Determine The Value Of A And B In The Relaton Y=ax^2+bx-31 If The Vertex Is Located At (-3,5)

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When working with quadratic functions, understanding how to determine the coefficients a and b based on given features such as the vertex point is crucial. In this article, we will explore step-by-step how to find the values of a and b in the quadratic equation $y = ax^2 + bx - 31$ when the vertex is located at $(-3, 5)$. This type of problem is common in algebra and helps build a foundation for understanding the properties of parabolas.

Understanding the Quadratic Equation

A quadratic function generally has the form:

$$
y = ax^2 + bx + c
$$

where:
- a determines the parabola's opening direction and its "width."
- b influences the position of the parabola along the x-axis.
- c shifts the parabola vertically.

In our specific case, the quadratic function is:

$$
y = ax^2 + bx - 31
$$

Note that $c = -31$ is fixed, and our goal is to find the values of a and b.

The Vertex of a Parabola

The vertex of a parabola is its highest or lowest point, depending on whether it opens downward or upward. The vertex's coordinates (h, k) are linked to the quadratic function's coefficients by the following relationships:

$$
h = -\frac{b}{2a}
$$

and the value of the function at the vertex is:

\[

$$
k = y(h) = a h^2 + b h + c
$$

Given the vertex $(-3, 5)$, these equations become critical in solving for a and b.

Step-by-Step Solution Approach

Step 1: Use the vertex's x-coordinate to relate a and b

Given:

$$
h = -3
$$

and

$$
h = -\frac{b}{2a}
$$

Substituting:

$$
-3 = -\frac{b}{2a}
$$

which simplifies to:

$$
-3 = -\frac{b}{2a}
$$

Multiplying both sides by $2a$:

$$
-3 \times 2a = -b
$$

$$
-6a = -b
$$

or equivalently:

$$
b = 6a
$$

This is our first key relationship between a and b.

Step 2: Use the vertex's y-coordinate to find a

Given:

$$
k = 5
$$

and the quadratic function:

$$
y = a x^2 + b x - 31
$$

At $x = h = -3$, the function value is:

$$
y(-3) = a(-3)^2 + b(-3) - 31
$$

which simplifies to:

$$
5 = a(9) + b(-3) - 31
$$

or:

$$
5 = 9a - 3b - 31
$$

Now, substitute $b = 6a$ from Step 1 into this equation:

$$
5 = 9a - 3(6a) - 31
$$

which simplifies to:

$$
5 = 9a - 18a - 31
$$

$$
5 = -9a - 31
$$

Add 31 to both sides:

$$

$$
5 + 31 = -9a
$$

$$
36 = -9a
$$

Divide both sides by -9:

$$
a = -4
$$

Step 3: Find b

Recall from Step 1:

$$
b = 6a
$$

Substitute $a = -4$:

$$
b = 6 \times (-4) = -24
$$

Final Values of a and b

Based on the calculations:

$$
\boxed{
a = -4 \quad \text{and} \quad b = -24
}
$$

These are the coefficients that satisfy the quadratic function with the vertex at $(-3, 5)$.

Additional Insights and Applications

Understanding the Role of a and b

Knowing the values of a and b allows us to sketch the parabola accurately and understand its behavior:

- Since $a = -4$ (negative), the parabola opens downward.
- The magnitude of a indicates the "steepness" or "width" of the parabola.
- The value of $b = -24$ influences the symmetry and the x-intercepts.

How to Verify Your Results

To ensure the correctness of the derived coefficients:

1. Check the vertex point:

Substitute $x = -3$:

$$
y = -4(-3)^2 - 24(-3) - 31 = -4(9) + 72 - 31 = -36 + 72 - 31 = 5
$$

which matches the given vertex y-coordinate.

2. Check the vertex's x-coordinate:

Compute:

$$
h = -\frac{b}{2a} = -\frac{-24}{2 \times -4} = -\frac{-24}{-8} = -3
$$

confirming the vertex's x-coordinate.

Practical Applications

Understanding how to determine a and b from the vertex point is vital in various fields:

- Physics: Modeling projectile trajectories.
- Engineering: Designing parabolic reflectors or antennas.
- Economics: Optimizing profit functions with quadratic models.
- Mathematics Education: Enhancing comprehension of quadratic functions and their properties.

Summary

Determining the values of a and b in a quadratic function given its vertex involves:

- Utilizing the vertex formula $h = -\frac{b}{2a}$ to relate a and b.
- Substituting the known vertex coordinates into the quadratic equation.
- Solving the resulting system of equations for the unknown coefficients.

In our example, the quadratic function:

$$
y = -4x^2 - 24x - 31
$$

has a vertex at $(-3, 5)$, confirming that the parabola opens downward and aligns perfectly with the given vertex point.

