

Evaluate The Integral. (Use C For The Constant Of Integration.) $\int \frac{x + 11}{x^2 + 4x + 8} dx$

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Introduction

Calculus is a fundamental branch of mathematics that deals with rates of change and the accumulation of quantities. One of its core concepts is integration, which allows us to find the total accumulation of a quantity given its rate of change. Integrals are widely applied across various disciplines such as physics, engineering, economics, and biology. They enable us to compute areas under curves, volumes, work done by forces, and more.

In this article, we will focus on evaluating a specific indefinite integral:

$$\int \frac{x + 11}{x^2 + 4x + 8} dx$$

where the constant of integration, denoted by (C) , is used to account for the family of antiderivatives. This integral appears to be straightforward at first glance but involves multiple techniques like algebraic manipulation and substitution to arrive at the solution. Understanding how to evaluate such integrals enhances your calculus skills and deepens your understanding of integral calculus.

Understanding the Integral

Before diving into the calculation, it is essential to analyze the structure of the integral:

$$\int \frac{x + 11}{x^2 + 4x + 8} dx$$

This integral involves a rational function where the numerator is linear, and the denominator is a quadratic expression. Typically, integrals of this form are tackled using techniques like:

- Polynomial division (if numerator degree $\geq$ denominator degree)
- Completing the square in the denominator

- Partial fractions decomposition
- Substitution methods

Given the quadratic in the denominator, the most suitable approach here is to complete the square and then utilize substitution to simplify the integral.

Step 1: Completing the Square in the Denominator

The quadratic expression in the denominator is:

$$\sqrt{X^2 + 4X + 8}$$

To facilitate substitution, rewrite it by completing the square:

$$\sqrt{X^2 + 4X + 8} = \sqrt{(X^2 + 4X + 4) + 4} = \sqrt{(X + 2)^2 + 4}$$

This form is advantageous because it resembles a standard derivative form involving a squared term plus a constant.

Step 2: Rewrite the Integral

Express the original integral with the completed square:

$$\int \frac{X + 11}{\sqrt{(X + 2)^2 + 4}} \, dx$$

To simplify, perform a substitution:

$$\begin{aligned} u &= X + 2 \\ du &= dx \\ X &= u - 2 \end{aligned}$$

Express the numerator in terms of u :

$$X + 11 = (u - 2) + 11 = u + 9$$

\]

Now, rewrite the integral in terms of (u) :

$$\int \frac{u + 9}{u^2 + 4} \, du$$

This separates naturally into two integrals:

$$\int \frac{u}{u^2 + 4} \, du + 9 \int \frac{1}{u^2 + 4} \, du$$

Step 3: Evaluate the First Integral

Integral 1:

$$I_1 = \int \frac{u}{u^2 + 4} \, du$$

This is a straightforward integral that can be solved via substitution:

Let:

$$w = u^2 + 4 \rightarrow dw = 2u \, du$$

Thus,

$$u \, du = \frac{1}{2} dw$$

Rewriting the integral:

$$I_1 = \int \frac{u}{w} \, du = \frac{1}{2} \int \frac{1}{w} \, dw = \frac{1}{2} \ln |w| + C = \frac{1}{2} \ln |u^2 + 4| + C$$

Step 4: Evaluate the Second Integral

Integral 2:

$$I_2 = 9 \int \frac{1}{u^2 + 4} \, du$$

This integral is a standard form:

$$\int \frac{1}{u^2 + a^2} \, du = \frac{1}{a} \arctangent \left(\frac{u}{a} \right) + C$$

Here, $(a^2 = 4 \Rightarrow a = 2)$. Therefore:

$$I_2 = 9 \times \frac{1}{2} \arctangent \left(\frac{u}{2} \right) + C = \frac{9}{2} \arctangent \left(\frac{u}{2} \right) + C$$

Step 5: Combine the Results and Re-Substitute

Adding the two parts:

$$\int \frac{u + 9}{u^2 + 4} \, du = \frac{1}{2} \ln |u^2 + 4| + \frac{9}{2} \arctangent \left(\frac{u}{2} \right) + C$$

Recall that:

$$u = X + 2$$

Thus, the indefinite integral becomes:

$$\boxed{\int \frac{X + 11}{X^2 + 4X + 8} \, dx = \frac{1}{2} \ln |(X + 2)^2 + 4| + \frac{9}{2} \arctangent \left(\frac{X + 2}{2} \right) + C}$$

where (C) is the constant of integration.

