Determine The Slope Of The Tangent Line To The Circle X^2+y^2 = 1 At The Point (1/2, 1/2).

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Understanding how to find the slope of a tangent line to a circle at a specific point is a fundamental skill in calculus and analytic geometry. In this article, we will explore the process step-by-step for the circle defined by the equation $x^2 + y^2 = 1$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$. By the end, you will be equipped with the methods to determine slopes of tangent lines to circles at various points, an essential concept in both pure and applied mathematics.

Understanding the Circle Equation and the Point of Interest

The Equation of the Circle

The circle given by the equation:

- $x^2 + y^2 = 1$

represents a circle centered at the origin (0,0) with a radius of 1. Every point (x, y) satisfying this equation lies on the circle.

The Point (1/2, 1/2)

The point $\left(\frac{1}{2}, \frac{1}{2}\right)$ is located on the circle because:

- $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\neq 1$

Wait, this indicates a miscalculation—let’s verify if the point is on the circle.

Verification:

$$
\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
$$

Since $\frac{1}{2} \neq 1$, the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not on the circle $x^2 + y^2 = 1$.

Correction:

It appears there is a discrepancy; perhaps the intended point is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, which is on the circle, since:

$$
\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1
$$

Alternatively, if the original point is $\left(\frac{1}{2}, \frac{1}{2}\right)$, then it lies on the circle only if the circle's equation is different, such as $x^2 + y^2 = \frac{1}{2}$.

Assuming the correct point is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, let's proceed with that point since it satisfies the circle equation $x^2 + y^2 = 1$.

Note: For consistency, assume the point of interest is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. If you wish to analyze the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ on a different circle, please specify.

Method 1: Using Implicit Differentiation to Find the Slope

What is Implicit Differentiation?

Implicit differentiation is a technique used to find the derivative dy/dx when x and y are related through an equation not explicitly solved for y. Since the circle's equation involves both x and y, this method is ideal.

Applying Implicit Differentiation to the Circle Equation

Starting with:

$$
x^2 + y^2 = 1
$$

Differentiate both sides with respect to x:

$$
\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)
$$

$$
2x + 2y \frac{dy}{dx} = 0
$$

Solve for $\frac{dy}{dx}$:

$$
2y \frac{dy}{dx} = -2x
$$

$$
\frac{dy}{dx} = -\frac{x}{y}
$$

Interpretation:
The slope of the tangent line at any point (x, y) on the circle is:
$$
\boxed{
\text{Slope} = -\frac{x}{y}
}
$$

Calculating the Slope at the Given Point

Substitute the Coordinates

Using the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, the slope becomes:
$$
\text{Slope} = -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1
$$

Result:
$$
\boxed{
\text{The slope of the tangent line at } \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ is }
-1
}
$$

This means the tangent line at that point has a slope of -1.

If the point was $\left(\frac{1}{2}, \frac{1}{2}\right)$, and it was on the circle $x^2 + y^2 = 1/2$, then:

$$
x^2 + y^2 = \frac{1}{2}
$$
Applying the same differentiation:
$$
2x + 2y \frac{dy}{dx} = 0
$$
$$
\frac{dy}{dx} = -\frac{x}{y}
$$
Substituting $x = \frac{1}{2}$ and $y = \frac{1}{2}$:
$$

$$
\text{Slope} = -\frac{\frac{1}{2}}{\frac{1}{2}} = -1
$$

Thus, the slope is -1 in this case as well.

Alternative Method: Using Derivatives of Explicit Functions

Expressing y as a Function of x

From the circle equation:

$$
y^2 = 1 - x^2
$$

$$
y = \pm \sqrt{1 - x^2}
$$

Differentiate $y = \sqrt{1 - x^2}$:

$$
\frac{dy}{dx} = \frac{1}{2} (1 - x^2)^{-1/2} \times (-2x) = -\frac{x}{\sqrt{1 - x^2}}
$$

Similarly for the negative branch:

$$
\frac{dy}{dx} = \frac{x}{\sqrt{1 - x^2}}
$$

At $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$:

$$
\frac{dy}{dx} = -\frac{\frac{\sqrt{2}}{2}}{\sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2}} = -\frac{\frac{\sqrt{2}}{2}}{\sqrt{1 - \frac{1}{2}}} = -\frac{\frac{\sqrt{2}}{2}}{\sqrt{\frac{1}{2}}}
$$

$$
= -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1
$$

Confirming the previous result.

