Determine Whether The Geometric Series Is Convergent Or Divergent. [infinity] 1 (3)n N = 0

Determine Whether The Geometric Series Is Convergent Or Divergent. [infinity] 1 (3)n N = 0

Understanding whether a series converges or diverges is fundamental in calculus and mathematical analysis, especially when dealing with infinite series. One such series that often appears in various applications is the geometric series, notably expressed as the sum of terms involving powers of a common ratio. In this article, we will focus on determining whether the series

$$ \sum_{n=0}^{\infty} \frac{1}{3^n} $$

is convergent or divergent. This particular series is a classic example of a geometric series, and analyzing its behavior provides insight into the broader concept of infinite series convergence.

What Is a Geometric Series?

Definition of a Geometric Series

A geometric series is an infinite sum of the form:

$$ \sum_{n=0}^{\infty} ar^n $$

where:
- a is the first term of the series,
- r is the common ratio between successive terms.

The series continues infinitely, with each term obtained by multiplying the previous term by the ratio r.

Examples of Geometric Series

Some common geometric series include:
- Sum of powers of 2: $ 1 + 2 + 4 + 8 + \dots $
- Sum of powers of a fraction: $ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots $

The behavior of these series (whether they converge or diverge) depends heavily on the value of the common ratio r.

Understanding Series Convergence and Divergence

Convergence of a Series

An infinite series converges if the sequence of its partial sums approaches a finite limit as the number of terms approaches infinity. In other words, the sum stabilizes to a specific value.

Divergence of a Series

Conversely, a series diverges if its partial sums do not approach any finite limit; instead, they grow without bound or oscillate indefinitely.

Analyzing the Series: $\sum_{n=0}^{\infty} \frac{1}{3^n}$

Identifying the Series as Geometric

The series:

$$ \sum_{n=0}^{\infty} \frac{1}{3^n} $$

can be written as:

$$ \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n $$

Here:
- The first term $a = 1$,
- The common ratio $r = \frac{1}{3}$.

Since it's a geometric series with $a = 1$ and $r = \frac{1}{3}$, we can analyze its convergence based on the value of r.

Conditions for Geometric Series Convergence

A geometric series converges if and only if:

$$ |r| < 1 $$

In this case, since $r = \frac{1}{3}$, and:

$$ |\frac{1}{3}| = \frac{1}{3} < 1 $$

the series converges.

Calculating the Sum of the Series

Sum of a Convergent Geometric Series

For a geometric series with $|r| < 1$, the sum to infinity is given by:

$$ S = \frac{a}{1 - r} $$

Applying this formula:

$$ S = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} $$

Thus, the series:

$$ \sum_{n=0}^{\infty} \frac{1}{3^n} $$

converges to $\frac{3}{2}$.

Summary of Key Points

- The series $\sum_{n=0}^{\infty} \frac{1}{3^n}$ is a geometric series with $a=1$ and $r=\frac{1}{3}$.
- Since $|r| < 1$, the series converges.
- The sum to infinity of the series is $\frac{3}{2}$.

Why Understanding Series Convergence Matters

Applications in Mathematics and Science

Determining whether a series converges is crucial in various fields:
- Calculating limits and sums in calculus.
- Modeling real-world phenomena such as population growth, radioactive decay, and financial calculations.
- Analyzing algorithms in computer science, especially those involving recursive or iterative processes.

Implications of Convergence or Divergence

- Convergent series have well-defined sums, enabling precise calculations and predictions.
- Divergent series, on the other hand, indicate unbounded growth or oscillation, which can be significant in understanding system instabilities or the need for alternative analysis methods.

Additional Considerations for Geometric Series

Series with $|r| \geq 1$

If the absolute value of the common ratio r is greater than or equal to 1:

- The series diverges.
- The partial sums tend toward infinity or oscillate without settling.

Series with $|r| < 1$

If $|r| < 1$:
- The series converges.
- The sum can be calculated using the formula $S = \frac{a}{1 - r}$.