Additional Insights and Applications

Why Completing the Square is Useful

Completing the square transforms the quadratic into a sum of a perfect square and a constant, simplifying the integral significantly. This technique is essential in calculus, especially when dealing with quadratic denominators in integrals, as it allows for standard substitution methods such as arctangent functions.

Practical Applications of the Integral

- Physics: Calculating work or energy where the force varies according to a quadratic function.
- Engineering: Signal processing involving quadratic filters.
- Economics: Marginal analysis with quadratic cost functions.
- Biology: Modeling growth rates with quadratic relationships.

Common Mistakes to Avoid

- Forgetting to update limits or variables after substitution.
- Neglecting the absolute value in logarithmic functions, especially when the domain is not specified.
- Failing to include the constant of integration (C) when presenting indefinite integrals.

Summary

In this comprehensive guide, we explored the process of evaluating the indefinite integral:

$$\int \frac{X + 11}{X^2 + 4X + 8} \, dx$$

The key steps involved:

1. Recognizing the quadratic in the denominator and completing the square.
2. Substituting $(u = X + 2)$ to simplify the integral.
3. Separating the integral into manageable parts.
4. Applying standard integral formulas, particularly for rational and arctangent functions.
5. Reverting back to the original variable (X) .

The final expression provides the antiderivative explicitly, incorporating natural logarithms and arctangent functions:

$$\boxed{\int \frac{X + 11}{X^2 + 4X + 8} \, dx = \frac{1}{2} \ln |(X + 2)^2 + 4| + \frac{9}{2} \arctangent \left(\frac{X + 2}{2} \right) + C}$$

Mastering such integral evaluations enhances your calculus toolkit, enabling you to tackle more complex problems involving rational functions and quadratic denominators efficiently. Whether for academic pursuits or practical applications, understanding the techniques outlined here is invaluable for any student or professional working with calculus.

Additional Resources

- Khan Academy Calculus Courses: Comprehensive tutorials on integration techniques.
- Paul's Online Math Notes: Detailed explanations and practice problems.
- Textbooks: "Calculus" by James Stewart and "Calculus: Early Transcendentals" by Howard Anton.
- Online Calculators: WolframAlpha and Symbolab for verifying integrals.

Final Thoughts

Evaluating integrals such as $\int \frac{X + 11}{X^2 + 4X + 8} dx$ demonstrates the importance of algebraic manipulation and substitution in calculus. These skills are fundamental for progressing toward more advanced topics like differential equations, multivariable calculus, and real-world problem-solving. Always approach integrals methodically, and remember to check your work by differentiation if needed. Happy integrating!

Frequently Asked Questions

How do you set up the integral for $\int (x + 11) / (x^2 + 4x + 8) dx$?

You write the integral as is, recognizing the numerator and denominator, and plan to use substitution or partial fractions if applicable. First, note that the denominator can be completed to a square: $x^2 + 4x + 8 = (x + 2)^2 + 4$.

What substitution can simplify the integral $\int (x + 11) / (x^2 + 4x + 8) dx$?

Use the substitution $t = x + 2$, which simplifies the denominator to $t^2 + 4$. Then, $dx = dt$, and $x + 11$ becomes $(t - 2) + 11 = t + 9$.

How do you split the integral after substitution for easier evaluation?

Rewrite the integral as $\int (t + 9) / (t^2 + 4) dt$, then split it into two

integrals: $\int t / (t^2 + 4) dt + 9 \int 1 / (t^2 + 4) dt$.

