Find A Parametric Equation Of The Line Of Intersection Of The Planes X+y = 4 And 2x Y Z = 2.

Find A Parametric Equation Of The Line Of Intersection Of The Planes X + y = 4 And 2x - y + z = 2.

Introduction

In three-dimensional geometry, the intersection of two planes generally results in a straight line. Finding the parametric equation of this line involves understanding the equations of the planes involved and deriving a set of parametric equations that describe all points lying on both planes simultaneously. This process is fundamental in analytical geometry, physics, and engineering applications, such as computing line-of-sight, designing mechanical parts, or analyzing spatial relationships.

In this article, we will explore how to find the parametric equation of the line of intersection between two specific planes:

- Plane 1: $x + y = 4$
- Plane 2: $2x - y + z = 2$

We will proceed step by step, starting from understanding the equations, finding points on the line, and finally expressing the line parametrically.

Understanding the Given Planes

Equation of the First Plane: $x + y = 4$

This is a plane in three-dimensional space where the sum of the x and y coordinates always equals 4. It can be rewritten as:

$$y = 4 - x$$

This equation indicates that for any value of x, y is determined, and z can vary freely because z does not appear in the equation. Therefore, this plane extends infinitely in the z-direction, forming a "sloped" plane in the x-y plane.

Equation of the Second Plane: $2x - y + z = 2$

This plane relates x, y, and z coordinates. Rearranging for z:

$$ z = 2 - 2x + y $$

This plane is inclined in space, and z depends on x and y.

Finding the Line of Intersection

The intersection line consists of all points (x, y, z) that satisfy both equations simultaneously. To find this line, we need to find parametric expressions for x, y, and z that satisfy both equations.

Step 1: Express variables to reduce the problem

Since the first plane gives:

$$ y = 4 - x $$

We can substitute this into the second plane's equation to eliminate y:

$$ z = 2 - 2x + y $$
$$ z = 2 - 2x + (4 - x) $$
$$ z = 2 - 2x + 4 - x $$
$$ z = (2 + 4) - (2x + x) $$
$$ z = 6 - 3x $$

Now, the three variables are expressed in terms of x:

- $y = 4 - x$
- $z = 6 - 3x$

x remains a free parameter. Let's denote:

$$ x = t $$

where t is a real parameter that can take any value.

Step 2: Write the parametric equations

From the above, the parametric equations are:

$$ x = t $$
$$ y = 4 - t $$
$$ z = 6 - 3t $$

These equations describe all points on the line of intersection.

Step 3: Write the parametric equations clearly

The parametric form of the line is:

\[
\boxed{
\begin{cases}
x = t \\
y = 4 - t \\
z = 6 - 3t
\end{cases}
}
\]
where \(t \in \mathbb{R} \).

Vector Equation of the Line

The parametric equations can be expressed as a vector equation:

\[
\vec{r}(t) = \vec{r}_0 + t \vec{d}
\]

where:

- \(\vec{r}_0 \) is a point on the line,
- \(\vec{d} \) is the direction vector of the line.

Finding a point on the line (\(\vec{r}_0 \))

Choose a convenient value of \(t \), for instance \(t=0 \):

\[
x=0, \quad y=4, \quad z=6
\]

So, one point on the line is:

\[
\vec{r}_0 = (0, 4, 6)
\]

Finding the direction vector (\(\vec{d} \))

The coefficients of \(t \) in the parametric equations give the direction vector:

\[
\vec{d} = (1, -1, -3)
\]

Thus, the vector equation of the line is:

\[
\boxed{

\vec{r}(t) = (0, 4, 6) + t(1, -1, -3)
}
\]

Summary of the Results

To summarize:

- Parametric equations:

$$
\begin{cases}
x = t \\
y = 4 - t \\
z = 6 - 3t
\end{cases}
$$

- Vector form:

$$
\boxed{
\vec{r}(t) = (0, 4, 6) + t(1, -1, -3)
}
$$

where $t \in \mathbb{R}$.

Conclusion

The process of finding the parametric equation of the line of intersection of two planes involves substitution and elimination. By expressing one variable in terms of the free parameter and substituting into the other equations, we obtain a set of parametric equations that describe all points on the line. The key steps include:

- Recognizing the relationship between the planes,
- Choosing a free parameter (commonly t),
- Expressing all variables in terms of this parameter,
- Writing the parametric and vector equations.

