Determine The Equation Of The Parabola With Focus (2 , 5) (2,5) And Directrix = 18 X=18.

Determine The Equation Of The Parabola With Focus (2, 5) And Directrix = 18, X=18

Understanding how to find the equation of a parabola given its focus and directrix is a fundamental concept in conic sections, with applications in physics, engineering, and geometry. In this article, we will walk through the process of determining the parabola's equation step-by-step, using the focus at (2, 5) and the directrix line x = 18. This comprehensive guide aims to clarify the concepts involved and provide a clear methodology for solving similar problems.

Understanding Parabolas: Basic Concepts

Before delving into the specific problem, it's essential to understand the basic properties of parabolas.

What Is a Parabola?

A parabola is a symmetric, U-shaped curve that can be defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix.

Key Components of a Parabola

- Focus: A fixed point outside the parabola, denoted as (h, k) in the coordinate plane.
- Directrix: A fixed line perpendicular to the axis of symmetry, associated with the parabola.
- Vertex: The point where the parabola changes direction, located midway between the focus and directrix.
- Axis of Symmetry: The line passing through the focus and vertex, about which the parabola is symmetric.

Given Data and Problem Setup

The problem provides:
- Focus: (2, 5)
- Directrix: x = 18

Our goal is to determine the algebraic equation of the parabola with these given components.

Step-by-Step Solution Approach

The process involves understanding the geometric setup, identifying the parabola's key features, and translating these into an algebraic equation.

Step 1: Recognize the Parabola's Orientation

Since the directrix is a vertical line (x=18), the parabola opens horizontally.

- Direction of opening: The parabola opens either left or right depending on the position of the focus relative to

the directrix.
- Focus: (2, 5) is to the left of the directrix at x=18, indicating the parabola opens to the left.

Step 2: Find the Vertex of the Parabola

The vertex lies midway between the focus and the directrix along the axis of symmetry.

- Compute the midpoint of x-coordinates:

$$
x_{vertex} = \frac{x_{focus} + x_{directrix}}{2} = \frac{2 + 18}{2} = \ \frac{20}{2} = 10
$$

- Y-coordinate of the vertex:

Since the focus and directrix are aligned horizontally (same y-coordinate), the vertex shares the same y-coordinate as the focus:

$$
y_{vertex} = 5
$$

Thus, the vertex is at (10, 5).

Step 3: Determine the Distance from the Vertex to the Focus (p)

- The distance from the vertex to the focus along the x-axis:

$$
p = x_{focus} - x_{vertex} = 2 - 10 = -8
$$

The negative sign indicates the parabola opens to the left.

- Absolute value of p:

$$
|p| = 8
$$

Step 4: Write the Standard Equation of the Parabola

Since the parabola opens horizontally, the general form with vertex at (h, k) is:

$$
(y - k)^2 = 4p (x - h)
$$

- $(h, k) = (10, 5)$
- $p = -8$

Plugging in the values:

$$
(y - 5)^2 = 4 \times (-8) \times (x - 10)
$$

$$
(y - 5)^2 = -32 (x - 10)
$$

This is the standard form of the parabola's equation.

Final Equation of the Parabola

$$
\boxed{
(y - 5)^2 = -32 (x - 10)
}
$$

This equation describes the parabola with focus at (2, 5) and directrix at $x=18$, opening to the left.

Additional Insights and Applications

Understanding this process allows for solving similar problems with different focus and directrix configurations.

Key Takeaways:

- The parabola's orientation depends on the position of the focus relative to the directrix.
- The vertex is always halfway between the focus and directrix along the axis of symmetry.
- The value of p indicates the distance from the vertex to the focus and determines the parabola's opening direction.

Practical Applications of Parabolas:

- Satellite Dish Design: Parabolic reflectors focus signals to a single point.
- Headlights and Torches: The parabola directs light beams efficiently.
- Physics: Path of objects under gravity with specific initial conditions.
- Architecture: Parabolic arches for structural stability.

Summary

Determining the equation of a parabola given its focus and directrix involves understanding the geometric relationships and translating them into algebraic form. For the focus at (2, 5) and directrix $x=18$, the key steps included identifying the parabola's orientation, finding the vertex, calculating the focal parameter p, and writing the standard form equation. The resulting parabola:

$$
(y - 5)^2 = -32 (x - 10)
$$

accurately captures the geometric properties specified.

Conclusion

Mastering how to determine the equation of a parabola from its focus and directrix enhances your understanding of conic sections and their applications. This method can be adapted to various configurations, whether the directrix is vertical or horizontal, and whether the parabola opens left, right, up, or down. By practicing these steps, you develop a solid foundation for solving complex geometric problems and applying these principles in real-world scenarios.

If you want to explore more about conic sections, or need solutions to similar problems, feel free to ask!

Frequently Asked Questions

How do you find the equation of a parabola given its focus at (2, 5) and directrix x = 18?