What is the integral of $t / (t^2 + a^2) dt$?

The integral of $t / (t^2 + a^2) dt$ is $(1/2) \ln |t^2 + a^2| + C$.

How do you evaluate $\int 1 / (t^2 + 4) dt$?

Use the standard form: $\int 1 / (t^2 + a^2) dt = (1/a) \arctangent(t / a) + C$. Here, $a = 2$, so the integral becomes $(1/2) \arctangent(t / 2) + C$.

What is the final expression for the integral $\int (x + 11) / (x^2 + 4x + 8) dx$ after substitution and simplification?

After substituting $t = x + 2$, the integral becomes $(1/2) \ln |t^2 + 4| + (9/2) \arctangent(t / 2) + C$. Replacing t back with $x + 2$, the answer is $(1/2) \ln |(x + 2)^2 + 4| + (9/2) \arctangent((x + 2) / 2) + C$.

Additional Resources

Evaluate The Integral. (Use C For The Constant Of Integration.) $X + 11 / X^2 + 4x + 8 Dx$

Introduction

The process of evaluating integrals is foundational to calculus, serving as the backbone for understanding areas, accumulated quantities, and the inverse operation of differentiation. The integral presented, which involves a rational function of the form $\int \frac{x + 11}{x^2 + 4x + 8} dx$, offers an intriguing challenge that combines algebraic manipulation with integral calculus techniques. This article aims to dissect the problem comprehensively, guiding readers through the steps of simplifying, integrating, and interpreting the solution, all while emphasizing the importance of constants of integration.

Understanding the Integral

The Mathematical Expression

The integral in question is:

$$\int \frac{x + 11}{x^2 + 4x + 8} dx$$

\]

Here, the numerator $(x + 11)$ and the denominator $(x^2 + 4x + 8)$ suggest that straightforward substitution might be insufficient unless we first manipulate the integrand into a more manageable form.

Significance of the Components

- Numerator $(x + 11)$: A linear expression, which hints that parts of the integrand may relate to the derivative of the denominator.
- Denominator $(x^2 + 4x + 8)$: Quadratic in form, which typically suggests completing the square or substitution techniques for integration.

Goal of the Evaluation

Our goal is to find an antiderivative $F(x)$ such that:

$$F'(x) = \frac{x + 11}{x^2 + 4x + 8}$$

and to include the constant of integration (C) to represent the family of all antiderivatives.

Step-by-Step Approach to Integration

1. Simplify the Denominator

The quadratic in the denominator can be rewritten by completing the square:

$$x^2 + 4x + 8 = (x^2 + 4x + 4) + 4 = (x + 2)^2 + 4$$

This form is advantageous because it resembles a standard integral involving quadratic denominators.

2. Decompose the Numerator

Since the numerator is $(x + 11)$, it can be expressed in terms of $(x + 2)$ to facilitate substitution:

$$x + 11 = (x + 2) + 9$$

This decomposition allows us to split the original integral into two parts:

\[

$$\int \frac{x + 11}{(x + 2)^2 + 4} \, dx = \int \frac{(x + 2)}{(x + 2)^2 + 4} \, dx + 9 \int \frac{1}{(x + 2)^2 + 4} \, dx$$

Techniques for Integration

3. Substitution Strategy

Let:

$$u = x + 2 \quad \Rightarrow \quad du = dx$$

Rewriting the integral in terms of u :

$$\int \frac{u}{u^2 + 4} \, du + 9 \int \frac{1}{u^2 + 4} \, du$$

Now, the problem reduces to standard forms:

- $\int \frac{u}{u^2 + a^2} \, du$
- $\int \frac{1}{u^2 + a^2} \, du$

4. Standard Integral Forms

- Integral of $\left(\frac{u}{u^2 + a^2}\right)$:

$$\int \frac{u}{u^2 + a^2} \, du = \frac{1}{2} \ln |u^2 + a^2| + C$$

- Integral of $\left(\frac{1}{u^2 + a^2}\right)$:

$$\int \frac{1}{u^2 + a^2} \, du = \frac{1}{a} \arctan \left(\frac{u}{a} \right) + C$$

where $(a > 0)$.

