Evaluate The Expression -4-4i/4i And Write The Result In The Form A + Bi. Submit Question

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Understanding Complex Numbers and Their Operations

Complex numbers are fundamental in advanced mathematics, engineering, physics, and many applied sciences. They extend the real number system by introducing the imaginary unit, denoted as i, where i satisfies the equation $i^2 = -1$. The general form of a complex number is expressed as:

$$A + Bi$$

where A and B are real numbers, and i is the imaginary unit.

In this article, we will evaluate the complex expression -4 - 4i / 4i and write the simplified result in the form A + Bi. This process involves understanding complex division, conjugates, and simplifying complex fractions.

Step-by-Step Breakdown of the Expression

Before diving into the calculation, let's clearly state the expression:

$$-4 - \frac{4i}{4i}$$

The main goal is to simplify this expression into the form A + Bi.

Step 1: Simplify the Fraction $\frac{4i}{4i}$

The first step involves simplifying the fraction $\frac{4i}{4i}$. Since both numerator and denominator are the same (assuming they are not zero), this simplifies directly to 1, provided that the denominator is not zero.

Important note: Zero in the denominator must be avoided because division by zero is undefined.

Given that $4i \neq 0$, the fraction simplifies as:

$$
\frac{4i}{4i} = 1
$$

Therefore:

$$
-4 - \frac{4i}{4i} = -4 - 1
$$

which simplifies to:

$$
-4 - 1 = -5
$$

Result so far: The expression simplifies to -5.

Verifying the Simplification

While the straightforward division indicates the answer is -5, it's essential to understand why this is valid in the context of complex numbers, especially when dealing with division involving complex numbers.

Why is $\frac{4i}{4i} = 1$?

The reasoning stems from the properties of complex numbers:

- Any non-zero complex number divided by itself equals 1.
- The key is to ensure the denominator is not zero.

Since $4i \neq 0$, the division is valid:

$$
\frac{4i}{4i} = 1
$$

Alternative Approach: Multiplying numerator and denominator by the conjugate

In cases where the denominator is a complex number, multiplying numerator and denominator by the conjugate of the denominator simplifies the expression.

Conjugate of $4i$: $-4i$

Applying this method:

$$
\frac{4i}{4i} \times \frac{-4i}{-4i} = \frac{4i \times -4i}{(4i) \times (-4i)} = \frac{-16i^2}{-16i^2}
$$

Since $i^2 = -1$:

$$
-16i^2 = -16 \times (-1) = 16
$$

Thus,

$$
\frac{16}{16} = 1
$$

which confirms the previous result.

Complete Simplification of the Expression

Now, let's revisit the entire original expression:

$$
-4 - \frac{4i}{4i}
$$

Substituting the simplified fraction:

$$
-4 - 1
$$

which equals:

$$
-5
$$

Since -5 is a real number, it can be written in the form A + Bi as:

$$
-5 + 0i
$$

where:

- A = -5
- B = 0

Final answer:

$$
\boxed{-5 + 0i}
$$

Understanding the Significance of the Result

The key takeaways from this problem are:

- Performing division involving complex numbers often simplifies to real numbers when the numerator and denominator are identical, provided the denominator is not zero.
- Expressing the final answer in the form A + Bi is essential for clarity, especially when dealing with complex numbers.
- Such problems reinforce the importance of understanding both algebraic manipulation and properties of imaginary numbers.

Additional Tips for Simplifying Complex Expressions

When working with complex numbers, keep these tips in mind:

- **Always check for zero denominators:** Dividing by zero is undefined, so verify that the denominator is non-zero before simplifying.
- **Use conjugates to rationalize denominators:** When the denominator is a complex number, multiply numerator and denominator by its conjugate to eliminate the imaginary part from the denominator.
- **Recall properties of i:** Remember that $i^2 = -1$, which is fundamental in simplifying powers of i.
- **Express results in standard form:** Always write the final answer as A + Bi for clarity, even if B is zero.

