Construct A 3 3 Nonzero Matrix A Such That The Vector 2 4 1 2 1 3 5 Is A Solution Of Ax D 0.

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Introduction

In linear algebra, understanding how matrices and vectors interact is fundamental. One common problem involves constructing a matrix with specific properties so that a given vector is a solution to the corresponding matrix equation. Specifically, this article focuses on constructing a nonzero 3×3 matrix A such that the vector $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 2 \\ 1 \\ 3 \\ 5 \end{bmatrix}$ satisfies the equation $A \mathbf{x} = \mathbf{0}$.

However, note that the vector provided appears to be a 7-dimensional vector, which does not conform to the standard 3×3 matrix multiplication context, as the matrix A must be compatible with the vector's dimensions to produce a zero vector. Therefore, before proceeding, we clarify the problem, interpret the vector as multiple vectors, or adjust the scenario accordingly.

This article will explore the interpretation, necessary conditions, and construction strategies for the matrix A, ensuring it is nonzero and that the given vector (or vectors) satisfy $A \mathbf{x} = \mathbf{0}$.

Understanding the Problem and Its Dimensions

Clarifying the Vector and Matrix Dimensions

The problem states: "Construct a 3×3 nonzero matrix A such that the vector $\begin{bmatrix} 2 \\ 4 \\ 1 \\ 2 \\ 1 \\ 3 \\ 5 \end{bmatrix}$ is a solution of $A \mathbf{x} = 0$."

- The matrix A is 3×3, so it acts on vectors in $\mathbb{R}^3$.
- The provided vector has 7 entries, which suggests a mismatch since $A \mathbf{x}$ for $A \in \mathbb{R}^{3 \times 3}$ must produce a vector in $\mathbb{R}^3$.

Possible interpretations:

1. The vector is meant to be a concatenation of multiple vectors: perhaps the problem is asking about multiple vectors, or perhaps, there's a typo.

2. The intention is to find a matrix A such that the components of the vector satisfy some relation, or perhaps each segment is considered

separately.

3. The problem is to construct a matrix A such that the given vector $\mathbf{x}$ (or a related vector) is in the null space of A, i.e., $A \mathbf{x} = \mathbf{0}$.

Given the standard matrix dimensions, it's most logical to interpret the vector as a 3-dimensional vector, perhaps the first three entries: $(2, 4, 1)$, or to consider the vector as multiple vectors, each of dimension 3.

Assumption:

Suppose the vector is actually a set of three 3-dimensional vectors:

$$
\mathbf{x}_1 = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}, \quad
\mathbf{x}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad
\mathbf{x}_3 = \begin{bmatrix} 5 \\ \text{(additional data?)} \end{bmatrix}
$$

But since the vector has 7 entries, perhaps the problem is about constructing a matrix that annihilates the entire vector, or perhaps the vector is simply misrepresented.

In conclusion, for clarity, the most straightforward interpretation is:

- The vector $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, a 3D vector.
- The goal is to construct a nonzero 3×3 matrix A such that $A \mathbf{x} = \mathbf{0}$.

Hence, the problem reduces to constructing a nonzero matrix A with $A \mathbf{x} = 0$, where $\mathbf{x} = (2, 4, 1)^T$.

This interpretation aligns with typical linear algebra problems and allows us to proceed with the construction.

Constructing the Matrix A

Key Concept: Null Space and Its Relation to A

To find a matrix A such that $A \mathbf{x} = 0$, we need to understand the null space (kernel) of A. The null space consists of all vectors that A sends to zero.

- Since A is 3×3, it is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.
- For A to be nonzero and satisfy $A \mathbf{x} = 0$, $\mathbf{x}$ must be in the null space of A.

Therefore:

- The null space of A must contain $\mathbf{x} = (2, 4, 1)^T$.
- A can be constructed as any matrix whose null space includes $\mathbf{x}$, but A must be nonzero.

General approach:

1. Identify a basis for the null space that includes $\mathbf{x}$.
2. Construct A such that its null space is exactly the subspace orthogonal to the row space of A.

Step-by-Step Construction Strategy

Step 1: Find a basis for the null space

Since $\mathbf{x} = (2, 4, 1)^T$, any matrix A with null space containing $\mathbf{x}$ must satisfy:

$$
A \mathbf{x} = \mathbf{0}
$$

Step 2: Choose A as a matrix whose rows are orthogonal to $\mathbf{x}$.

- The rows of A must be orthogonal to $\mathbf{x}$, i.e., each row $\mathbf{r}_i$ satisfies:

$$
\mathbf{r}_i \cdot \mathbf{x} = 0
$$

- Because A is 3×3, it has 3 rows, each orthogonal to $\mathbf{x}$.

Step 3: Find the set of vectors orthogonal to $\mathbf{x}$.

To find vectors orthogonal to $\mathbf{x} = (2, 4, 1)$:

Solve:

$$
2a + 4b + 1c = 0
$$

for (a, b, c).

- Pick two linearly independent solutions to get a basis for the orthogonal subspace.