To find the equation, first note that the parabola opens horizontally because the directrix is vertical (x=18). The focus is at (2, 5), so the vertex is midway between the focus and directrix, at ((2 + 18)/2, 5) = (10, 5). The distance from the vertex to the focus (or directrix) is 8 units. The parabola's standard form is (y - k)^2 = 4p(x - h), where (h,k) is the vertex and p is the distance from vertex to focus. Here, p = -8 (since the parabola opens to the left). Therefore, the equation is (y - 5)^2 = -32(x - 10).

What is the vertex of the parabola with focus at (2, 5) and directrix x=18?

The vertex is located midway between the focus and the directrix along the x-axis, at ((2 + 18)/2, 5) = (10, 5).

In the parabola equation (y - 5)^2 = -32(x - 10), what does the coefficient -32 represent?

The coefficient -32 equals 4p, where p is the distance from the vertex to the focus. Since p = -8, the negative indicates the parabola opens to the left.

How does the focus at (2, 5) influence the orientation of the parabola?

Since the focus is to the left of the vertex at (10, 5), and the directrix is to the right, the parabola opens horizontally toward the focus, i.e., to the left.

Can you verify the parabola equation (y - 5)^2 = -32(x - 10) by plugging in the focus point?

Yes. Plugging in the focus (2, 5): (5 - 5)^2 = -32(2 - 10) => 0 = -32(-8) => 0 = 256, which is false, but note that the focus lies on the parabola if it satisfies the focus condition: the parabola consists of points equidistant from focus and directrix. Alternatively, check that the focus satisfies the parabola's definition: the distance from (2, 5) to (10, 5) is 8, matching p, confirming the equation's correctness.

Why is the parabola's equation written in the form (y - k)^2 = 4p(x - h) in this case?

Because the parabola opens horizontally with its axis along the x-axis, and the standard form for such a parabola is (y - k)^2 = 4p(x - h), where (h, k) is the vertex. Since the focus and directrix are vertical, this form applies.

What is the significance of the directrix x=18 in determining the parabola's equation?

The directrix x=18 is a vertical line that, together with the focus at (2, 5), helps define the parabola as the set of points equidistant from the focus and the directrix. Its position relative to the focus determines the parabola's orientation and shape.

How do you confirm that the derived parabola equation is correct?

You can verify by checking that the focus point (2, 5) satisfies the equation when substituted into the parabola's distance property, or by confirming that the distance from any point on the parabola to the focus equals the perpendicular distance to the directrix. Alternatively, plotting the parabola and checking the focus and directrix positions can confirm correctness.

Additional Resources

Determine The Equation Of The Parabola With Focus (2, 5) And Directrix x = 18

Understanding the geometric and algebraic properties of parabolas is fundamental in both pure mathematics and applied sciences such as physics, engineering, and computer graphics. The problem of finding the equation of a parabola given a focus point and a directrix line is a classic exercise that delves into the core principles of conic sections. In this article, we will explore the problem: Determine the equation of the parabola with focus (2, 5) and directrix x = 18, providing a comprehensive, detailed, and analytical approach suitable for students, educators, and enthusiasts alike.

Introduction to Parabolas: Focus and Directrix

Before diving into the problem specifics, it is essential to understand the fundamental definition and properties of a parabola.

What Is a Parabola?

A parabola is a symmetric, U-shaped conic section that can be described as the set of all points equidistant from a fixed point called the "focus" and a fixed line called the "directrix." This geometric property provides the foundation for deriving the algebraic equation of the parabola.

Focus and Directrix: The Core Elements

- Focus (F): A point outside or inside the parabola that, along with the directrix, defines the parabola.
- Directrix (D): A fixed line perpendicular or inclined relative to the axes; the parabola comprises points equidistant from both the focus and the directrix.

The unique property:
> For any point P (x, y) on the parabola, the distance to the focus is equal to the perpendicular distance to the directrix.

This geometric condition is pivotal in deriving the parabola's equation.

Problem Statement and Given Data

The problem provides:

- Focus (F): (2, 5)
- Directrix: $x = 18$

Our goal is to find the algebraic equation of the parabola satisfying these conditions.

Understanding the Geometry of the Given Parabola

In this specific case, the directrix is a vertical line at $x = 18$, and the focus is at (2, 5).

Key Observations:

- The parabola opens horizontally because the directrix is vertical.
- The focus is located to the left of the directrix since its x-coordinate (2) is less than that of the directrix (18).
- The vertex of the parabola lies midway between the focus and the directrix along the axis of symmetry.

Determining the Axis of Symmetry

Since the directrix is vertical, and the focus has an x-coordinate of 2, the parabola's axis of symmetry is a vertical line passing through the focus and the vertex. The axis of symmetry is the line $x = x_v$, where x_v is the x-coordinate of the vertex.

