A Line's Y-intercept Is 3, And Its Slope Is 1. What Is Its Equation In Slope-intercept Form?

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Understanding the equation of a line is fundamental in algebra and coordinate geometry. When given specific characteristics such as the slope and y-intercept, you can quickly determine the equation that describes the line. In this article, we will explore how to find the equation of a line in slope-intercept form when the slope is 1 and the y-intercept is 3. We will cover the concepts of slope and y-intercept, their significance, how to derive the equation, and practical applications of this knowledge.

Introduction to Line Equations

Before delving into the specific problem, it's important to understand the general form of the equation of a line and the terminology involved.

What Is the Equation of a Line?

The equation of a line mathematically describes all points (x, y) that lie on that line. Depending on the form, the equation can be expressed in various ways, including:

- Slope-intercept form
- Point-slope form
- Standard form

For this discussion, we focus primarily on the slope-intercept form because it directly involves the slope and y-intercept.

Why Is the Slope-Intercept Form Important?

The slope-intercept form provides a straightforward way to graph a line and understand its behavior. It is written as:

$$ y = mx + b $$

where:

- $ y $ is the y-coordinate of a point on the line
- $ x $ is the x-coordinate
- $ m $ is the slope of the line
- $ b $ is the y-intercept, i.e., the point where the line crosses the y-axis

Knowing the slope and y-intercept allows for quick graphing and analysis of the line's properties.

Understanding the Components: Slope and Y-intercept

What Is the Slope?

The slope $ m $ measures the steepness of the line. It is calculated as the ratio of the vertical change to the horizontal change between two points on the line:

$$ m = \frac{\Delta y}{\Delta x} $$

- A positive slope indicates the line rises as x increases.
- A negative slope indicates the line falls.
- A slope of zero indicates a horizontal line.
- An undefined slope indicates a vertical line.

In our case, the slope is 1, meaning for every unit increase in x, y increases by the same amount.

What Is the Y-intercept?

The y-intercept b is the point where the line crosses the y-axis (where $x = 0$). It provides a starting point for graphing the line.

For our problem, the y-intercept is 3, meaning the line crosses the y-axis at the point (0, 3).

Deriving the Equation of the Line

Given the slope $m = 1$ and y-intercept $b = 3$, we can directly substitute these values into the slope-intercept form.

$$
\boxed{
y = mx + b
}
$$

$$
\Rightarrow y = 1 \times x + 3
$$

$$
\Rightarrow y = x + 3
$$

This is the equation of the line in slope-intercept form.

Step-by-Step Breakdown

1. Identify the slope: $m = 1$
2. Identify the y-intercept: $b = 3$
3. Substitute into slope-intercept form:

$$
y = mx + b
$$

$$
y = 1 \times x + 3
$$

4. Simplify:

$$
y = x + 3
$$

Thus, the line's equation is $y = x + 3$.

Graphing the Line

Graphing helps visualize the line described by the equation $y = x + 3$.

Steps to Graph the Line

1. Plot the y-intercept: Mark the point (0, 3) on the coordinate plane.
2. Use the slope: Since the slope is 1, rise over run is 1/1.
- From (0, 3), move up 1 unit and right 1 unit to reach (1, 4).
- Alternatively, move down 1 unit and left 1 unit to reach (-1, 2).
3. Draw the line: Connect these points with a straight line, extending across the graph.

Key Points on the Line

- (0, 3)
- (1, 4)
- (-1, 2)

Having these points ensures accurate graphing and confirms the line’s slope and intercept.

Applications of Knowing a Line's Equation

Understanding the equation $y = x + 3$ has many practical uses:

- Predicting values: Given a value of x, find y.
- Analyzing relationships: Recognize how changes in x affect y.
- Solving real-world problems: Such as cost calculations, physics, and engineering.

For example, if y represents the total cost of items based on quantity x, and the fixed cost is 3, with a per-item cost of 1, then the total cost equation is $y = x + 3$.

Related Concepts and Variations

While the slope-intercept form is most straightforward, understanding related forms enhances your overall grasp of line equations.

Point-Slope Form

Used when a point on the line and the slope are known:

$$
y - y_1 = m(x - x_1)
$$

For example, with point (0, 3) and slope 1:

$$
y - 3 = 1(x - 0) \Rightarrow y - 3 = x
$$

which simplifies to $y = x + 3$.

