A Sequence Is Defined Recursively Bya1=4an=34an1What Is The Explicit Form Of This Sequence?

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Understanding recursive sequences is fundamental in advanced mathematics, particularly in areas like algebra, calculus, and discrete mathematics. When a sequence is defined recursively, each term depends on one or more previous terms, making it essential to derive an explicit formula to analyze its behavior effectively. In this article, we will explore the recursive sequence given by the initial term $a_1 = 4$ and the recurrence relation $a_n = 3 \times 4 \times a_{n-1}$. Our goal is to find the explicit formula for this sequence, providing a comprehensive explanation suitable for students, educators, and enthusiasts alike.

Understanding Recursive Sequences

Before delving into the specific sequence, it's important to understand what recursive sequences are and how they differ from explicit formulas.

What Is a Recursive Sequence?

A recursive sequence defines each term based on previous terms, often using a recurrence relation. For example:

- The Fibonacci sequence: $F_n = F_{n-1} + F_{n-2}$, with initial terms $F_0 = 0, F_1 = 1$.
- The sequence in question: $a_n = 3 \times 4 \times a_{n-1}$, with a given initial term.

The recursive approach provides a way to generate subsequent terms but doesn't immediately reveal the general pattern or formula for the nth term.

Why Find an Explicit Formula?

An explicit formula allows you to compute any term directly without computing all previous terms. This is especially useful for large n, optimizing calculations, and analyzing the sequence's long-term behavior.

Given Sequence Details

Let's carefully examine the sequence described:

$$
a_1 = 4
$$

$$
a_n = 3 \times 4 \times a_{n-1}
$$

The recurrence relation can be simplified:

$$
a_n = (3 \times 4) \times a_{n-1} = 12 \times a_{n-1}
$$

This simplifies our recursive formula to:

$$
a_n = 12 \times a_{n-1}
$$

with the initial condition $a_1 = 4$.

Deriving the Explicit Formula

To find the explicit formula for the sequence, we analyze the recurrence relation:

$$
a_n = 12 \times a_{n-1}
$$

This is a first-order linear homogeneous recurrence relation with constant coefficients, which generally has solutions of the form:

$$
a_n = k \times r^{n-1}
$$

where:

- k is a constant determined by initial conditions.
- r is the common ratio.

Step 1: Recognize the Pattern

Starting from the initial term:

$$
a_1 = 4
$$

Compute subsequent terms:

$$
a_2 = 12 \times a_1 = 12 \times 4 = 48
$$
$$
a_3 = 12 \times a_2 = 12 \times 48 = 576
$$
$$
a_4 = 12 \times a_3 = 12 \times 576 = 6912
$$

We observe that each term is obtained by multiplying the previous term by 12, indicating the sequence is geometric with ratio $r = 12$.

Step 2: Write the General Term

Since the sequence is geometric:

$$
a_n = a_1 \times r^{n-1}
$$

Substituting the known initial term and ratio:

$$
a_n = 4 \times 12^{n-1}
$$

Step 3: Final Explicit Formula

The explicit form of the sequence is:

$$
\boxed{
a_n = 4 \times 12^{n-1}
}
$$

This formula allows you to directly compute the nth term without recursive calculations.

Verification of the Explicit Formula

It's important to verify the derived formula against the initial terms:

- For $n=1$:

$$
a_1 = 4 \times 12^{0} = 4 \times 1 = 4
$$

- For $n=2$:

$$
a_2 = 4 \times 12^{1} = 4 \times 12 = 48
$$

- For $n=3$:

$$
a_3 = 4 \times 12^{2} = 4 \times 144 = 576
$$

- For $n=4$:

$$
a_4 = 4 \times 12^{3} = 4 \times 1728 = 6912
$$

All computed terms match the recursive sequence, confirming the correctness of the explicit formula.

Applications and Significance of the Sequence

Formula

The explicit formula $a_n = 4 \times 12^{n-1}$ has various applications:

- Efficient calculation of terms: Quickly compute large n terms without iterative calculations.
- Growth rate analysis: The exponential nature indicates rapid growth, which can model phenomena like population growth, financial investments, or radioactive decay in reverse.
- Mathematical modeling: Used in computer science algorithms, such as analyzing recursive functions or iterative processes with exponential complexity.

Summary of Key Points

To encapsulate the essential aspects of deriving the explicit formula for the recursive sequence:

1. The sequence starts with $a_1 = 4$.
2. The recurrence relation simplifies to $a_n = 12 \times a_{n-1}$.
3. This is a geometric sequence with common ratio $r=12$.
4. The explicit formula is $a_n = 4 \times 12^{n-1}$.
5. This formula enables direct computation of any term in the sequence.