Example:

Let’s choose values for a and b, then compute c:

- For $a = 1$, $b = 0$:

$$
2(1) + 4(0) + c = 0 \Rightarrow c = -2
$$

- For $a = 0$, $b = 1$:

$$
2(0) + 4(1) + c = 0 \Rightarrow c = -4
$$

Thus, two basis vectors orthogonal to $\mathbf{x}$:

$$
\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix},
\quad
\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}
$$

Step 4: Construct A with rows $\mathbf{v}_1$, $\mathbf{v}_2$, and a third vector orthogonal to both (or simply repeat a linear combination).

- To ensure A is nonzero and has rank 3, choose three linearly independent vectors orthogonal to $\mathbf{x}$. But we have only two; the third row can be any vector not in the span of these two but orthogonal to $\mathbf{x}$, or we can include the third vector as a linear combination.

Alternatively, construct A as:

$$
A = \begin{bmatrix}
1 & 0 & -2 \\
0 & 1 & -4 \\
p & q & r
\end{bmatrix}
$$

with (p, q, r) chosen such that A is invertible (nonzero) and the last row is orthogonal to $\mathbf{x}$:

$$
2p + 4q + r = 0
$$

Choose convenient values, for example:

- $p = 0$, $q = 0$, then $r = 0$. But this would make the third

Frequently Asked Questions

What does it mean for a vector to be a solution to the matrix equation Ax = 0?

It means that when the matrix A multiplies the vector x, the result is the zero vector, indicating that x is in the null space of A.

How can I construct a 3x3 nonzero matrix A such that a given vector is its solution to Ax = 0?

You can choose matrix A so that its rows are orthogonal to the given vector, ensuring that A times the vector results in the zero vector. This involves solving for A's rows to satisfy the dot product with the vector being zero.

Given the vector [2, 4, 1, 2, 1, 3, 5], how do I interpret it as a solution to Ax = 0?

Since the vector has 7 elements, it cannot directly be the solution to a 3x3 matrix equation. Instead, it might be a concatenation of multiple vectors or a typo. Ensure the vector is 3-dimensional to serve as a solution for a 3x3 matrix.

What are the steps to find a matrix A such that Ax = 0 for a specific vector x?

First, ensure x is in the null space of A. Then, construct A with rows orthogonal to x. For example, choose two vectors orthogonal to x to form the rows of A, and fill in the third row to make a nonzero matrix.

Can the vector [2, 4, 1] be a solution to Ax = 0 for some nonzero 3x3 matrix A?

Yes. Any nonzero vector in the null space of A can serve as a solution, provided the rows of A are orthogonal to the vector. For example, if A's rows are orthogonal to [2, 4, 1], then Ax = 0.

How do I verify if a constructed matrix A satisfies the condition Ax = 0 for a given vector?

Multiply the matrix A by the vector x. If the result is the zero vector, then x is indeed a solution to Ax = 0 for that matrix.

Is it possible to have multiple different matrices A that make a given nonzero vector a solution to Ax = 0?

Yes. Since the null space can have many different bases, there are infinitely many matrices with different rows orthogonal to the vector, all satisfying Ax = 0.

What constraints must the entries of matrix A satisfy to ensure a specific vector is in its null space?

The entries of A must be chosen so that each row vector of A is orthogonal (dot product zero) to the given vector. This ensures that multiplying A by the vector yields the zero vector.

Can I construct a matrix A with random entries that satisfies Ax = 0 for a given vector?

Yes. By ensuring each row of A is orthogonal to the given vector, you can randomly assign the entries of A's rows under the orthogonality constraint to satisfy Ax = 0.

Additional Resources

Constructing a 3×3 Nonzero Matrix A Given a Solution Vector for the Equation Ax = 0

When delving into linear algebra, one fundamental concept is understanding the relationship between matrices, vectors, and systems of equations. In particular, the problem of constructing a matrix \(A\) with specific properties, such as being nonzero and having a particular vector as a solution to the homogeneous system \(A x = 0\), is both educational and illustrative of core linear algebra principles. Here, we explore how to construct such a matrix \(A\), given a specific vector, and analyze the implications and techniques involved.

Introduction to the Problem

The problem at hand is:

> Construct a 3×3 nonzero matrix \(A\) such that the vector \(\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \\ 2 \\ 1 \\ 3 \\ 5 \end{bmatrix}\) is a solution of \(A \mathbf{x} = 0\).

At first glance, there are several issues to clarify:

1. Dimensionality Mismatch: The vector provided has 7 entries, but the matrix \(A\) is specified as 3×3, meaning it should multiply vectors in \(\mathbb{R}^3\). Since the vector \(\mathbf{x}\) has 7 components, it cannot directly be multiplied by a 3×3 matrix.

2. Possible Typo or Misinterpretation: It is plausible that the vector was intended to be a 3-dimensional vector, and perhaps a formatting issue occurred. Alternatively, the vector might be a concatenation or a typo.

Given the context and typical linear algebra exercises, it is reasonable to assume the vector intended is a 3-dimensional vector, possibly:

\[
\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}
\]

or similar.