Step-by-Step Derivation of the Parabola Equation

The process involves several key steps:

1. Identify the Distance Between the Focus and the Directrix
2. Find the Coordinates of the Vertex
3. Determine the Orientation and Standard Form of the Equation
4. Derive the Equation Based on the Geometric Properties

Let's proceed through each step carefully.

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1. Distance Between Focus and Directrix

Calculate the horizontal distance:

$$
d = |x_{directrix} - x_{focus}| = |18 - 2| = 16
$$

This distance is critical as it relates to the parabola's parameters.

---

2. Coordinates of the Vertex (V)

Since the parabola opens horizontally, the vertex lies midway between the focus and the directrix along the x-axis, with the same y-coordinate as the focus (since the directrix is vertical).

- The x-coordinate of the vertex:

$$
x_v = \frac{x_{focus} + x_{directrix}}{2} = \frac{2 + 18}{2} = 10
$$

- The y-coordinate of the vertex:

$$
y_v = y_{focus} = 5
$$

Thus, the vertex is at:

$$
V = (10, 5)
$$

This point is equidistant from both the focus and the directrix and serves as the origin for the parabola's standard form.

---

3. The Parabola's Standard Equation

Since the parabola opens horizontally to the left (toward decreasing x-values), and the vertex is at (10, 5), the general form of the parabola's equation is:

$$
(y - y_v)^2 = 4p (x - x_v)
$$

where:

- p is the distance from the vertex to the focus (positive if the parabola opens to the right, negative if to the left).

- For a parabola opening to the left, p is negative.

---

4. Determining the Parameter p

Calculate the distance from the vertex to the focus:

$$
p = x_{focus} - x_v = 2 - 10 = -8
$$

- Since the focus is at (2, 5), and the vertex at (10, 5), the focus is 8 units to the left of the vertex, confirming the negative value of p.

Now, plug the values into the standard form:

$$
(y - 5)^2 = 4 \times (-8) \times (x - 10)
$$

which simplifies to:

$$
(y - 5)^2 = -32 (x - 10)
$$

This is the algebraic equation of the parabola in its vertex form.

Final Equation of the Parabola

$$
\boxed{
(y - 5)^2 = -32 (x - 10)
}
$$

This equation encapsulates the parabola's geometric properties:

- Focus at (2, 5)
- Directrix at x = 18
- Vertex at (10, 5)
- Opening to the left

Analytical Validation of the Derived Equation

To ensure the correctness of this equation, we can verify the focus and directrix conditions.

1. Distance from a point on the parabola to the focus

Pick a test point $P = (x, y)$ on the parabola, satisfying the equation.

- Distance to focus $F = (2, 5)$:

$$
d_{F} = \sqrt{(x - 2)^2 + (y - 5)^2}
$$

- Distance to directrix $x = 18$:

$$
d_{D} = |x - 18|
$$

For the parabola, $d_{F} = d_{D}$.

Test with the vertex point:

$$
V = (10, 5)
$$

Compute:

$$
d_{F} = \sqrt{(10 - 2)^2 + (5 - 5)^2} = \sqrt{8^2 + 0} = 8
$$

$$
d_{D} = |10 - 18| = 8
$$

Equal distances confirm the vertex lies on the parabola.

Similarly, at the focus:

$$
d_{F} = 0
$$

$$
d_{D} = |2 - 18| = 16
$$

Since the focus is not on the parabola but defines it, the key is that any point (x, y) satisfying the equation verifies the geometric property.

Applications and Significance of the Derived Equation

Understanding the explicit equation of a parabola with given focus and directrix has broad applications:

- Optics: Parabolic reflectors focus light or sound waves efficiently.

- Satellite Dishes: Parabolic shapes direct signals towards the focus.
- Engineering: Parabolic arches distribute stress uniformly.
- Mathematical Modeling: Conic sections serve as foundational models in various scientific disciplines.

The derived equation:

$$
(y - 5)^2 = -32 (x - 10)
$$

encapsulates the geometric essence of the parabola, enabling precise plotting, analysis, and application.

Extended Considerations: Variations and Generalizations

While this specific problem involves a parabola opening horizontally, similar methods extend to:

- Vertical parabolas: when directrix is horizontal.
- Oblique parabolas: with inclined directrices.
- Parametric equations: useful for plotting and computational modeling.
- Focus-directrix form: more general and applicable to various orientations.

Understanding these variations enhances the versatility of conic section analysis.

Conclusion

Determining the equation of a parabola given a focus and directrix involves geometric insights and algebraic manipulations. For the case where the focus is at $(2, 5)$ and the directrix is the vertical line $x = 18$, the parabola opens to the left, with its vertex at $(10, 5)$. The derived equation:

$$
(y - 5)^2 = -32 (x - 10)
$$

faithfully represents the

Determine The Equation Of The Parabola With Focus 2 5 2 5 And Directrix 18 X 18

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