Performing the Integration

5. Computing the First Integral

$$\int$$

$$\int \frac{u}{u^2 + 4} \, du = \frac{1}{2} \ln |u^2 + 4| + C$$

6. Computing the Second Integral

$$\int \frac{1}{u^2 + 4} \, du = \frac{1}{2} \arctan \left(\frac{u}{2} \right) + C = \frac{9}{2} \arctan \left(\frac{u}{2} \right) + C$$

Final Expression of the Integral

7. Reverting to Original Variable

Recall that $u = x + 2$. Substituting back, the indefinite integral becomes:

$$F(x) = \frac{1}{2} \ln |(x + 2)^2 + 4| + \frac{9}{2} \arctan \left(\frac{x + 2}{2} \right) + C$$

where C is the constant of integration, representing the family of all antiderivatives.

Analytical Insights and Interpretations

8. Significance of the Result

The obtained antiderivative combines logarithmic and inverse tangent functions, reflecting the integral's structure:

- The logarithmic component arises from integrating the linear term over a quadratic denominator.
- The arctangent component emerges naturally from the integral involving the quadratic form in the denominator.

This combination underscores the interconnectedness of algebraic manipulation and integral calculus techniques, such as completing the square and standard integral formulas.

9. Applications of the Integral

Understanding such integrals has broad implications across various scientific fields:

- Physics: Calculating work, energy, or fields where rational functions model physical phenomena.
- Engineering: Signal processing and control systems often involve similar integrals.
- Mathematics: Analyzing functions' behaviors, solving differential equations, or modeling real-world problems.

Additional Considerations

10. Partial Fraction Decomposition

While the approach used here bypassed partial fractions, in cases with more complex denominators, such decompositions are invaluable. For this specific quadratic, partial fractions would involve complex roots or factorization, but completing the square provided a more straightforward path.

11. Numerical Evaluation

For specific values of x , numerical methods can approximate the integral's value, especially when dealing with definite integrals or when exact expressions become cumbersome.

Broader Context and Educational Significance

12. Teaching and Learning Implications

This integral exemplifies several essential calculus concepts:

- The importance of algebraic manipulation before integration.
- Recognizing standard integral forms.
- The utility of substitution techniques.
- The significance of constants of integration in indefinite integrals.

Educators often use similar problems to reinforce the interconnectedness of algebra and calculus, fostering deeper understanding.

13. Common Mistakes and Pitfalls

Students might encounter pitfalls such as:

- Forgetting the absolute value in the logarithm.
- Misapplying the arctangent integral formula.
- Failing to revert substitution correctly.
- Overlooking the constant of integration.

Awareness of these can improve problem-solving accuracy.

Conclusion

Evaluating the integral $\int \frac{x + 11}{x^2 + 4x + 8} \, dx$ exemplifies the elegant interplay between algebra and calculus. By completing the square, decomposing the numerator, applying substitution, and utilizing standard integral formulas, we derive a comprehensive antiderivative:

$$\boxed{F(x) = \frac{1}{2} \ln |(x + 2)^2 + 4| + \frac{9}{2} \arctan \left(\frac{x + 2}{2} \right) + C}$$

This solution reflects the rich structure underlying rational functions and showcases the power of calculus techniques. Understanding such integrals deepens one's grasp of mathematical analysis, with broad applications across science, engineering, and mathematics.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Apostol, T. M. (1967). Calculus, Volume 1. Wiley.
- Thomas, G. B., & Finney, R. L. (2002). Calculus and Analytic Geometry. Pearson Education.

Note: The integral evaluated here is indefinite, and the constant (C) captures the family of all antiderivatives.

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Evaluate The Integral Use C For The Constant Of Integration X¹¹ X² 4x 8 Dx

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