Additional Tips for Working with Recursive Sequences

- Always verify the recurrence relation and initial conditions before deriving the explicit formula.
- Recognize geometric sequences by the form of the recurrence relation.
- Use algebraic manipulation to simplify recurrence relations when possible.
- Confirm the explicit formula by plugging in initial terms and comparing with recursive calculations.

Conclusion

In conclusion, the recursive sequence defined by $a_1=4$ and $a_n=3 \times 4 \times a_{n-1}$ simplifies to a geometric sequence with ratio 12. Its explicit formula $a_n=4 \times 12^{n-1}$ provides an efficient way to compute any term directly. Understanding how to derive explicit formulas from recursive relations is a crucial skill in mathematics, enabling deeper insights into the behavior of sequences and their applications across various scientific fields. By mastering these techniques, students and professionals can better analyze complex systems exhibiting exponential growth or decay, enhancing their mathematical toolkit for tackling real-world problems.

Keywords: recursive sequence, explicit formula, geometric sequence, recurrence relation, sequence analysis, mathematical sequences, exponential growth, sequence derivation, algebra, discrete mathematics

Frequently Asked Questions

What is the recursive formula for the sequence given as $a_1 = 4$ and $a_n = 3a_{n-1}$?

The recursive formula is $a_1 = 4$ and $a_n = 3a_{n-1}$ for $n \geq 2$.

How do you find the explicit formula of a sequence defined recursively by $a_1 = 4$ and $a_n = 3a_{n-1}$?

Since the recurrence is geometric, the explicit formula is $a_n = a_1 r^{(n-1)}$, where r is the common ratio. Here, $a_1 = 4$ and $r = 3$, so $a_n = 4 3^{n-1}$.

What is the explicit formula for the sequence with $a_1 = 4$ and $a_n = 3a_{n-1}$?

The explicit formula is $a_n = 4 3^{n-1}$.

How can I verify the explicit formula for this recursive sequence?

You can verify by plugging in values of n into the explicit formula and checking if it matches the recursive sequence, e.g., for n=2, $a_2=43^{2-1}=43=12$, which matches the recursive calculation $a_2=3a_1=34=12$.

What is the growth pattern of the sequence defined by $a_1=4$ and $a_n=3a_{n-1}$?

The sequence grows exponentially with a common ratio of 3, meaning each term is three times the previous term.

Can you explain why the sequence is geometric based on its recursive formula?

Yes, because each term is obtained by multiplying the previous term by a constant ratio (3), which is characteristic of geometric sequences.

What is the value of the 5th term in the sequence $a_1=4$, $a_n=3a_{n-1}$?

Using the explicit formula a_n=43^{n-1}, the 5th term is a_5=43^{4}=481=324.

How is the explicit formula related to the recursive formula in this sequence?

The explicit formula provides a direct way to compute any term without recursion, derived from the recursive relation by recognizing it as a geometric sequence.

What is the general method to convert a recursive geometric sequence to an explicit formula?

Identify the initial term a_1 and the common ratio r; then the explicit formula is a_n= a_1 r^{n-1}.

If $a_1=4$ and $a_n=3a_{n-1}$, what is the explicit formula and the value of the 10th term?

The explicit formula is a_n=43^{n-1}. The 10th term is a_{10}=43^{9}=419683=78732.

Additional Resources

Recursion

Understanding the Sequence: An In-Depth Exploration

When delving into the fascinating world of sequences and series, one encounters a variety of definitions and representations. Among these, recursive sequences stand out due to their elegant and often intricate structure. Today, we explore a particular recursive sequence defined as:

$$
a_1 = 4,\quad a_{n} = 3a_{n-1}
$$

Our goal is to determine the explicit form of this sequence, transforming its recursive definition into a closed-form expression that allows us to compute any term directly without referencing previous terms. Let's embark on this mathematical journey step-by-step, dissecting the sequence's properties and uncovering its explicit formula.

The Foundations: What Is a Recursive Sequence?

Before tackling the specific sequence at hand, it's essential to understand what recursive sequences entail.

Definition of a Recursive Sequence

A recursive sequence is a sequence where each term is defined in terms of one or more preceding terms. Typically, it involves:

- An initial term (or terms), serving as the starting point.
- A recurrence relation, describing how to generate subsequent terms from earlier ones.

Example:

$$
a_1 = 4,\quad a_{n} = 3a_{n-1}
$$

Here, the sequence begins with $a_1 = 4$, and each subsequent term is obtained by multiplying the previous term by 3.

Dissecting the Recursive Definition

Let's analyze the recursive formula:

$$
a_1 = 4
$$
$$
a_{n} = 3a_{n-1} \quad \text{for } n > 1
$$

This is a classic geometric recurrence relation, where each term is a constant multiple of its predecessor.

