 'm', quantifies the rate at which the line rises or falls as you move along it. Mathematically, the slope between two points, (x_1, y_1) and (x_2, y_2), is expressed as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This ratio measures the change in the y-coordinates relative to the change in the x-coordinates. A positive slope indicates an upward trend from left to right, while a negative slope indicates a downward trend.

Key Points:

- The slope is constant for a straight line, making it a linear function.
- It dictates the angle of the line relative to the x-axis.
- It is essential in deriving the line's equation and solving for unknown coordinates.

The Equation of a Line in Slope-Intercept Form

Once the slope is known, the line's equation can be expressed in the slope-intercept form:

$$y = mx + b$$

where:

- m is the slope.
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful when two points on the line are known, as it allows for straightforward calculation of the line's equation, facilitating the determination of unknown points.

Analyzing the Given Data and Problem Statement

Information Provided

The problem states:

- The line has a slope $m = -10$.
- The line passes through two points: (4, W) and (3, 0).

The unknown in this scenario is W, the y-coordinate corresponding to x = 4.

Objective

Our goal is to find the value of W. Since the line passes through both points, the key is to apply the slope formula to these points and solve for W.

Step-by-Step Solution Methodology

1. Applying the Slope Formula

Given two points:

- $(x_1, y_1) = (3, 0)$
- $(x_2, y_2) = (4, W)$

and the slope $m = -10$, the slope formula becomes:

$$m = \frac{W - 0}{4 - 3}$$

which simplifies to:

$$-10 = \frac{W}{1}$$

since $4 - 3 = 1$.

2. Solving for W

Rearranging the equation:

$$W = -10 \times 1 = -10$$

This calculation directly yields:

$$W = -10$$

Conclusion: The value of W is -10.

3. Validating the Result

To ensure the accuracy, one can verify the calculation by checking if the slope between the points indeed equals -10:

$$\text{Slope} = \frac{-10 - 0}{4 - 3} = \frac{-10}{1} = -10$$

which confirms our solution.

Deeper Insights and Broader Implications

The Significance of Slope in Geometric Contexts

Understanding how the slope relates to specific points on a line provides crucial insights into the geometric configurations and relationships. For instance:

- It allows for the construction of the line's equation.
- It aids in understanding the line's inclination relative to axes.
- It facilitates the prediction of other points lying on the same line.

In the context of this problem, knowing the slope and one point enables the precise calculation of the other point's coordinate.

Implications in Real-World Situations

While the problem is abstract, its principles are widely applicable:

- Engineering: Determining the gradient of a ramp or a slope in terrain modeling.
- Economics: Calculating rate changes, such as marginal cost or revenue.
- Physics: Analyzing velocity or acceleration along a trajectory.
- Computer Graphics: Rendering lines with specific inclinations on digital displays.

The ability to manipulate and interpret the slope and points on a line is foundational across disciplines.

Complexity and Limitations of the Problem

This problem exemplifies a straightforward application of the slope formula, but real-world scenarios often involve additional complexities:

- Non-linear relationships requiring quadratic or higher-order functions.
- Multiple lines intersecting, necessitating systems of equations.
- Data inaccuracies leading to approximate solutions.

Given these complexities, understanding the basics, as illustrated here, becomes vital for tackling more advanced problems.

Further Explorations and Related Concepts

Extensions to the Basic Problem

- Finding the equation of the line: With the slope and a point, the line's equation can be written as:

$$ y - y_1 = m(x - x_1) $$

Using $(3, 0)$ and $m = -10$:

$$ y - 0 = -10(x - 3) $$
$$ y = -10x + 30 $$

- Predicting other points: For any x, the y coordinate can be computed.

- Graphical representation: Plotting the line confirms the linear relationship and the calculated point.

Related Mathematical Concepts

- Perpendicular lines: Lines with slopes that are negative reciprocals.
- Parallel lines: Lines with identical slopes.
- Distance between points: Using the distance formula:

$$ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

- Midpoint calculation: The average of the x- and y-coordinates.

Understanding these concepts enriches the analysis of lines and their properties.

Summary and Final Remarks

This comprehensive examination reveals that the problem of determining the value of W hinges on applying the fundamental principles of slope in coordinate geometry. By recognizing that the slope between two points is a fixed value, we can derive the unknown coordinate confidently. The key takeaway is that the slope's negative value of -10 indicates a steep downward inclination, and the specific points provided allow for direct calculation of the missing coordinate, which is -10.

Such problems exemplify the elegance of algebraic methods in solving geometric queries, emphasizing the interconnectedness of numerical data and spatial reasoning. Mastery of these concepts not only facilitates solving textbook exercises but also equips individuals with analytical tools applicable in various scientific, engineering, and technological contexts. As mathematical understanding deepens, so does the capacity to interpret and manipulate the world through the lens of lines, slopes, and coordinates.

In essence, the value of \(W \) for the line with a slope of -10 passing through points (4, W) and (3, 0) is \(-10\). This conclusion underscores the power of fundamental algebraic techniques and highlights the importance of slope in understanding linear relationships.

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