

A Line Passes Through The Points (4, 6) And (8, 7). What Is Its Equation In Point-slope Form?

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Understanding the properties of lines and their equations is fundamental in algebra and coordinate geometry. When analyzing a line passing through specific points, calculating its equation provides insight into its slope and position in the coordinate plane. In this article, we will explore how to determine the equation of a line passing through the points (4, 6) and (8, 7), specifically in the point-slope form. We will also discuss the significance of the point-slope form, how to derive it, and its applications in various mathematical contexts.

Introduction to Line Equations in Coordinate Geometry

Coordinate geometry, also known as analytic geometry, involves studying geometric figures through algebraic equations. Among these figures, lines are fundamental objects characterized by their slope and position.

The most common forms of a line's equation include:

- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

Each form offers unique advantages depending on the context. The point-slope form is particularly useful when you know the slope of the line and a specific point through which it passes.

Understanding the Given Points (4, 6) and (8, 7)

Before deriving the equation, it's essential to understand the provided points:

- Point 1: $(x_1, y_1) = (4, 6)$
- Point 2: $(x_2, y_2) = (8, 7)$

These points lie on the same straight line, and our goal is to find the equation of that line in point-slope form.

Calculating the Slope of the Line

The first step in formulating the line's equation is to calculate its slope (m) . The slope measures the steepness of the line and is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Applying the coordinates:

$$m = \frac{7 - 6}{8 - 4} = \frac{1}{4}$$

This result indicates that for every 4 units increase in (x) , (y) increases by 1 unit. The line has a gentle positive slope.

Understanding the Point-slope Form of a Line Equation

The point-slope form of a line's equation is expressed as:

$$y - y_1 = m(x - x_1)$$

Where:

- (x_1, y_1) is any point on the line
- (m) is the slope of the line

This form is especially useful when you know the slope and a specific point through which the line passes. It provides a straightforward way to write the equation without immediately converting to slope-intercept form.

Deriving the Equation in Point-slope Form

Using the known point $(4, 6)$ and the calculated slope $(\frac{1}{4})$, we substitute into the

point-slope formula:

$$y - 6 = \frac{1}{4}(x - 4)$$

This is the equation of the line passing through points (4, 6) and (8, 7) in point-slope form.

Alternatively, using the other point (8, 7):

$$y - 7 = \frac{1}{4}(x - 8)$$

Both equations describe the same line, and choosing either point is acceptable.

Significance and Applications of the Point-slope Form

The point-slope form is a flexible and practical way to express a line's equation, especially in the following contexts:

- When the slope and a point are known
- For quick graphing of the line
- To derive the slope-intercept form easily
- In solving systems of equations involving lines

It provides a clear relationship between the variables, emphasizing the geometric interpretation of slope and position.

Converting the Equation to Slope-intercept Form

While the question specifically asks for the point-slope form, understanding how to convert it to slope-intercept form is useful. Starting from:

$$y - 6 = \frac{1}{4}(x - 4)$$

Distribute the slope:

$$y - 6 = \frac{1}{4}x - 1$$

\]

Add 6 to both sides:

\[

$$y = \frac{1}{4}x - 1 + 6$$

\]

\[

$$y = \frac{1}{4}x + 5$$

\]

This form makes it easier to identify the slope $(\frac{1}{4})$ and the y-intercept (5) .

Visualizing the Line and Its Graph

Graphing the line helps solidify understanding. To plot:

- Plot the point $(4, 6)$
- Use the slope $(\frac{1}{4})$ to find another point:
- From $(4, 6)$, move 4 units right (x-direction), and 1 unit up (y-direction):

\[

$$(4 + 4, 6 + 1) = (8, 7)$$

\]

- Confirm that the second point is $(8, 7)$, matching the original data.

Plotting these points and drawing a line through them visualizes the line described by the equation.

Summary of the Derived Equation

The complete, accurate point-slope form of the line passing through points $(4, 6)$ and $(8, 7)$ is:

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$$\textbf{y} - 6 = \frac{1}{4}(x - 4)}$$

\]

\]

This equation succinctly captures the line's slope and passing point, providing a foundation for further analysis or graphing.

