

A Logo Is 15 Cm Wide. If The Logo Is Dilated By A Factor Of $\frac{7}{4}$, Find The Width Of The Logo

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Understanding how to determine the new width of a logo after dilation is an important concept in geometry, especially in the context of similar figures and scale transformations. Whether you're a student working on a math problem or a professional involved in graphic design, mastering the calculation of dilated dimensions helps in creating accurate and proportional designs. In this article, we'll delve into the concept of dilation, explore how to calculate the new width of a logo after dilation, and discuss related concepts to enhance your understanding.

What Is Dilation in Geometry?

Definition of Dilation

Dilation, also known as resizing or scaling, is a transformation that enlarges or reduces a figure by a specific scale factor while maintaining the shape's proportions. The key characteristics of dilation include:

- The shape remains similar to the original shape.
- The size changes according to the scale factor.
- The center of dilation is a fixed point in the plane.

Scale Factor in Dilation

The scale factor, often denoted as k , determines how much the figure is enlarged or reduced:

- If $k > 1$, the figure is enlarged.
- If $0 < k < 1$, the figure is reduced.
- If $k = 1$, the figure remains the same size.

In our case, the scale factor is $\frac{7}{4}$, which indicates an enlargement since the factor is greater than 1.

Understanding the Problem

The problem states:

- Original width of the logo = 15 cm
- The logo is dilated by a factor of $\frac{7}{4}$
- Find the width of the dilated logo

This is a straightforward application of dilation principles. The key is to understand how the scale factor affects the original measurement.

Calculating the New Width of the Logo

Step-by-Step Solution

To find the new width after dilation, follow these steps:

1. **Identify the original measurement:** The original width is 15 cm.
2. **Determine the scale factor:** $\left(\frac{7}{4}\right)$.
3. **Apply the scale factor:** Multiply the original width by the scale factor.

Mathematically, the new width (W_{new}) is calculated as:

$$W_{\text{new}} = W_{\text{original}} \times k$$

Where:

$$\begin{aligned} - (W_{\text{original}} &= 15 \text{ cm}) \\ - (k &= \frac{7}{4}) \end{aligned}$$

Substituting the values:

$$W_{\text{new}} = 15 \times \frac{7}{4}$$

$$W_{\text{new}} = 15 \times \frac{7}{4} = \frac{15 \times 7}{4} = \frac{105}{4}$$

Converting to decimal:

$$\frac{105}{4} = 26.25 \text{ cm}$$

Therefore, the width of the dilated logo is 26.25 centimeters.

Understanding the Significance of the Scale Factor

Impact on Dimensions

The scale factor directly influences the size of the figure:

- A scale factor of $\frac{7}{4}$ (or 1.75) means the logo's width increases by 75%.
- For other measurements, such as height or area, similar scaling applies, but area scales by the square of the scale factor.

Practical Applications

Knowing how to calculate scaled dimensions is useful in:

- Graphic design: resizing logos, images, or layouts proportionally.
- Architecture and engineering: scaling models or plans.
- Mathematics education: understanding similarity and proportional reasoning.

Related Concepts in Geometry

Similar Figures

Figures that are the same shape but different in size are called similar figures. Dilation is a transformation that creates similar figures, with corresponding sides proportional.

Scale Factor and Area

While linear dimensions (like width or height) scale directly with the scale factor, areas scale with the square of the scale factor. For example:

- If the linear scale factor is k ,
- Then the area scale factor is k^2 .

Example: Area Scaling

Suppose the original logo has an area of A_{original} . After dilation:

$$A_{\text{new}} = A_{\text{original}} \times k^2$$

If the original area was, say, 50 cm^2 , then:

$$A_{\text{new}} = 50 \times \left(\frac{7}{4}\right)^2 = 50 \times \frac{49}{16} = \frac{2450}{16} = 153.125 \text{ cm}^2$$

Visualizing the Dilated Logo

Visual aids can significantly enhance understanding:

- Diagram showing original and dilated logos side by side.
- Labels indicating the original and new widths.
- Illustration of the center of dilation, if specified.

Such visuals help in grasping the proportional change and reinforce comprehension of the concept.

Common Mistakes to Avoid

When solving dilation problems, avoid these pitfalls:

- Forgetting to multiply by the scale factor; some might mistakenly add or subtract.
- Confusing the scale factor with its reciprocal or inverse.
- Misapplying the scale factor to areas or volumes without considering the square or cube relationship.
- Neglecting to convert fractions or decimals properly.