Standard Form

Expressed as:

$$
Ax + By = C
$$

For our line:

$$
y = x + 3 \Rightarrow -x + y = 3
$$

or

$$
x - y = -3
$$

Practice Problems and Examples

To reinforce understanding, here are some practice exercises:

1. Find the equation of a line with a slope of 2 and y-intercept of -1.

Solution:

$$
y = 2x - 1
$$

2. Determine the slope-intercept form of the line passing through points (2, 5) and (4, 7).

Solution:

- Calculate slope:

$$
m = \frac{7 - 5}{4 - 2} = \frac{2}{2} = 1
$$

- Use point-slope form with point (2, 5):

$$
y - 5 = 1(x - 2) \Rightarrow y - 5 = x - 2
$$

- Convert to slope-intercept form:

$$

$$
y = x + 3
$$

3. Write the equation of a line with slope -3 that passes through the point (1, 2).

Solution:

- Use point-slope form:

$$
y - 2 = -3(x - 1) \Rightarrow y - 2 = -3x + 3
$$

- Simplify:

$$
y = -3x + 5
$$

Summary and Key Takeaways

- The equation of a line in slope-intercept form is $y = mx + b$.
- Given the slope $m = 1$ and y-intercept $b = 3$, the equation is $y = x + 3$.
- The slope indicates the line's steepness; here, a slope of 1 means a 45-degree angle with the x-axis.
- The y-intercept indicates the point where the line crosses the y-axis at (0, 3).
- Graphing involves plotting the intercept and using the slope to find additional points.
- Variations of the line equation include point-slope and standard forms, which are useful in different contexts.

Conclusion

Knowing how to find the equation of a line from its slope and y-intercept is a fundamental skill in algebra. It enables quick graphing, analysis, and application in real-world scenarios. The specific line with a slope of 1 and y-intercept of 3 is elegantly expressed as $y = x + 3$, embodying the simplicity and power of the slope-intercept form. Mastery of this concept facilitates deeper understanding of linear relationships and prepares you for more complex mathematical topics.

If you want to explore further, consider practicing with different slopes and intercepts, or delve into other forms of line equations to broaden your mathematical toolkit.

Frequently Asked Questions

What is the slope of the line if its slope-intercept form has a slope of 1?

The slope of the line is 1.

Given the y-intercept is 3 and the slope is 1, what is the equation of the line in slope-intercept form?

The equation is $y = 1x + 3$, which simplifies to $y = x + 3$.

How do you write the equation of a line when the y-intercept is 3 and the slope is 1?

You write it as y = 1x + 3, or simply y = x + 3.

What does the y-intercept of 3 tell us about the line's graph?

It tells us the line crosses the y-axis at the point (0, 3).

If a line has a slope of 1 and y-intercept of 3, what is its graph's key point?

The key point is (0, 3), where the line crosses the y-axis.

Can the equation y = x + 3 be rewritten in another form?

Yes, it can be written as y - x = 3 or in standard form as x - y = -3.

What is the significance of the slope being 1 in the line's equation?

A slope of 1 indicates the line rises one unit vertically for every one unit horizontally.

How does knowing the y-intercept and slope help in graphing the line?

Knowing the y-intercept and slope allows you to plot the point (0,3) and then use the slope to find additional points.

Is the equation y = x + 3 linear, and why?

Yes, because it is in slope-intercept form, which represents a straight line.

What real-world situations could be modeled by the line y = x + 3?

Any scenario where the relationship between two variables increases at a constant rate, starting from a baseline of 3, such as savings increasing at $1 per day starting with an initial amount of $3.

Additional Resources

A Line's Y-intercept Is 3, And Its Slope Is 1. What Is Its Equation In Slope-intercept Form?

Understanding the fundamental principles of algebra and coordinate geometry is crucial for deciphering the equations of lines. Among the various forms used to represent a line, the slope-intercept form stands out for its simplicity and direct interpretability. When given specific parameters such as the y-intercept and the slope of a line, deriving its equation becomes an exercise in applying basic algebraic concepts. In this article, we delve into the process of determining the equation of a line given its y-intercept and slope, explore the significance of these parameters, and analyze how they shape the line's behavior on the Cartesian plane.