Key observations:

- The sequence is geometric because each term is obtained by multiplying the previous one by a fixed ratio.
- The initial term is 4, which sets the sequence's starting point.

From Recursion to Explicit Formula: The Mathematical Challenge

While recursive definitions are straightforward to understand, they are often cumbersome for calculating large n-th terms directly. An explicit (or closed-form) formula provides a way to compute any term without iterative calculations.

Why Seek an Explicit Formula?

- Efficiency: Direct computation avoids iterative steps.
- Clarity: It reveals underlying patterns and growth behavior.
- Analytical tools: Facilitates summing the sequence, finding limits, and analyzing properties.

Deriving the Explicit Formula: Step-by-Step

Let's now derive the explicit formula for the sequence defined by:

$$
a_1 = 4,\quad a_{n} = 3a_{n-1}
$$

Step 1: Recognize the Pattern

Given the recurrence relation:

$$
a_n = 3a_{n-1}
$$

we see that each term is obtained by multiplying the previous term by 3. Starting from a_1:

$$
a_2 = 3a_1 = 3 \times 4
$$
$$
a_3 = 3a_2 = 3 \times (3 \times 4) = 3^2 \times 4
$$

\]
\[
a_4 = 3a_3 = 3 \times (3^2 \times 4) = 3^3 \times 4
\]

This pattern suggests:

\[
a_n = 3^{n-1} \times 4
\]

Step 2: Formalize the Pattern

By induction or pattern recognition, the explicit formula is:

\[
\boxed{
a_n = 4 \times 3^{n-1}
}
\]

This formula allows us to compute any term directly, given its position \(n
\).

Verifying the Explicit Formula

It's essential to validate the derived formula to ensure correctness.

Example:

Calculate \(a_5 \):

\[
a_5 = 4 \times 3^{5-1} = 4 \times 3^4 = 4 \times 81 = 324
\]

Check via recursion:

\[
a_1 = 4
\]
\[
a_2 = 3 \times 4 = 12
\]
\[
a_3 = 3 \times 12 = 36
\]
\[
a_4 = 3 \times 36 = 108

$$
a_5 = 3 \times 108 = 324
$$

The explicit formula matches the recursive calculation, confirming its correctness.

Exploring the Sequence's Properties

Having a closed-form expression opens avenues to analyze the sequence's behavior and properties.

Growth Rate

Since $a_n = 4 \times 3^{n-1}$, the sequence grows exponentially with base 3. As $n \to \infty$, $a_n \to \infty$ rapidly.

Sum of the First n Terms

The sum S_n of the first n terms of a geometric sequence is given by:

$$
S_n = a_1 \times \frac{r^n - 1}{r - 1}
$$

where:

- $a_1 = 4$
- $r = 3$

Therefore,

$$
S_n = 4 \times \frac{3^n - 1}{3 - 1} = 4 \times \frac{3^n - 1}{2} = 2 \times (3^n - 1)
$$

This formula allows us to compute the sum efficiently.

Practical Applications and Relevance

Sequences defined recursively and their explicit forms are not just abstract constructs—they have real-world applications across various fields:

- Computer Science: Modeling exponential growth in algorithms or population

simulations.
- Economics: Compound interest calculations and investment growth.
- Physics: Radioactive decay or population dynamics.

Understanding how to convert recursive definitions into explicit formulas enhances analytical capabilities, streamlines computations, and deepens comprehension of growth patterns.

Summary and Key Takeaways

Aspect	Explanation
Recursive Definition	$a_1 = 4$, $a_n = 3a_{n-1}$
Nature of Sequence	Geometric sequence with ratio $r = 3$
Explicit Formula	$a_n = 4 \times 3^{n-1}$
Validation	Matches recursive calculations for multiple n
Growth Behavior	Exponential growth, rapidly increasing terms

Final Remarks: Mastering Recursion and Explicit Forms

Transforming recursive sequences into explicit formulas is a fundamental skill in mathematics and related disciplines. It simplifies analysis, facilitates calculations, and reveals the inherent structure of sequences. The sequence examined here exemplifies a classic geometric progression, and understanding its explicit form provides insights into exponential growth phenomena.

Whether you're analyzing algorithms, modeling natural processes, or exploring mathematical theory, mastering the art of converting recursion to explicit expressions unlocks new levels of understanding and problem-solving prowess.

In essence, the sequence defined by $a_1 = 4$ and $a_n = 3a_{n-1}$ is explicitly expressed as:

$$
\boxed{
a_n = 4 \times 3^{n-1}
}
$$

This formula stands as a testament to the power of pattern recognition and mathematical reasoning, transforming a simple recursive rule into a direct, elegant expression—an essential tool in the mathematician's toolkit.

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