

# Consider The Three Points: A=(9,2) B=(2,1) C=(4,9). Determine The Angle Between Ab And Ac.

**Consider The Three Points: A=(9,2) B=(2,1) C=(4,9). Determine The Angle Between AB And AC.**

Understanding how to calculate the angle between two vectors originating from the same point is a fundamental concept in coordinate geometry. Given points A, B, and C with their respective coordinates, this problem involves finding the angle between vectors AB and AC, which originate from point A. This process not only enhances your grasp of vector operations but also deepens your understanding of geometric relationships within the coordinate plane. In this comprehensive guide, we will walk through the steps to determine the angle between these two vectors, supported by detailed explanations and calculations.

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## Understanding the Problem: Points and Vectors

Before diving into calculations, it's essential to understand the problem's components:

- **Points:** A=(9, 2), B=(2, 1), C=(4, 9)
- **Vectors:** AB and AC
- **Goal:** Find the angle between AB and AC

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## Step 1: Find the Vectors AB and AC

Vectors are directed line segments that have both magnitude and direction. To find the vector from point A to point B (denoted as AB), subtract the coordinates of A from B:

1. **Calculate vector AB:**

$$\vec{AB} = \vec{B} - \vec{A} = (x_B - x_A, y_B - y_A)$$

\]

\[

$$\vec{AB} = (2 - 9, 1 - 2) = (-7, -1)$$

\]

## 2. Calculate vector AC:

\[

$$\vec{AC} = \vec{C} - \vec{A} = (x_C - x_A, y_C - y_A)$$

\]

\[

$$\vec{AC} = (4 - 9, 9 - 2) = (-5, 7)$$

\]

Summary:

- $AB = (-7, -1)$
- $AC = (-5, 7)$

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## Step 2: Calculate the Dot Product of Vectors AB and AC

The dot product (also called the scalar product) of two vectors  $u = (u_1, u_2)$  and  $v = (v_1, v_2)$  is given by:

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$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

\]

Applying this to vectors AB and AC:

\[

$$\vec{AB} \cdot \vec{AC} = (-7)(-5) + (-1)(7) = 35 - 7 = 28$$

\]

Result:

\[

$$\boxed{\vec{AB} \cdot \vec{AC} = 28}$$

\]

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## Step 3: Find the Magnitudes of Vectors AB and AC

The magnitude (or length) of a vector  $u = (u_1, u_2)$  is:

\[

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2}$$

\]

Calculate for AB:

\[

$$|\vec{AB}| = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50} \approx 7.07$$

\]

Calculate for AC:

\[

$$|\vec{AC}| = \sqrt{(-5)^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.60$$

\]

Summary:

- $|\vec{AB}| \approx 7.07$
- $|\vec{AC}| \approx 8.60$

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## Step 4: Use the Dot Product Formula to Find the Angle

The angle  $\theta$  between vectors AB and AC can be found using the dot product formula:

\[

$$\cos \theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \times |\vec{AC}|}$$

\]

Substituting the known values:

\[

$$\cos \theta = \frac{28}{7.07 \times 8.60} = \frac{28}{60.862} \approx 0.460$$

\]

To find the angle:

\[

$$\theta = \cos^{-1}(0.460)$$

\]

Using a calculator:

\[

$$\theta \approx 62.4^\circ$$

\]

Final answer:

\[

\boxed{

\text{The angle between } \vec{AB} \text{ and } \vec{AC} \text{ is approximately } 62.4^\circ

}

\]

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## Additional Insights: Why Is This Important?

Understanding the angle between two vectors originating from a common point is crucial in various fields such as:

- **Geometry and Trigonometry:** Solving problems involving angles and distances.
- **Physics:** Analyzing forces and directions.
- **Computer Graphics:** Calculating angles for rendering and modeling.
- **Robotics and Engineering:** Determining joint angles and orientations.

Mastering this calculation enhances spatial reasoning and problem-solving skills.

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## Common Mistakes to Avoid

When calculating the angle between vectors, ensure:

- **Correct vector subtraction:** Always subtract the coordinates of the starting point from the ending point (A to B, A to C).
- **Accurate calculations:** Keep track of signs and use a calculator for square roots and inverse cosine.
- **Unit consistency:** Use the same units and double-check your calculations for rounding errors.