This approach is versatile and can be applied to other pairs of planes or surfaces to find their line or curve of intersection.

Understanding this method enhances spatial visualization skills and analytical problem-solving in three-dimensional geometry, which are essential in advanced mathematics, physics, and engineering disciplines.

Frequently Asked Questions

How do you find the parametric equation of the line of intersection of the planes x + y = 4 and 2x + y + z = 2?

First, express one variable in terms of another from one plane, then substitute into the second plane to find parametric equations. For example, from $x + y = 4$, let $y = 4 - x$. Substitute into $2x + y + z = 2$ to find z in terms of x, then parametrize x as a parameter t.

What steps are involved in deriving the parametric equations of the line where the planes x + y = 4 and 2x + y + z = 2 intersect?

1. Solve one plane for a variable (e.g., $y = 4 - x$). 2. Substitute into the second plane to find z in terms of x. 3. Choose x as a parameter t, then write y and z in terms of t to get the parametric equations.

Can you provide the parametric equations for the line of intersection of the planes x + y = 4 and 2x + y + z = 2?

Yes. Let t be a parameter. From $x + y = 4$, set $x = t$. Then $y = 4 - t$. Substitute into $2x + y + z = 2$: $2t + (4 - t) + z = 2 \rightarrow t + 4 + z = 2 \rightarrow z = -t - 2$. Thus, the parametric equations are: $x = t$, $y = 4 - t$, $z = -t - 2$.

Why is it useful to parametrize the line of intersection of two planes?

Parametrizing the line provides a clear, algebraic description of the intersection, which is helpful for graphing, further analysis, or solving related geometry problems involving the line.

What is the geometric significance of the parametric equations derived for the intersection line of the planes?

The parametric equations represent all points that lie simultaneously on both planes, describing the line where the two planes intersect in three-dimensional space.

Additional Resources

Parametric Equation of the Line of Intersection of Two Planes

Understanding the intersection of planes is a fundamental concept in three-dimensional geometry, with applications spanning engineering, physics, computer graphics, and more. In this article, we delve into deriving the parametric equation of the line formed by the intersection of two specific planes:

- Plane 1: $x + y = 4$
- Plane 2: $2x + y + z = 2$

Our approach will be systematic, thorough, and designed to not only find the solution but also deepen your understanding of the geometric and algebraic principles involved.

Understanding the Geometry of Planes and Their Intersections

The Significance of Plane Intersections

In three-dimensional space, planes are flat, two-dimensional surfaces extending infinitely in all directions. When two planes intersect, their intersection is generally a line—unless they are parallel (no intersection) or coincident (infinite intersections).

The line of intersection is characterized by points that satisfy the equations of both planes simultaneously. Finding its parametric form allows us to describe every point on this line efficiently, which is vital in applications like computer-aided design (CAD), collision detection, and geometric modeling.

Why Find a Parametric Equation?

A parametric equation provides a clear, flexible way to represent a line through a point and a direction vector:

$$
\begin{cases}
x = x_0 + at \\
y = y_0 + bt \\
z = z_0 + ct
\end{cases}
$$

where (x_0, y_0, z_0) is a point on the line, $\vec{d} = \langle a, b, c \rangle$ is the direction vector, and t is a parameter that varies over all real numbers.

Step-by-Step Derivation of the Line of

Intersection

Step 1: Write Down the Equations

Given the planes:

1. $x + y = 4$ — (Equation 1)
2. $2x + y + z = 2$ — (Equation 2)

Our goal: Find the set of points (x, y, z) satisfying both equations, then express this set parametrically.

Step 2: Express One Variable in Terms of Others

Start with Equation 1:

$$
x + y = 4 \Rightarrow y = 4 - x
$$

This allows us to express y in terms of x.