Practice Problems to Reinforce Learning

To strengthen understanding of complex number operations, consider practicing with these problems:

1. Evaluate $(3 + 2i) + (4 - 5i)$ and express in $A + Bi$ form.
2. Simplify $\frac{5 + 7i}{1 - 2i}$ and write the answer in standard form.
3. Calculate $(2i)^3$ and express the result as $A + Bi$.
4. Find the modulus and argument of the complex number $-3 + 4i$.
5. Express the product $(1 + i)(1 - i)$ in standard form.

Each of these exercises helps develop proficiency in complex number arithmetic, preparing you for more advanced mathematical problems.

Conclusion

In summary, the expression $-4 - \frac{4i}{4i}$ simplifies straightforwardly to -5, which can be written in the standard complex number form as −5 + 0i. This problem exemplifies key concepts in complex number algebra, including division, the use of conjugates, and the importance of expressing answers in the standard form A + Bi. Mastery of these techniques is essential for tackling a wide range of problems involving complex numbers in mathematics and engineering.

Remember, always verify the denominator is not zero, simplify step-by-step, and express your final answer clearly. Practice with similar problems to deepen your understanding and become proficient in complex number operations.

Frequently Asked Questions

What is the value of the expression (−4 − 4i) divided by 4i?

The value of the expression is 1 − i in the form A + Bi.

How do you simplify (−4 − 4i) / 4i into the form A + Bi?

By multiplying numerator and denominator by the conjugate of the denominator and simplifying, the expression simplifies to 1 − i.

What are the steps to convert (−4 − 4i) / 4i into A + Bi format?

First, write the division as a fraction, then multiply numerator and denominator by the conjugate of the denominator, simplify, and separate the real and imaginary parts to get A + Bi.

Is the expression (−4 − 4i) / 4i equal to 1 − i?

Yes, after simplification, (−4 − 4i) / 4i equals 1 − i.

What is the importance of expressing complex numbers in A + Bi form?

Expressing complex numbers in A + Bi form makes it easier to perform arithmetic operations and understand their components clearly.

Can you verify the result of (−4 − 4i) / 4i by

substitution?

Yes, by simplifying the expression to 1 – i, which is in A + Bi form, confirms the result.

What is the numerator after multiplying numerator and denominator by the conjugate of 4i?

The numerator becomes (-4 – 4i) (-4i), which simplifies to a real part and an imaginary part upon expansion.

How do you handle division involving imaginary units in complex expressions?

You multiply numerator and denominator by the conjugate of the denominator to eliminate the imaginary unit from the denominator, then simplify.

What is the simplified form of (-4 – 4i) / 4i in complex number notation?

The simplified form is 1 – i.

Why is it useful to write complex numbers in the form A + Bi?

Writing complex numbers as A + Bi clearly separates the real and imaginary parts, facilitating easier calculations and understanding.

Additional Resources

Evaluate the Expression -4 – 4i / 4i and Write the Result in the Form A + Bi

In the realm of complex numbers, the ability to manipulate and evaluate expressions involving real and imaginary parts is fundamental. The expression -4 – 4i / 4i presents an intriguing opportunity to explore complex division, a process that extends real-number algebra into the complex plane. By carefully analyzing and simplifying this expression, we can not only arrive at a simplified form but also deepen our understanding of how complex fractions behave. This article delves into the step-by-step evaluation of the expression, explaining underlying concepts thoroughly, and culminates in presenting the result in the standard form A + Bi. Such exercises are vital in fields ranging from engineering to physics, where complex numbers underpin many calculations.

Understanding the Components of the Expression

Before diving into the division, it’s essential to understand the components involved:

- −4: This is a real number, situated entirely on the real axis in the complex plane.
- −4i: Represents an imaginary number with a magnitude of 4, lying on the imaginary axis, pointing downward since the coefficient is negative.
- 4i: Also an imaginary number with magnitude 4, but pointing upward on the imaginary axis.

The entire expression:

$$
\frac{-4 - 4i}{4i}
$$

involves dividing a complex number by another complex number. The key challenge here is to simplify the fraction into the form A + Bi, where A and B are real numbers.

Step-by-Step Evaluation of the Expression

The process of evaluating a complex division involves several crucial steps:

Step 1: Recognize the Division of Complex Numbers

Given two complex numbers, $z_1 = a + bi$ and $z_2 = c + di$, the division $\frac{z_1}{z_2}$ is performed by multiplying numerator and denominator by the conjugate of the denominator. This process eliminates the imaginary component in the denominator, simplifying the expression.