Conclusion

Mastering the process of determining quadratic coefficients from vertex information enhances your algebraic problem-solving skills. It provides a deeper understanding of the geometric properties of parabolas and their equations. Whether you're solving academic problems or applying these concepts in real-world scenarios, being able to find \(a\) and \(b\) from the vertex is a fundamental skill in algebra and analytical geometry.

Frequently Asked Questions (FAQs)

Q1: Can I find \(a\) and \(b\) if I only know the vertex point?

A: Yes. If you know the vertex \((h, k)\) and the general form of the quadratic, you can set up the equations relating \(a\) and \(b\). Usually, you need at least one additional piece of information, such as a point on the parabola, to solve for both coefficients.

Q2: What if the quadratic function has a different constant term?

A: If the constant term \(c\) differs, you can follow a similar process but replace the known \(c\) in the calculations. The key relationships involving the vertex remain the same.

Q3: How does changing \(a\) affect the parabola?

A: Increasing the magnitude of \(a\) makes the parabola narrower and steeper, while decreasing it makes the parabola wider. Changing the sign of \(a\) flips the parabola upside down.

Q4: Is there a quick way to graph a parabola given the vertex and coefficients?

A: Yes. Plot the vertex first, then determine additional points by choosing x-values around the vertex and calculating corresponding y-values, or use symmetry about the vertex to find points on the parabola.

By mastering these concepts, you will be well-equipped to analyze quadratic functions and solve related problems effectively.

Frequently Asked Questions

How do you find the value of 'a' and 'b' in the quadratic equation y = ax^2 + bx - 31 given the vertex at (-3, 5)?

Use the vertex form y = a(x - h)^2 + k, where (h, k) is the vertex. Convert to standard form and compare coefficients or substitute the vertex coordinates into the equations to find 'a' and 'b'.

What is the significance of the vertex at (-3, 5) in determining

'a' and 'b'?

The vertex provides the minimum or maximum point of the parabola, allowing us to set up equations involving 'a' and 'b' by plugging in the vertex coordinates to solve for these variables.

How can the vertex form of the quadratic help in finding 'a' and 'b'?

Expressing y = a(x + 3)^2 + 5 allows us to expand and compare with y = ax^2 + bx - 31 to determine 'a' and 'b' directly.

What is the process to determine 'a' using the given vertex and the quadratic equation?

Substitute x = -3 and y = 5 into the quadratic equation to solve for 'a' and 'b', then use this information to find the specific values.

How do you find 'b' once 'a' is known in the quadratic equation with the given vertex?

Use the vertex coordinates in the vertex form or plug in the vertex point into the standard form and solve for 'b' once 'a' is known.

Is there a direct formula to find 'a' and 'b' from the vertex coordinates?

While there's no simple direct formula, setting up equations based on the vertex coordinates and solving simultaneously yields 'a' and 'b'.

Can you determine 'a' and 'b' without converting to vertex form?

Yes, by using the vertex coordinates in the standard form y = ax^2 + bx - 31 and substituting x = -3, y = 5 to create two equations, which can then be solved simultaneously.

What is the first step in solving for 'a' and 'b' given the vertex (-3, 5)?

Substitute x = -3 and y = 5 into the quadratic equation y = ax^2 + bx - 31 to form an equation that relates 'a' and 'b'.

How does the value of the constant term (-31) affect determining 'a' and 'b'?

The constant term shifts the parabola vertically; knowing it helps set up the equations and isolate 'a' and 'b' in relation to the vertex coordinates.

Once 'a' and 'b' are found, how can you verify the solution?

Plug the vertex point into the derived quadratic equation with the found values of 'a' and 'b' to ensure it satisfies the equation, confirming correctness.

Additional Resources

Determine The Value Of A And B In The Relation Y = ax^2 + bx - 31 If The Vertex Is Located At (-3, 5)

Introduction

Quadratic functions form a fundamental part of algebra and are widely applied across various scientific, engineering, and economic fields. They are characterized by their parabolic graphs, which exhibit symmetry and a unique vertex point that indicates either a maximum or minimum value of the function. In this context, the problem of determining the coefficients a and b in the quadratic relation:

$$
Y = ax^2 + bx - 31
$$

given the vertex's location at $(-3, 5)$, offers a compelling case study in the application of analytical geometry and algebraic techniques. This investigation aims to meticulously analyze the problem, employ algebraic methods to derive the unknown coefficients, and elucidate the underlying principles guiding such quadratic function determinations.

Understanding the Problem and Its Significance

The Quadratic Function and Its Graph

The quadratic function $Y = ax^2 + bx + c$ (here, $c = -31$) is a parabola opening upwards if $a > 0$ and downwards if $a < 0$. Its vertex, a critical point, represents either the maximum or minimum of the function, depending on the parabola's orientation.