Additional Considerations and Related Topics

Understanding the line's equation in different forms enhances problem-solving skills. Here are some related concepts:

- Standard Form of a Line: Converting the point-slope or slope-intercept form into $(Ax + By = C)$
- Finding the Equation of a Line Given Two Points: Using the slope formula and point-slope form
- Parallel and Perpendicular Lines: Deriving equations based on the slopes
- Line Equations in Real-World Contexts: Applications in physics, engineering, and data analysis

Conclusion

In this comprehensive exploration, we have determined that the equation of the line passing through the points (4, 6) and (8, 7) in point-slope form is:

$$\boxed{y - 6 = \frac{1}{4}(x - 4)}$$

This process involved calculating the slope, selecting a point, and applying the point-slope formula. Mastering this method is essential for solving various problems in algebra and coordinate geometry, enabling students and professionals to analyze and interpret linear relationships effectively. Whether for graphing, solving systems, or understanding geometric properties, the point-slope form remains a fundamental tool in mathematical analysis.

Frequently Asked Questions

How do you find the slope of the line passing through the points (4, 6) and (8, 7)?

The slope is calculated using the formula $m = (y_2 - y_1) / (x_2 - x_1)$. Substituting the points: $m = (7 - 6) / (8 - 4) = 1 / 4$.

What is the point-slope form of a line equation?

The point-slope form is written as $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the line and m is the slope.

Using the point (4, 6) and the slope, what is the equation of the line in point-slope form?

The equation is $y - 6 = (1/4)(x - 4)$.

Can the point (8, 7) be used to write the point-slope form? If so, what is it?

Yes. Using (8, 7) and the slope $1/4$, the equation is $y - 7 = (1/4)(x - 8)$.

Why is it important to find the slope before writing the point-slope form?

Because the slope determines the steepness of the line and is essential for constructing the correct point-slope equation.

What is the significance of the point-slope form in graphing the line?

It provides a straightforward way to graph the line using a known point and the slope, making it easy to plot and understand the line's orientation.

How can you verify that the line passes through both points (4, 6) and (8, 7)?

By substituting each point into the point-slope equation $y - 6 = (1/4)(x - 4)$ (or the equivalent form) and confirming both satisfy the equation.

Additional Resources

A line passes through the points (4, 6) and (8, 7). What is its equation in point-slope form?

Introduction: The Significance of Understanding Line Equations

In the realm of mathematics, particularly coordinate geometry, the ability to determine the equation of a line given specific points is fundamental. Whether in academic pursuits, engineering, computer graphics, or everyday problem-solving, understanding how to formulate the equation of a line enables us to analyze relationships between variables, predict outcomes, and interpret data visually.

The question at hand—finding the equation of a line passing through points (4, 6) and (8, 7)—serves as an excellent illustration of core principles in coordinate geometry. It not only reinforces the concepts of slope calculation and point-slope form but also demonstrates practical methods to derive and interpret linear equations. This comprehensive review aims to explore these concepts in detail, providing clarity for students, educators, and anyone interested in mathematical analysis.

Understanding the Coordinates: Points (4, 6) and (8, 7)

Before delving into the equation itself, it is essential to understand the significance of the points involved.

Coordinates in the Cartesian Plane

- Each point is represented as an ordered pair: (x, y).
- In our case:
- Point A: (4, 6), where 4 is the x-coordinate and 6 is the y-coordinate.
- Point B: (8, 7), where 8 is the x-coordinate and 7 is the y-coordinate.

Visual Interpretation

Plotting these points on a Cartesian plane reveals their relative positions:

- Point A is located at 4 units along the x-axis and 6 units along the y-axis.
- Point B is located at 8 units along the x-axis and 7 units along the y-axis.

Connecting these points with a straight line illustrates the linear relationship between the variables.

Calculating the Slope: The Foundation of Line Equations

The slope (denoted as m) measures the steepness or inclination of a line. It indicates how much the y-value changes for a unit increase in x.

The Slope Formula

To compute the slope between two points, (x₁, y₁) and (x₂, y₂), the standard formula is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This formula calculates the ratio of the change in y to the change in x, often called "rise over run."

Applying the Formula to Our Points

Given:

- Point A: (4, 6) → (x₁=4, y₁=6)
- Point B: (8, 7) → (x₂=8, y₂=7)

Calculating the slope:

$$m = \frac{7 - 6}{8 - 4} = \frac{1}{4}$$

Thus, the slope of the line passing through these points is $\frac{1}{4}$.

Interpretation of the Slope

A slope of $\frac{1}{4}$:

- Indicates that for every 4 units increase in x, y increases by 1 unit.
- Signifies a gently upward-sloping line from left to right.
- Reflects the rate of change between the two points, which is crucial for understanding the linear relationship.

Deriving the Equation in Point-Slope Form

Once the slope is known, the next step is to derive the equation of the line in point-slope form.

What Is Point-Slope Form?