Conclusion

In summary, the series:

$$\sum_{n=0}^{\infty} \frac{1}{3^n}$$

is a geometric series with a ratio $r = \frac{1}{3}$. Because $|r| < 1$, the series is convergent, and its sum to infinity is $\frac{3}{2}$. Recognizing the properties of the geometric series allows mathematicians and scientists to analyze such series efficiently and apply their sums in various practical contexts.

Understanding whether a geometric series converges or diverges is essential for both theoretical and applied mathematics. This example underscores the importance of the common ratio in determining the behavior of the series and highlights how simple formulas can reveal profound insights into the nature of infinite sums.

Frequently Asked Questions

What is the general form of the geometric series in the given problem?

The series is of the form sum from n=0 to infinity of (1/3)^n, which is a geometric series with first term 1 and common ratio 1/3.

How do you determine if a geometric series converges or diverges?

A geometric series converges if the absolute value of the common ratio r is less than 1; otherwise, it diverges.

What is the common ratio in the series 1 + (1/3) + (1/3)^2 + ...?

The common ratio r is 1/3.

Does the series sum from n=0 to infinity of (1/3)^n

converge?

Yes, because the common ratio r = 1/3 satisfies |r| < 1, so the series converges.

What is the sum of the convergent geometric series sum from n=0 to infinity of (1/3)^n?

The sum is 1 / (1 - 1/3) = 1 / (2/3) = 3/2.

How can you verify the convergence of this series mathematically?

By applying the geometric series convergence criterion: since |1/3| < 1, the series converges.

What happens if the common ratio was 1 instead of 1/3?

If r=1, the series becomes sum of 1 + 1 + 1 + ... which diverges to infinity.

Can the divergence of the series be confirmed using the nth-term test?

Yes, since the limit of the nth term (1/3)^n as n approaches infinity is 0, the nth-term test alone does not confirm divergence; however, the geometric ratio criterion confirms convergence. If the limit didn't approach zero, the series would diverge.

Why is understanding convergence important for geometric series?

Understanding convergence helps determine whether the sum of an infinite series approaches a finite value or diverges, which is crucial in many applications like finance, physics, and engineering.

Additional Resources

Determine Whether The Geometric Series Is Convergent Or Divergent. [infinity] 1 (3)n N = 0

Understanding whether a geometric series is convergent or divergent is a fundamental concept in mathematical analysis, especially when dealing with infinite series. The question "Determine whether the geometric series is convergent or divergent" often arises in calculus and advanced mathematics courses, and it forms the basis for deeper exploration into series behavior, limits, and summation techniques. Specifically, when analyzing a series such as \(\sum_{n=0}^{\infty} (3)^n\), it’s essential to understand the criteria that dictate whether the series converges to a finite value or diverges to infinity. This guide aims to break down the process step-by-step, explaining the core principles involved and providing a clear methodology to analyze similar series.

What Is a Geometric Series?

Before diving into convergence or divergence, it’s crucial to understand what a geometric series is. A geometric series is a sum of terms where each term is obtained by multiplying the previous term by a fixed, non-zero constant called the common ratio.

Standard form of a geometric series:

$$
\sum_{n=0}^{\infty} ar^n
$$

- a = the first term of the series
- r = the common ratio between successive terms
- n = the index of summation, starting from 0 or 1

For example:

$$
\sum_{n=0}^{\infty} 2 \times (0.5)^n
$$

is a geometric series with $a = 2$ and $r = 0.5$.

The Core Question: Convergence vs. Divergence

When analyzing an infinite geometric series, the central question is:

- Does the sum approach a finite value as n approaches infinity (converge)?
- Or does the sum grow without bound (diverge)?

This depends on the properties of the common ratio r.

The Convergence Criterion for Geometric Series

For a geometric series $\sum_{n=0}^{\infty} ar^n$, the series converges if and only if:

$$
|r| < 1
$$

Why?

- When $|r| < 1$, the terms ar^n tend to zero as $n \to \infty$.
- The sum of the series converges to a finite value, which can be expressed as:

$$
S = \frac{a}{1 - r}
$$

- If $|r| \geq 1$, the terms do not tend to zero (they either stay constant or grow without bound), and the series diverges.