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## Conclusion

By following a systematic approach—finding vectors, calculating their dot product and magnitudes, and applying the cosine formula—you can accurately determine the angle between two vectors in a coordinate plane. For the specific points  $A=(9, 2)$ ,  $B=(2, 1)$ , and  $C=(4, 9)$ , the angle between vectors  $AB$  and  $AC$  is approximately 62.4 degrees. This process underscores the importance of vector operations in geometry and their applications across diverse fields. With practice, solving such problems becomes more intuitive, equipping you with essential mathematical tools for advanced studies and professional work.

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## Further Practice and Resources

To deepen your understanding, consider practicing with different points and vectors:

- Calculate the angle between other pairs of vectors in the coordinate plane.
- Explore how changing points affects the angle.
- Use graphing tools or software to visualize vectors and angles.

Recommended resources:

- Online vector calculators for quick computations
- Geometric visualization tools like GeoGebra
- Textbooks on coordinate geometry and vector algebra

Mastering the calculation of angles between vectors enhances your mathematical literacy and problem-solving capabilities, essential for advanced mathematics, physics, engineering, and computer science.

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Remember: Accuracy and clarity in each step are key to solving vector angle problems effectively. Keep practicing, and you'll develop a strong intuition for spatial relationships in the coordinate plane!

## Frequently Asked Questions

### How do I find the angle between vectors AB and AC given points A(9,2), B(2,1), and C(4,9)?

First, find the vectors AB and AC by subtracting the coordinates:  $AB = B - A = (-7, -1)$  and  $AC = C - A = (-5, 7)$ . Then, use the dot product formula to find the angle  $\theta$ :  $\cos \theta = (AB \cdot AC) / (|AB| |AC|)$ . Calculate the dot product and magnitudes, then find  $\theta$  using arccos.

### What is the step-by-step method to compute the angle between vectors AB and AC from the given points?

1. Calculate vectors  $AB = B - A = (-7, -1)$  and  $AC = C - A = (-5, 7)$ . 2. Compute the dot product:  $AB \cdot AC = (-7)(-5) + (-1)(7) = 35 - 7 = 28$ . 3. Find magnitudes:  $|AB| = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50}$ ,  $|AC| = \sqrt{(-5)^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74}$ . 4. Use the formula  $\cos \theta = (28) / (\sqrt{50} \sqrt{74})$ . 5. Calculate  $\theta = \arccos$  of that value.

### What is the numerical value of the angle between vectors AB and AC for the given points?

Calculating step-by-step:  $AB \cdot AC = 28$ ,  $|AB| \approx 7.07$ ,  $|AC| \approx 8.60$ . Then,  $\cos \theta \approx 28 / (7.07 \cdot 8.60) \approx 28 / 60.83 \approx 0.460$ . Therefore,  $\theta \approx \arccos(0.460) \approx 62.4$  degrees.

### Can I use a calculator to find the angle between two vectors in coordinate geometry? How?

Yes. After calculating the dot product and magnitudes of the vectors, use a scientific calculator to compute the arccosine of their ratio ( $\cos \theta$ ). This gives the angle in degrees or radians, depending on your calculator settings.

### Why is it important to find the angle between vectors AB and AC in coordinate geometry problems?

Finding the angle helps to understand the spatial relationship between points, such as determining if points form specific angles, solving geometric problems involving triangles, or analyzing the

orientation of vectors in space, which is crucial in fields like physics, engineering, and computer graphics.

## Additional Resources

Consider The Three Points:  $A=(9,2)$   $B=(2,1)$   $C=(4,9)$ . Determine The Angle Between AB And AC.

Understanding the spatial relationships between points in a coordinate plane is fundamental in geometry, with applications spanning architecture, engineering, computer graphics, and more. In this article, we delve into a classic geometric problem: given three points  $A$ ,  $B$ , and  $C$ , how do we determine the angle formed at point  $A$  by the lines  $AB$  and  $AC$ ? Specifically, with the points  $A=(9,2)$ ,  $B=(2,1)$ , and  $C=(4,9)$ , our goal is to find the measure of the angle between the vectors  $\vec{AB}$  and  $\vec{AC}$ .

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### Understanding the Problem: Points and Vectors

At the outset, it's crucial to understand what the problem entails. The points are given in a two-dimensional coordinate plane:

- Point  $A$  at  $(9, 2)$
- Point  $B$  at  $(2, 1)$
- Point  $C$  at  $(4, 9)$

The core task involves calculating the angle  $\theta$  between two vectors originating from  $A$ :

- $\vec{AB}$ , the vector from  $A$  to  $B$
- $\vec{AC}$ , the vector from  $A$  to  $C$

This is a common problem in vector geometry, and solving it requires a clear understanding of vector operations, especially the dot product and how it relates to angles.