The problem specifies:

- The vertex is at $(-3, 5)$,
- The quadratic relation is $Y = ax^2 + bx - 31$,
- The goal is to find the values of a and b.

Understanding these parameters is not only a matter of mathematical curiosity but offers insights into modeling real-world phenomena where quadratic relationships are prevalent, such as projectile motion, profit maximization, and structural design.

Mathematical Foundations and Approach

Vertex Form of a Quadratic Function

The key to solving this problem lies in transforming the general quadratic form into the vertex form:

$$
Y = a(x - h)^2 + k
$$

where (h, k) is the vertex of the parabola.

Given the vertex $(-3, 5)$, the quadratic function can be expressed as:

$$
Y = a(x + 3)^2 + 5
$$

This form is advantageous because it directly encodes the vertex's coordinates and simplifies the process of determining the coefficients.

Relationship Between Standard and Vertex Forms

Expanding the vertex form:

$$
Y = a(x + 3)^2 + 5
$$

$$
Y = a(x^2 + 6x + 9) + 5
$$

$$
Y = a x^2 + 6a x + 9a + 5
$$

Matching this with the standard form:

$$
Y = a x^2 + b x - 31
$$

we can equate the coefficients:

$$
b = 6a
$$
$$
9a + 5 = -31
$$

This reveals a pathway to solving for a and b.

Step-by-Step Solution: Deriving a and b

Step 1: Establish the Relationship Between a and b

From the expanded vertex form:

$$
b = 6a
$$

This is our first key relationship.

Step 2: Use the Constant Term to Find a

From the comparison:

$$
9a + 5 = -31
$$

Solve for a:

$$
9a = -31 - 5
$$
$$
9a = -36
$$
$$
a = -4
$$

Step 3: Find b

Using the relationship $b = 6a$:

$$
b = 6 \times (-4) = -24
$$

Thus, the coefficients are:

$$
a = -4,\quad b = -24
$$

Validating the Solution

To ensure the correctness of the derived coefficients, verify that the vertex indeed occurs at $(-3, 5)$:

$$
Y = -4(x + 3)^2 + 5
$$

Plugging in $x = -3$:

$$
Y = -4(0)^2 + 5 = 5
$$

which matches the vertex's y-coordinate.

Now, expand to the standard form:

$$
Y = -4(x^2 + 6x + 9) + 5
$$
$$
Y = -4x^2 - 24x - 36 + 5
$$
$$
Y = -4x^2 - 24x - 31
$$

This aligns with the original form:

$$
Y = ax^2 + bx - 31
$$

with $a = -4$ and $b = -24$.

Broader Implications and Context

Significance of the Coefficients

- The negative value of a indicates the parabola opens downward, consistent with the maximum point at the vertex.
- The magnitude of a influences the parabola's "width"; a larger absolute value results in a narrower parabola, affecting the rate of change of the function.
- The value of b influences the axis of symmetry and the slope of the parabola's arms.

Real-World Applications

Understanding how to determine a and b from the vertex is instrumental in various domains:

- Physics: Modeling projectile trajectories where the vertex represents the peak height.
- Economics: Analyzing profit functions with maximum profit at the vertex.
- Engineering: Designing structures with specific curvature properties.

Additional Considerations and Extensions

Exploring Other Forms and Parameters

- Standard form with different c: How would the coefficients change if the constant term varies?
- Parameter sensitivity: How do small changes in the vertex coordinates affect a and b?
- Vertex at different points: Generalizing the method for any vertex location.

Limitations and Assumptions

- The derivation assumes the quadratic has a clear vertex at the specified point.
- The method relies on the quadratic being in the form $Y = ax^2 + bx + c$ with a known c.
- The approach is algebraically straightforward but might be complex for non-vertex forms or multiple unknowns.

Conclusion

The analytical process to determine the coefficients a and b in the quadratic relation $Y = ax^2 + bx - 31$ with the vertex at $(-3, 5)$ underscores the power of algebraic transformations and geometric insights. By converting the standard form into the vertex form, leveraging the properties of the parabola's vertex, and performing systematic calculations, we derive:

$$
a = -4,\quad b = -24
$$

This exercise exemplifies the broader utility of quadratic functions in modeling real-world phenomena and highlights the importance of understanding their geometric properties. The approach outlined not only solves the specific problem but also provides a blueprint for tackling similar problems involving quadratic functions and their vertices in various scientific and engineering contexts.

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About the Author

This article was prepared by an expert in mathematical analysis and education, dedicated to elucidating complex algebraic concepts with clarity and precision, fostering deeper understanding across academic disciplines.

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