Assumption for Clarity: For the purpose of this comprehensive exploration, we will assume the vector is:

\[

\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}
\]

and the goal is to construct a nonzero 3×3 matrix \(A\) such that:

\[
A \mathbf{x} = \mathbf{0}
\]

This aligns with typical problems in linear algebra involving the null space and kernel of matrices.

Understanding the Null Space and Its Connection to the Problem

Before constructing the matrix \(A\), it is essential to understand the null space (kernel) of a matrix:

- The null space of an \(m \times n\) matrix \(A\), denoted as \(\mathcal{N}(A)\), is the set of all vectors \(x \in \mathbb{R}^n\) such that \(A x = 0\).

- When a vector \(\mathbf{x}\) satisfies \(A \mathbf{x} = 0\), it belongs to the null space of \(A\).

- Designing a matrix \(A\) with specific null space vectors involves ensuring that those vectors are in the kernel of \(A\).

In this problem, since \(\mathbf{x}\) is a known solution, any such matrix \(A\) must have \(\mathbf{x}\) in its null space.

Constructing a 3×3 Nonzero Matrix \(A\) with a Given Null Vector

Given a vector \(\mathbf{x}\), the general approach to constructing matrices \(A\) such that \(A \mathbf{x} = 0\) involves understanding the null space's structure.

Step 1: Confirm the Vector is Nonzero

To ensure \(A\) is nonzero, the null space must be a subspace of \(\mathbb{R}^3\) with dimension at least 1, but \(A\) itself should not be the zero matrix.

Since \(\mathbf{x}\neq 0\), and \(A \mathbf{x} = 0\), the null space contains at least \(\mathbf{x}\).

Step 2: Find the Null Space

The null space is a subspace of $\mathbb{R}^3$. To construct A, we need the null space to include $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$.

This means:

$$
A \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = 0
$$

which implies that the rows of A are orthogonal to $\mathbf{x}$, as the matrix-vector multiplication is a dot product of each row with $\mathbf{x}$.

Step 3: Construct Rows of A

Recall that:

$$
A = \begin{bmatrix}
\text{row}_1 \\
\text{row}_2 \\
\text{row}_3
\end{bmatrix}
$$

and each row must satisfy:

$$
\text{row}_i \cdot \mathbf{x} = 0
$$

for $i = 1, 2, 3$.

This condition ensures that:

$$
A \mathbf{x} = \mathbf{0}
$$

Step 4: Find Vectors Orthogonal to $\mathbf{x}$

To construct such rows, find vectors orthogonal to $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$.

Set:

$$
\text{row} = \begin{bmatrix} a & b & c \end{bmatrix}
$$

and impose:

$$
2a + 4b + c = 0
$$

This is a linear equation in three variables, with infinitely many solutions.

Explicit Construction of the Matrix A

To generate specific rows, select arbitrary values for two variables and solve for the third.

Example 1: Using Parametric Forms

- Let's choose:

- $a = 1$

- $b = 0$

- Then:

$$
2(1) + 4(0) + c = 0 \Rightarrow c = -2
$$

- First row:

$$
\text{row}_1 = \begin{bmatrix} 1 & 0 & -2 \end{bmatrix}
$$

- Choose a second row with different parameters:

- $a = 0$

- $b = 1$

- Then:

$$
2(0) + 4(1) + c = 0 \Rightarrow c = -4
$$

- Second row:

$$
\text{row}_2 = \begin{bmatrix} 0 & 1 & -4 \end{bmatrix}
$$

- For the third row, pick:

- $a = 1$

- $b = 1$

- Then:

$$
2(1) + 4(1) + c = 0 \Rightarrow c = -6
$$

- Third row:

$$
\text{row}_3 = \begin{bmatrix} 1 & 1 & -6 \end{bmatrix}
$$

Constructed Matrix A:

$$
A = \begin{bmatrix}
1 & 0 & -2 \\
0 & 1 & -4 \\
1 & 1 & -6
\end{bmatrix}
$$

This matrix is nonzero and satisfies:

$$
A \mathbf{x} = \mathbf{0}
$$

since each row dot product with $\mathbf{x}$ equals zero.

Verifying the Construction

Let’s verify explicitly:

$$
A \mathbf{x} = \begin{bmatrix}
1 & 0 & -2 \\
0 & 1 & -4 \\
1 & 1 & -6
\end{bmatrix}
\begin{bmatrix}
2 \\ 4 \\ 1
\end{bmatrix}
$$

Calculations:

- Row 1: $1 \times 2 + 0 \times 4 + (-2) \times 1 = 2 - 2 = 0$

- Row 2: $0 \times 2 + 1 \times 4 + (-4) \times 1 = 4 - 4 = 0$

- Row 3: $1 \times 2 + 1 \times 4 + (-6) \times 1 = 2 + 4 - 6 = 0$

Confirmed! The matrix A annihilates $\mathbf{x}$.

Properties of the Constructed Matrix \(A\)

The matrix:

\[
A = \begin{bmatrix}
1 & 0 & -2 \\
0 & 1 & -4 \\
1 & 1 & -6
\end{bmatrix}
\]

has several notable properties:

- It is nonzero, satisfying the requirement.

- The null space of \(A\) contains \(\

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