Step 3: Substitute into the Second Equation to Find z

Substitute $y = 4 - x$ into Equation 2:

$$
2x + (4 - x) + z = 2
$$

Simplify:

$$
2x + 4 - x + z = 2
$$
$$
(2x - x) + 4 + z = 2
$$
$$
x + 4 + z = 2
$$

Solve for z:

$$
z = 2 - x - 4 = -x - 2
$$

Now, both y and z are expressed in terms of x:

\[

\begin{cases}
y = 4 - x \\
z = -x - 2
\end{cases}
\]

Step 4: Choose a Parameter and Express All Coordinates

Let’s designate the parameter \(t \) as \(x \):

\[
x = t
\]

Substitute into the expressions for \(y \) and \(z \):

\[
\begin{cases}
x = t \\
y = 4 - t \\
z = -t - 2
\end{cases}
\]

This parametrization captures every point on the line of intersection.

Formulating the Parametric Equation

Point on the Line

You can select any specific value of \(t \) to find a particular point on the line. For the parametric form, it's typical to express the line as a function of \(t \):

\[
\boxed{
\begin{cases}
x = t \\
y = 4 - t \\
z = -t - 2
\end{cases}
}
\]

Alternatively, in vector form:

\[
\vec{r}(t) = \langle t, 4 - t, -t - 2 \rangle
\]

or

$$
\vec{r}(t) = \langle 0, 4, -2 \rangle + t \langle 1, -1, -1 \rangle
$$

where:

- $\boxed{\textbf{Point } P} = (0, 4, -2)$ — a specific point on the line (found by setting $t=0$)
- $\boxed{\textbf{Direction vector } \vec{d}} = \langle 1, -1, -1 \rangle$

Note: The point $P = (0, 4, -2)$ satisfies both equations:

- $x + y = 0 + 4 = 4$
- $2 \times 0 + 4 + (-2) = 0 + 4 - 2 = 2$

Interpretation and Validation

Geometric Significance of the Direction Vector

The direction vector $\vec{d} = \langle 1, -1, -1 \rangle$ indicates that:

- Moving one unit in x increases x by 1.
- Simultaneously, y decreases by 1, and z decreases by 1.

This aligns with how changing t affects all coordinates, maintaining the line as the intersection of the two planes.

Verification of the Parametric Equation

To verify, pick an arbitrary t, say $t=2$:

$$
x = 2, \quad y = 4 - 2 = 2, \quad z = -2 - 2 = -4
$$

Check if this point lies on both planes:

- Plane 1: $x + y = 2 + 2 = 4$
- Plane 2: $2x + y + z = 4 + 2 - 4 = 2$

Thus, the parametric equations are consistent and correctly describe the line of intersection.

Additional Insights and Applications

Alternative Representations

While the parametric form is often the most straightforward, other forms include:

- Symmetric equations:

$$
\frac{x - 0}{1} = \frac{y - 4}{-1} = \frac{z + 2}{-1}
$$

This form is useful for visualizations and understanding the line’s orientation.

- Vector form:

$$
\vec{r} = \vec{r}_0 + t \vec{d}
$$

where $\vec{r}_0 = \langle 0, 4, -2 \rangle$, $\vec{d} = \langle 1, -1, -1 \rangle$.

Applications include:

- Determining points of intersection in 3D modeling.
- Analyzing the spatial relationship between planes.
- Calculating angles between intersecting lines or planes.
- Engineering design where intersection lines define boundaries or edges.

Limitations and Extensions

This derivation assumes the planes are not parallel or coincident. If the planes were parallel (e.g., identical or no intersection), the method would need adjustment.

Extensions of this process involve:

- Finding the intersection of more than two planes (which can form complex lines or points).
- Analyzing intersections in higher-dimensional spaces.
- Using matrix methods for systems with multiple equations.

Conclusion

Deriving the parametric equation of the line of intersection of two planes is a foundational skill in analytical geometry. Starting from the equations, expressing variables in terms of a parameter, and verifying the solution ensures a robust understanding of the geometric relationship.

```
For the given planes:

- \( x + y = 4 \)
- \( 2x + y + z = 2 \)

The parametric equations are:

\[
\boxed{
\begin{cases}
x = t \\
y = 4 - t \\
z = -t - 2
\end{cases}
}
\]

This line passes through the point \( (0, 4, -2) \) and extends infinitely in
the direction of \( \langle 1, -1, -1 \rangle \).

Understanding such intersections is not merely an academic exercise but a
practical tool in multiple disciplines requiring spatial reasoning. Mastery
of this process enhances your ability to analyze complex 3D systems, design
efficient algorithms, and interpret the spatial relationships that underpin
our physical and digital worlds.
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