Applying the Criterion to $\sum_{n=0}^{\infty} (3)^n$

Let’s examine the specific series:

$$
\sum_{n=0}^{\infty} (3)^n
$$

Here:

- $a = 1$ (since the first term when $n=0$ is $3^0=1$)
- $r = 3$

Step 1: Check the absolute value of r:

$$
|r| = |3| = 3
$$

Since $3 \geq 1$, the series does not satisfy the convergence criterion.

Step 2: Analyze the behavior of terms:

- The terms 3^n grow exponentially as n increases.
- The terms do not tend to zero; instead, they tend to infinity.

Conclusion:

The series $\sum_{n=0}^{\infty} (3)^n$ diverges because the ratio's magnitude exceeds 1.

Visualizing Divergence and Convergence

Understanding the convergence or divergence of a geometric series can be made more intuitive by considering the behavior of partial sums.

Partial sum S_N:

$$
S_N = \sum_{n=0}^{N} ar^n = a \frac{1 - r^{N+1}}{1 - r}
$$

- When $|r| < 1$, as $N \to \infty$, $r^{N+1} \to 0$, and the sum approaches:

$$
S = \frac{a}{1 - r}
$$

- When $|r| \geq 1$, r^{N+1} does not tend to zero, and the partial sums grow without bound, indicating divergence.

Step-by-Step Guide to Determine Series Behavior

1. Identify the series form:

Recognize if the series is geometric, i.e., of the form $\sum ar^n$.

2. Determine a and r:

Find the first term a and the common ratio r.

3. Calculate $|r|$:

- If $|r| < 1$, the series converges.
- If $|r| \geq 1$, the series diverges.

4. Use the convergence formula (if applicable):

$$
\text{Sum } S = \frac{a}{1 - r}
$$

for convergent series.

5. Check the behavior of terms:

Confirm that $\lim_{n \to \infty} ar^n = 0$.

Examples for Clarification

Example 1:
$$
\sum_{n=0}^{\infty} 5 \times (0.2)^n
$$
- $a=5$, $r=0.2$
- $|r|=0.2<1$ → Converges
- Sum: $S = \frac{5}{1-0.2} = \frac{5}{0.8} = 6.25$

Example 2:
$$
\sum_{n=0}^{\infty} 4 \times (2)^n
$$
- $a=4$, $r=2$
- $|r|=2 \geq 1$ → Diverges
- The partial sums grow exponentially; the series diverges.

Additional Considerations

While the geometric series provides a straightforward convergence test, some series may not be geometric but still convergent. In those cases, more advanced tests like the ratio test, root test, or comparison test are employed. However, for geometric series, the ratio test simplifies to the criterion $|r|<1$.

Summary

- Determine the ratio r.
- Check if $|r|<1$.
- If yes, the series converges, and the sum is $\frac{a}{1-r}$.
- If no, the series diverges.
- For the series $\sum_{n=0}^\infty (3)^n$, since $|r|=3>1$, the series diverges.

Final Thoughts

Understanding whether a geometric series converges or diverges is a foundational skill in calculus and series analysis. Recognizing the key role played by the ratio r simplifies the process significantly. Whether dealing with financial models, physics, or pure mathematics, the ability to quickly determine the behavior of an infinite geometric series is invaluable. Remember, the crux lies in the magnitude of the ratio: if it’s less than one, the series converges; if it’s equal to or greater than one, it diverges.

Determine Whether The Geometric Series Is Convergent Or Divergent Infinity 1 3 N N 0

Determine Whether The Geometric Series Is Convergent Or Divergent Infinity 1 3 N N 0

Related Articles

- Language And Communication Skills Assessment Conducted By Behavior Analysis Will Focus On
- How Many Stereoisomers Will Be Formed From The Addition Of Phenyllithium To This Molecule
- How Long Does It Take For The Energy To Increase By A Factor Of 5.0 From The Initial Value?

Back to Home