Practice Problems for Mastery

To solidify your understanding, try solving these problems:

- **Problem 1:** The original height of a rectangle is 12 cm. If it's dilated by a factor of $\frac{3}{2}$, what is the new height?
- **Problem 2:** A circle has a radius of 8 cm. After dilation by a scale factor of 2, what is the new radius?
- **Problem 3:** The area of a square is 49 cm². If the square is scaled by a factor of $\frac{3}{7}$, what is the new area?

Answers:

1. $12 \times \frac{3}{2} = 18 \text{ cm}$
2. $8 \times 2 = 16 \text{ cm}$
3. New linear scale factor: $\frac{3}{7}$, area scale factor: $\left(\frac{3}{7}\right)^2 = \frac{9}{49}$

New area: $49 \times \frac{9}{49} = 9 \text{ cm}^2$

Summary and Key Takeaways

- Dilation is a transformation that scales figures proportionally.
- The scale factor determines how much the figure enlarges or reduces.
- To find the new measurement after dilation, multiply the original measurement by the scale factor.
- Linear measurements scale directly, while areas scale with the square of the scale factor.

- Visual aids and practice problems enhance understanding.

Final Thoughts

Understanding how to calculate the dimensions of dilated figures is fundamental in geometry and practical applications like design and engineering. In our specific example, the original logo width of 15 cm, when dilated by a factor of $\frac{7}{4}$, results in a new width of 26.25 cm. Recognizing and applying the principles of dilation ensures accurate scaling, which is crucial in various fields. Keep practicing with different figures and scale factors to deepen your comprehension and become proficient in this essential mathematical concept.

Frequently Asked Questions

What is the original width of the logo?

The original width of the logo is 15 cm.

By what factor is the logo dilated?

The logo is dilated by a factor of $\frac{7}{4}$.

How do you find the new width after dilation?

Multiply the original width by the dilation factor: new width = original width $\times$ dilation factor.

What is the calculation to find the dilated width?

New width = 15 cm $\times$ $(\frac{7}{4})$ = $(15 \times 7) / 4$ = $105 / 4$ = 26.25 cm.

What is the dilated width of the logo?

The dilated width of the logo is 26.25 cm.

If the original logo width was 15 cm, what is the size after dilation by $\frac{7}{4}$?

The size after dilation is 26.25 cm.

Why do we multiply the original width by the dilation factor?

Because dilation scales the size of the object proportionally, so the new width is the original width times the scale factor.

Can the dilation factor be less than 1? What does that mean?

Yes, if the dilation factor is less than 1, it means the logo is being reduced in size.

How would the answer change if the dilation factor was $\frac{4}{7}$ instead of $\frac{7}{4}$?

The new width would be $15 \text{ cm} \times (\frac{4}{7}) = \frac{60}{7} \approx 8.57 \text{ cm}$, indicating a reduction in size.

Is the calculation to find the new width different if the dilation is negative?

Yes, a negative dilation factor indicates a reflection and scaling; the magnitude is used to determine size, but the negative indicates a flip or reflection.

Additional Resources

A Logo Is 15 Cm Wide. If The Logo Is Dilated By A Factor Of $\frac{7}{4}$, Find The Width Of The Logo

In the realm of graphic design, branding, and advertising, logos serve as the visual cornerstone of a company's identity. Their dimensions, proportions, and scalability are critical components that influence both aesthetic appeal and functional application across various media. Today, we delve into a fundamental yet essential mathematical problem related to logo scaling, which exemplifies the importance of understanding geometric transformations in practical design scenarios.

Our focus centers on a specific question: A logo is 15 cm wide. If the logo is dilated by a factor of $\frac{7}{4}$, what is the width of the logo after dilation? This problem may seem straightforward at first glance, but it opens a window into the broader principles of similarity transformations, scale factors, and their applications in visual branding.

In this comprehensive review, we will explore the mathematical concepts underlying dilation, examine the implications of scaling in logo design, and provide a step-by-step solution to the problem. We will also discuss the significance of precise measurements in branding, the applications of dilation in digital and print media, and how understanding these principles ensures consistency across different platforms.

The Significance of Logo Dimensions in Branding

Before tackling the mathematical problem, it's important to understand why precise control over logo dimensions matters. Logos often need to be scaled to fit various mediums—business cards, billboards, websites, and merchandise—each requiring different sizes while maintaining visual integrity.