The conjugate of $z_2 = c + di$ is $c - di$.

Step 2: Identify Numerator and Denominator

In our case:

- Numerator $N = -4 - 4i$
- Denominator $D = 4i$

Note that D is purely imaginary, which simplifies the process somewhat, but we will still follow the standard conjugate method.

Step 3: Multiply Numerator and Denominator by the Conjugate of the Denominator

Since $D = 4i$, its conjugate is $-4i$.

Multiplying numerator and denominator by $-4i$:

$$
\frac{-4 - 4i}{4i} \times \frac{-4i}{-4i} = \frac{(-4 - 4i)(-4i)}{(4i)(-4i)}
$$

This step ensures the denominator becomes a real number.

Calculating the Numerator

Let's focus on the numerator:

\[
(-4 - 4i)(-4i)
\]

Distribute:

\[
(-4)(-4i) + (-4i)(-4i)
\]

Calculate each term:

- \(-4 \times -4i = 16i \)
- \(-4i \times -4i = 16i^2 \)

Recall that \(i^2 = -1 \), so:

\[
16i^2 = 16 \times -1 = -16
\]

Adding both terms:

\[
16i - 16
\]

Thus, the numerator simplifies to:

\[
16i - 16
\]

Calculating the Denominator

Now, evaluate:

\[
(4i)(-4i)
\]

Multiply:

\[
4i \times -4i = -16i^2
\]

Since \(i^2 = -1 \):

\[

$$
-16 \times -1 = 16
$$

The denominator simplifies to:

$$
16
$$

Final Simplification

Putting it all together, the division becomes:

$$
\frac{16i - 16}{16}
$$

Separate numerator components:

$$
\frac{16i}{16} - \frac{16}{16}
$$

Simplify each term:

- $\frac{16i}{16} = i$
- $\frac{16}{16} = 1$

Therefore, the entire expression simplifies to:

$$
i - 1
$$

Expressed in the standard form A + Bi, this is:

$$
-1 + 1i
$$

or more conventionally:

$$
-1 + i
$$

Final Result and Interpretation

The evaluation of the complex expression -4 - 4i / 4i yields the result -1 + i. This form clearly delineates the real part (A = -1) and the imaginary part

(B = 1). Such a result is not just a numerical conclusion but also offers insights into the geometric interpretation in the complex plane:

- The real part, -1, indicates a shift one unit to the left along the real axis.
- The imaginary part, 1, indicates a displacement one unit upward along the imaginary axis.

The process underscores the importance of conjugates in complex division, a technique rooted in the algebraic properties of imaginary units. This method ensures that the denominator becomes a real number, allowing straightforward simplification.

Broader Implications and Applications

Understanding how to evaluate complex expressions like this is pivotal in various scientific and engineering disciplines:

- Electrical Engineering: Complex impedance calculations often involve dividing complex numbers.
- Control Systems: Stability analysis utilizes complex plane representations, requiring manipulation of complex fractions.
- Quantum Physics: Wave functions and probability amplitudes are expressed as complex numbers, demanding precise algebraic handling.
- Mathematics Education: Mastery of complex division reinforces foundational algebra and prepares students for advanced topics like complex analysis.

Furthermore, the technique exemplified here—the use of conjugates—serves as a cornerstone in complex number arithmetic, ensuring that operations such as division are manageable and mathematically rigorous.

Conclusion

The comprehensive evaluation of the expression -4 - 4i / 4i demonstrates the elegance and utility of complex algebra. By systematically multiplying numerator and denominator by the conjugate of the denominator, the division simplifies to a form that neatly encapsulates both real and imaginary components. Arriving at -1 + i not only provides a clear, standardized answer but also exemplifies the broader principles of complex number manipulation. Mastering such techniques is essential for students, professionals, and researchers who rely on complex analysis to solve real-world problems across diverse scientific domains. As complex numbers continue to underpin modern technological advances, the ability to evaluate and interpret such expressions remains a fundamental skill in the mathematical toolkit.

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