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### Step 1: Constructing Vectors $\vec{AB}$ and $\vec{AC}$

The first step is to compute the vectors  $\vec{AB}$  and  $\vec{AC}$ . Recall that for points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the vector from the first to the second is:

$$\vec{XY} = (x_2 - x_1, y_2 - y_1)$$

Applying this to our points:

- $\vec{AB} = (x_B - x_A, y_B - y_A) = (2 - 9, 1 - 2) = (-7, -1)$
- $\vec{AC} = (x_C - x_A, y_C - y_A) = (4 - 9, 9 - 2) = (-5, 7)$

These vectors encode the directions from point  $A$  to points  $B$  and  $C$ , respectively.

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## Step 2: The Dot Product and Its Role in Finding Angles

The key to finding the angle between two vectors is the dot product, a fundamental operation in vector algebra. For any two vectors  $\vec{u} = (u_x, u_y)$  and  $\vec{v} = (v_x, v_y)$ , the dot product is:

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y$$

The significance of the dot product lies in its relation to the angle  $\theta$  between  $\vec{u}$  and  $\vec{v}$ :

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

where:

- $|\vec{u}|$  and  $|\vec{v}|$  are the magnitudes (lengths) of the vectors
- $\theta$  is the angle between them

Rearranged, to find  $\theta$ :

$$\theta = \arccos \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right)$$

Applying this to our vectors  $\vec{AB}$  and  $\vec{AC}$ , we proceed to compute their dot product and magnitudes.

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## Step 3: Calculating the Dot Product

Using the vectors:

- $\vec{AB} = (-7, -1)$
- $\vec{AC} = (-5, 7)$

The dot product is:

$$\vec{AB} \cdot \vec{AC} = (-7)(-5) + (-1)(7) = 35 - 7 = 28$$

This scalar value reflects the degree to which the vectors point in similar directions—the larger the value, the smaller the angle (closer to  $0^\circ$ ).

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Step 4: Determining the Magnitudes of  $\vec{AB}$  and  $\vec{AC}$

Next, we compute the lengths of the vectors:

$$|\vec{AB}| = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49 + 1} = \sqrt{50} \approx 7.07$$

$$|\vec{AC}| = \sqrt{(-5)^2 + 7^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.60$$

These lengths are vital for normalizing the dot product and accurately calculating the angle.

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Step 5: Computing the Angle  $\theta$

Now, plugging the dot product and magnitudes into the formula:

$$\cos \theta = \frac{28}{7.07 \times 8.60} \approx \frac{28}{60.82} \approx 0.460$$

Finally, calculating  $\theta$ :

$$\theta = \arccos(0.460) \approx 62.4^\circ$$

Thus, the angle between the vectors  $\vec{AB}$  and  $\vec{AC}$  — and consequently between lines  $\vec{AB}$  and  $\vec{AC}$  originating from point  $A$  — is approximately 62.4 degrees.

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## Practical Significance and Applications

Understanding how to compute such an angle has broad practical implications. For instance, in computer graphics, determining the angle between vectors helps in shading, lighting calculations, and object orientation. Engineers rely on these calculations to design structures with precise angular relationships. Even in navigation systems, understanding the angles between vectors aids in path planning and orientation.

## Additional Considerations: Error Margins and Precision

While the above calculations give a good approximation, real-world applications often demand higher precision. Using software tools like graphing calculators, computer algebra systems, or programming languages with mathematical libraries (e.g., Python with NumPy) ensures accuracy. Additionally, when working with real data, accounting for measurement errors and uncertainties

becomes crucial.

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### Conclusion: A Synthesis of Geometry and Algebra

By translating geometric problems into algebraic operations—constructing vectors, calculating dot products, and applying inverse cosine functions—we can accurately determine angles in the coordinate plane. The problem involving points  $A=(9,2)$ ,  $B=(2,1)$ , and  $C=(4,9)$  exemplifies this process, culminating in an approximate angle of 62.4 degrees between the lines  $AB$  and  $AC$  at point  $A$ .

This methodological approach underscores the power of vector algebra in solving geometric problems, bridging the gap between abstract mathematics and real-world applications. Whether designing complex structures, programming animations, or analyzing spatial data, understanding how to determine angles between points remains a fundamental skill in both academic and professional contexts.

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## **Consider The Three Points A 9 2 B 2 1 C 4 9 Determine The Angle Between Ab And Ac**

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