Summary of the Process to Find the Slope of the Tangent Line

Step-by-Step Guide:

1. Identify the equation of the circle and verify the point lies on it.
2. Use implicit differentiation to derive $\frac{dy}{dx}$, the slope of the tangent line.
3. Substitute the x and y coordinates of the point into the derivative to find the slope at that point.
4. Interpret the slope: a positive value indicates an upward slope, a negative value indicates a downward slope.

Conclusion: Key Takeaways

- The slope of the tangent line to the circle $x^2 + y^2 = 1$ at any point (x, y) is $-\frac{x}{y}$.
- For the point \(\left(\frac{\sqrt{2}}{2},

Frequently Asked Questions

How do you find the slope of the tangent line to the circle at a specific point?

You differentiate the equation of the circle implicitly to find dy/dx, which gives the slope of the tangent line at any point.

What is the equation of the circle given in the problem?

The circle is given by $x^2 + y^2 = 1$.

How do you find the slope of the tangent line at the point (1/2, 1/2)?

Differentiate the circle's equation implicitly to find dy/dx, then substitute x=1/2 and y=1/2 to compute the slope.

What is the implicit differentiation of $x^2 + y^2 = 1$?

Differentiating both sides with respect to x gives $2x + 2y(dy/dx) = 0$, which simplifies to $dy/dx = -x/y$.

What is the slope of the tangent line at (1/2, 1/2)?

Using $dy/dx = -x/y$, substitute $x=1/2$ and $y=1/2$ to get slope $= - (1/2) / (1/2) = -1$.

How can you verify that the point (1/2, 1/2) lies on the circle $x^2 + y^2 = 1$?

Calculate $(1/2)^2 + (1/2)^2 = 1/4 + 1/4 = 1/2$, which is not equal to 1, so the point (1/2, 1/2) does not lie on the circle. (Note: The point is given in the problem; ensure it's on the circle or clarify the point.)

Is the point (1/2, 1/2) on the circle $x^2 + y^2 = 1$?

No, substituting (1/2, 1/2) into $x^2 + y^2$ gives $1/4 + 1/4 = 1/2$, which is not equal to 1. Therefore, the point is not on the circle.

Additional Resources

Determine The Slope Of The Tangent Line To The Circle $x^2 + y^2 = 1$ At The Point $\left(\frac{1}{2}, \frac{1}{2}\right)$

Introduction

Understanding the behavior of curves and their tangent lines forms a fundamental aspect of calculus and analytical geometry. Among the most iconic shapes studied in these fields is the circle, defined by the equation $x^2 + y^2 = r^2$. Determining the slope of the tangent line at a specific point on a circle not only deepens comprehension of geometric principles but also provides insights into the application of differentiation techniques. This article explores the process of finding the slope of the tangent line to the circle $x^2 + y^2 = 1$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

The Geometric Context: The Circle and Its Properties

The Equation of the Circle

The given circle is described by the equation:

$$
x^2 + y^2 = 1
$$

This is the standard form of a circle centered at the origin $(0,0)$ with a radius of 1. The circle’s symmetry and simplicity make it an ideal candidate for applying calculus techniques to determine tangent lines.

Significance of the Point $\left(\frac{1}{2}, \frac{1}{2}\right)$

The point $\left(\frac{1}{2}, \frac{1}{2}\right)$ lies on the circle because substituting these coordinates into the equation yields:

$$
\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
$$

which is not equal to 1, suggesting an initial inconsistency. Therefore, let's verify the point carefully.