The point-slope form of a line's equation is a versatile and straightforward way to express a line that passes through a specific point with a known slope. It is written as:

$$y - y_1 = m(x - x_1)$$

Where:

- (x_1, y_1) is a point on the line.
- m is the slope.

Advantages of Point-Slope Form

- It allows direct insertion of a known point and the slope without the need for initial calculations.
- It provides a clear visualization of the line's inclination and a specific point it passes through.
- It is a stepping stone to other forms like slope-intercept or standard form.

Using Point (4, 6) as an Example

Given:

- Slope $m = \frac{1}{4}$
- Point: (4, 6)

The point-slope form becomes:

$$y - 6 = \frac{1}{4}(x - 4)$$

This is the equation of the line passing through the point (4, 6) with the slope $\frac{1}{4}$.

Using the Other Point (8, 7)

Alternatively, we can verify or derive the same line equation using the second point:

$$y - 7 = \frac{1}{4}(x - 8)$$

Both forms should theoretically describe the same line, confirming consistency.

Verifying the Equation: Consistency and Accuracy

It is essential to verify that the equation derived in point-slope form accurately represents the line passing through both points.

Substituting the Known Points

1. For (4, 6):

$$y - 6 = \frac{1}{4}(x - 4)$$

Plugging in $(x=4)$:

$$6 - 6 = \frac{1}{4}(4 - 4) \rightarrow 0 = 0$$

The point satisfies the equation.

2. For (8, 7):

$$7 - 6 = \frac{1}{4}(8 - 4) \rightarrow 1 = \frac{1}{4} \times 4 \rightarrow 1 = 1$$

Again, the point satisfies the equation, confirming correctness.

Converting to Other Forms: Slope-Intercept and Standard Form

While the point-slope form is useful, it often helps to convert the equation into slope-intercept form $(y = mx + b)$ for easier graphing and interpretation.

Slope-Intercept Form

Starting with:

$$y - 7 = \frac{1}{4}(x - 8)$$

$$y - 6 = \frac{1}{4}(x - 4)$$

Distribute:

$$y - 6 = \frac{1}{4}x - 1$$

Add 6 to both sides:

$$y = \frac{1}{4}x - 1 + 6$$

Simplify:

$$y = \frac{1}{4}x + 5$$

Equation in slope-intercept form:

$$\boxed{y = \frac{1}{4}x + 5}$$

Standard Form

Rearranging to standard form ($Ax + By = C$):

$$y = \frac{1}{4}x + 5$$

Multiply through by 4 to clear fractions:

$$4y = x + 20$$

Rearranged:

$$x - 4y = -20$$

Standard form:

$$\boxed{x - 4y = -20}$$

\]

Practical Implications and Applications

Understanding the equation of a line passing through two points has multiple real-world applications:

- Data Analysis: In statistics, fitting a line to data points helps identify trends.
- Engineering: Designing structures requires understanding linear relationships.
- Economics: Linear models describe cost, revenue, or profit functions.
- Computer Graphics: Drawing lines between points relies on equations and algorithms.
- Navigation and GPS: Calculating straight routes involves linear equations.

In educational settings, mastering these derivations enhances problem-solving skills and conceptual understanding.

Broader Context: The Importance of Precise Mathematical Communication

Expressing the equation of a line in various forms—point-slope, slope-intercept, and standard—serves different purposes. The point-slope form, in particular, emphasizes the specific point and the slope, making it especially useful when dealing with known points, such as in real-world measurements or initial data points.

Furthermore, the process of deriving these equations fosters critical thinking, attention to detail, and the ability to verify solutions, which are essential skills beyond mathematics.

Conclusion: Mastery of Line Equations as a Mathematical Tool

In summary, the line passing through (4, 6) and (8, 7) is characterized by a gentle slope of $\frac{1}{4}$. Its equation in point-slope form is:

$$\boxed{y - 6 = \frac{1}{4}(x - 4)}$$

This form directly relates the line's inclination and a specific point it traverses, offering clarity and precision for analysis. Converting it to slope-intercept and standard forms further enhances its utility and interpretability.

Understanding these derivations and their underlying principles equips students and professionals with a foundational skill set critical for tackling more complex problems in mathematics, science, and engineering. Mastery of such concepts underpins advances across numerous fields, underscoring the importance of thorough comprehension and analytical proficiency in coordinate geometry.

In essence, the ability to find the equation of a line given two points is not only a fundamental mathematical skill but also a gateway to understanding the interconnectedness of data, relationships, and representations in the world around us.

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