Consistency and Visual Impact

A well-scaled logo ensures that brand recognition remains consistent regardless of size. An unbalanced or improperly scaled logo can diminish brand professionalism and impact.

Practical Constraints

Designers and marketers frequently encounter situations where they must enlarge or reduce logos without distorting their proportions. This is where mathematical tools like dilation and scale factors become essential.

Understanding Geometric Dilation

What Is Dilation?

Dilation, also known as scaling, is a transformation that enlarges or reduces a figure by a specific scale factor relative to a fixed point called the center of dilation. In the case of logos, the center of dilation is typically the geometric center, and the transformation involves proportionally increasing or decreasing dimensions.

Scale Factor

The scale factor determines how much the figure is enlarged or reduced. A scale factor greater than 1 results in an enlargement, while a factor between 0 and 1 causes a reduction.

Mathematical Representation

If a figure is dilated by a scale factor (k) , then every linear measurement (L) of the original figure is multiplied by (k) :

$$L_{\text{new}} = L_{\text{original}} \times k$$

Deep Dive: Applying Dilation to the Logo

Let's analyze the specific problem:

> A logo is 15 cm wide. If the logo is dilated by a factor of $\frac{7}{4}$, find the width of the logo after dilation.

Step 1: Identify Known Values

- Original width $(W_{\text{original}} = 15, \text{cm})$
- Scale factor $(k = \frac{7}{4})$

Step 2: Apply the Dilation Formula

$$W_{\text{new}} = W_{\text{original}} \times k$$

Substituting the known values:

$$W_{\text{new}} = 15 \times \frac{7}{4}$$

Step 3: Simplify the Expression

$$W_{\text{new}} = 15 \times \frac{7}{4} = \frac{15 \times 7}{4} = \frac{105}{4}$$

Step 4: Convert to a Decimal or Mixed Number

$$W_{\text{new}} = \frac{105}{4} = 26.25, \text{ cm}$$

Thus, after dilation, the logo's width becomes 26.25 centimeters.

Broader Implications and Applications of Scaling in Logo Design

Understanding how to manipulate logo dimensions mathematically is vital for various real-world applications:

Digital Media

- Resizing logos for websites, social media profiles, and mobile applications requires precise scaling to prevent pixelation or distortion.
- Vector graphics, which are based on geometric formulas, facilitate such transformations without quality loss.

Print Media

- Large-format printing (billboards, banners) demands accurate scaling to maintain clarity and proportionality.
- Knowing the scale factor helps in preparing printable files that match the intended dimensions.

Merchandise and Packaging

- When logos are applied to products, they must be scaled appropriately, often involving proportional dilation to ensure brand consistency.

The Importance of Scale Factors in Design Accuracy

Using the correct scale factor ensures that aspects like logos, diagrams, and other graphical elements are proportionally accurate. Misapplication can lead to:

- Distorted images that harm brand perception.

- Compromised readability or visual balance.
- Increased costs due to reprinting or reworking designs.

In our problem, the scale factor $\frac{7}{4}$ represents an enlargement by 75%, a significant increase that maintains proportionality.

Additional Considerations in Logo Scaling

While the problem simplifies to linear dimensions, real-world applications often involve complex considerations:

- Aspect Ratio Preservation: Ensuring the height scales proportionally to width.
- Resolution and Pixel Density: Especially in raster images, scaling necessitates attention to resolution to prevent pixelation.
- Design Constraints: Sometimes, constraints on maximum or minimum dimensions influence scaling choices.

Summary and Final Thoughts

In conclusion, A Logo Is 15 Cm Wide. If The Logo Is Dilated By A Factor Of $\frac{7}{4}$, Find The Width Of The Logo showcases a fundamental application of geometric principles in practical design contexts. The straightforward calculation underscores the importance of understanding scale factors and dilation—skills that are essential for designers, marketers, and anyone involved in visual branding.

The calculated width of 26.25 centimeters highlights how mathematical precision translates into real-world applications, ensuring logos retain their visual integrity across different sizes and media. Mastery over these concepts not only facilitates technical accuracy but also supports the creation of consistent, professional brand imagery that resonates with audiences and upholds brand standards.

By integrating mathematical reasoning with design principles, professionals can confidently scale visual elements, ensuring their branding remains impactful, recognizable, and visually harmonious in every context.

References

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Note: The principles discussed here extend beyond simple problems, emphasizing the importance of mathematical literacy in professional design and branding.

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A Logo Is 15 Cm Wide If The Logo Is Dilated By A Factor Of 7 4 Find The Width Of The Logo

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