Verifying the Point on the Circle

Step 1: Substitution into the Circle Equation

Calculate:

$$
\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
$$

Since $\frac{1}{2} \neq 1$, the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ does not lie on the circle $x^2 + y^2 = 1$.

Clarification and Correct Point Selection

Given the initial problem statement, there might be a misinterpretation. To find the tangent slope at a point on the circle $x^2 + y^2 = 1$, the point must satisfy this equation.

Possible correction: The point should satisfy $x^2 + y^2 = 1$.

Let’s verify the point $\left(\frac{1}{2}, \frac{1}{2}\right)$:

$$
\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
$$

which is not on the circle.

Alternative points on the circle include:

- $(1, 0)$
- $(0, 1)$

- $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
- $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, etc.

Correct Point on the Circle: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Since the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ does not lie on the circle $x^2 + y^2 = 1$, perhaps the intended point is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. Let's verify:

$$
\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1
$$

Yes, this point lies on the circle.

Clarification of the Task

To proceed, we will assume the problem intends to find the slope of the tangent line to the circle $x^2 + y^2 = 1$ at the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

Differentiation Techniques for Finding the Slope

The main method to determine the slope of the tangent line at a point on a curve defined implicitly is implicit differentiation. This technique involves differentiating both sides of the equation with respect to x, treating y as a function of x, i.e., $y = y(x)$.

Implicit Differentiation of the Circle Equation

Step 1: Write the equation

$$
x^2 + y^2 = 1
$$

Step 2: Differentiate both sides with respect to x:

$$
\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)
$$

Applying the chain rule:

$$

2x + 2y \frac{dy}{dx} = 0
\]

Solving for \(\frac{dy}{dx}\): The Slope of the Tangent Line

Rearranged, the derivative becomes:

\[
2y \frac{dy}{dx} = -2x
\]

\[
\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}
\]

This expression gives the slope of the tangent line at any point \((x, y)\) on the circle.

Calculating the Slope at the Specific Point

Now, substitute the coordinates \(\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)\):

\[
\frac{dy}{dx} = -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1
\]

Thus, the slope of the tangent line at this point is \(-1\).

Interpretation and Significance

The slope of \(-1\) indicates that at the point \(\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)\), the tangent line is descending at a 45-degree angle relative to the x-axis. This slope reflects the symmetry of the circle and the uniform rate at which the y-coordinate changes with x at that specific point.

Generalization: Slope of the Tangent Line on a Circle

The formula \(\frac{dy}{dx} = -\frac{x}{y}\) is valid at any point on the circle \(x^2 + y^2 = 1\), except at points where \(y=0\) (which would cause division by zero). This formula highlights the intrinsic relationship between the coordinates and the tangent's inclination, emphasizing the circle's geometric properties.

Geometrical Insights

The Tangent Line's Slope and the Circle's Geometry

- The slope of the tangent line is negative of the ratio of the x-coordinate to the y-coordinate at the point.
- When x and y are equal (as in $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$), the slope becomes -1, consistent with the line passing through the origin with a 45-degree angle.

The Normals and Radial Lines

- The radius vector from the origin to the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ has a slope of 1.
- The tangent slope being -1 is perpendicular to the radius's slope, confirming the fundamental property that the tangent line is perpendicular to the radius at the point of contact.

Practical Applications and Broader Context

Understanding how to determine tangent slopes has numerous applications:

- Physics: Analyzing motion along curved paths.
- Engineering: Designing components that follow specific curvature constraints.
- Computer Graphics: Rendering smooth curves and calculating normals.
- Mathematical Analysis: Studying curvature and geometric properties.

In the context of the circle $x^2 + y^2 = 1$, the explicit formula for the tangent slope at any point simplifies many calculations involving tangent lines and normals.

Summary and Conclusions

- The equation of the circle $x^2 + y^2 = 1$ describes all points at a distance of 1 from the origin.
- To find the slope of the tangent line at a

Determine The Slope Of The Tangent Line To The Circle X 2 Y 2 1 At The Point 1 2 1 2

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