Understanding the Slope-Intercept Form

Definition and Significance

The slope-intercept form of a linear equation is expressed as:

$$ y = mx + b $$

where:

- m is the slope of the line, representing its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly advantageous because it immediately reveals two critical characteristics of the line:

1. Its inclination or decline – whether it rises, falls, or remains horizontal.
2. Its position relative to the y-axis – where it intersects.

The slope-intercept form provides a straightforward way to graph the line, predict its behavior, and understand its relationship with other lines.

Why Use the Slope-Intercept Form?

- Ease of graphing: Knowing the y-intercept and slope allows for simple plotting.
- Clarity of parameters: The form directly displays the line's key features.
- Solution to real-world problems: Many applications require understanding the rate of change (slope) and initial value (intercept).

Given Parameters: Y-intercept and Slope

Y-intercept = 3

The y-intercept, denoted as b, is the point where the line crosses the y-axis. When the y-intercept is 3, it means:

- The line passes through the point (0, 3).
- On the Cartesian plane, when $x = 0$, $y = 3$.

Slope = 1

The slope, denoted as m, indicates the rate at which y changes with respect to x. A slope of 1 signifies:

- For every 1 unit increase in x, y increases by 1.
- The line has a 45-degree inclination relative to the x-axis, assuming a standard scale.

Understanding these parameters provides a clear starting point for formulating the equation.

Deriving the Equation in Slope-Intercept Form

Step-by-Step Process

Given:

- $b = 3$
- $m = 1$

The slope-intercept form is:

$$y = mx + b$$

Substituting the known values:

$$ y = 1 \times x + 3 $$

which simplifies to:

$$ y = x + 3 $$

This is the equation of the line in slope-intercept form.

Verification of the Equation

To confirm the correctness:

- Y-intercept check: When $x = 0$:

$$ y = 0 + 3 = 3 $$

which matches the given y-intercept.

- Slope check: For any increase of 1 in x, y increases by 1 as well, confirming the slope.

Graphical Interpretation and Visualization

Plotting the Line

Visualizing the line helps in understanding its position and behavior.

- The line crosses the y-axis at (0, 3).
- For $x = 1$:

$$y = 1 + 3 = 4$$

- For $x = -1$:

$$y = -1 + 3 = 2$$

Plotting these points:

- (0, 3)
- (1, 4)
- (-1, 2)

Connecting these points yields a straight line with a slope of 1 passing through (0, 3).

Characteristics of the Line

- Inclination: The line rises at a 45-degree angle.
- Intercept: It crosses the y-axis at 3.
- Direction: Moves upward from left to right, indicating a positive slope.

Applications and Real-World Contexts

Understanding the equation of a line with given slope and intercept is fundamental in various fields:

- Economics: Modeling cost functions where the intercept reflects fixed costs and the slope indicates variable costs.
- Physics: Describing constant velocity motion, where the slope represents speed.
- Biology: Tracking growth rates over time.

In each case, the parameters provide insights into the underlying processes and facilitate predictions.

Extensions and Related Concepts

Alternative Forms of Line Equations

While the slope-intercept form is convenient, other forms exist:

- Point-slope form:

$$ y - y_1 = m(x - x_1) $$

which is useful when a point on the line and slope are known.

- Standard form:

$$ Ax + By = C $$

which is often used in algebraic manipulations and solving systems of equations.

Calculating the Equation from Different Data

If only two points on the line are known, the process involves:

1. Calculating the slope:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

2. Using one point and the slope in the point-slope form to find the equation.

Conclusion: The Complete Equation and Its Significance

Having analyzed the parameters and the process of deriving the equation, the final form of the line in slope-intercept form is:

$$ y = x + 3 $$

This simple yet powerful equation encapsulates the line's behavior:

- It crosses the y-axis at 3.
- It rises diagonally with a slope of 1.

Understanding how to derive and interpret such equations is fundamental for students, educators, and professionals dealing with linear relationships. Such knowledge not only aids in graphing and analysis but also fosters a deeper comprehension of the underlying concepts that govern linear functions

across various disciplines.

In summary, the line with a y-intercept of 3 and a slope of 1 is succinctly represented by the equation $y = x + 3$. This equation serves as a foundation for more complex analyses and exemplifies the elegance of algebraic representations in describing the geometric properties of lines on the Cartesian plane.

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