Find All Solutions Of The Equation In The Interval [0,2 Pi) . 5cos Theta = - 2sin^2 Theta 4

Find All Solutions Of The Equation In The Interval [0,2 Pi) . 5cos Theta = - 2sin^2 Theta 4 in the first paragraph to introduce the problem. This article provides a comprehensive step-by-step guide to solving the trigonometric equation within the specified interval, ensuring clarity for learners and enthusiasts alike.

Understanding the Trigonometric Equation

Before diving into the solution, it’s essential to understand the given equation:

$5\cos\theta = -2\sin^2\theta\ 4$

At first glance, the equation appears complex due to the mixture of cosine and sine functions, along with the constant and the number 4. Our goal is to find all angles θ within the interval $[0, 2\pi)$ that satisfy this equation.

Step 1: Clarify and Simplify the Equation

The initial step involves rewriting the equation into a more manageable form.

Identify and interpret the components:

- The term $5\cos\theta$ involves cosine directly.
- The term $-2\sin^2\theta\ 4$ suggests a multiplication: $-2\sin^2\theta$ multiplied by 4, resulting in $-8\sin^2\theta$.

Thus, the equation simplifies to:

$5\cos\theta = -8\sin^2\theta$

Note: It’s important to verify whether the original problem includes any typographical errors or additional context, but based on the provided expression, this is the logical simplification.

Step 2: Use Pythagorean Identity to Convert All Terms to Sin or Cos

Given that the equation contains both sine and cosine, leveraging the Pythagorean identity:

$\sin^2 \theta + \cos^2 \theta = 1$

allows us to express one function in terms of the other. Here, since the equation involves both, it's convenient to express everything in terms of sine or cosine.

Express $\cos \theta$ in terms of $\sin \theta$:

From the identity:

$\cos \theta = \pm\sqrt{(1 - \sin^2 \theta)}$

but taking the square root introduces ±, which complicates the solution. Alternatively, express $\sin^2 \theta$ in terms of $\cos^2 \theta$:

$\sin^2 \theta = 1 - \cos^2 \theta$

Substituting into the equation:

$5\cos \theta = -8(1 - \cos^2 \theta)$

which simplifies to:

$5\cos \theta = -8 + 8\cos^2 \theta$

Step 3: Formulate a Quadratic Equation in terms of cos θ

Rearranging the above:

$8\cos^2 \theta - 5\cos \theta - 8 = 0$

Let's set:

$x = \cos \theta$

so the quadratic becomes:

- $8x^2 - 5x - 8 = 0$

Quadratic Equation:

$8x^2 - 5x - 8 = 0$

Solve for x using the quadratic formula:

$x = [5 \pm \sqrt{((-5)^2 - 4\ 8\ (-8))}] / (2\ 8)$

Calculate the discriminant:

$(-5)^2 = 25$

$4\ 8\ (-8) = 4\ 8\ (-8) = 4\ (-64) = -256$

So,

$\sqrt{(25 - (-256))} = \sqrt{(25 + 256)} = \sqrt{281}$

Thus, the solutions:

$x = [5 \pm \sqrt{281}] / 16$

Calculate approximate values:

- $\sqrt{281} \approx 16.76$

So,

$x_1 \approx (5 + 16.76) / 16 \approx 21.76 / 16 \approx 1.36$

$x_2 \approx (5 - 16.76) / 16 \approx (-11.76) / 16 \approx -0.735$

Note: Since cosine values are always in [-1, 1], the solution $x_1 \approx 1.36$ is invalid because it falls outside this range.

Therefore, the only valid solution for cos θ is:

$x = \cos \theta \approx -0.735$

Step 4: Find θ Corresponding to cos θ ≈ -0.735 within [0, 2π)

Now, determine the angles θ where the cosine is approximately -0.735.

Use the inverse cosine function:

$\theta = \cos^{-1}(-0.735)$

Calculating:

$\theta \approx \arccos(-0.735) \approx 2.37$ radians

Since cosine is negative in the second and third quadrants, the solutions within $[0, 2\pi)$ are:

- $\theta_1 \approx 2.37$ radians (second quadrant)
- $\theta_2 \approx 2\pi - 2.37 \approx 6.283 - 2.37 \approx 3.91$ radians (third quadrant)

Note: These approximate solutions are sufficient for most purposes, but if exact values are needed, express θ in terms of $\arccos(-0.735)$.

Step 5: Verify the Solutions

It is good practice to verify whether these solutions satisfy the original equation.

Recall the original simplified form in terms of sine:

$5\cos\theta = -8\sin^2\theta$

or, equivalently:

$5\cos\theta + 8\sin^2\theta = 0$

Calculate for $\theta \approx 2.37$ radians:

- $\cos(2.37) \approx -0.735$

- $\sin(2.37) \approx \sqrt{(1 - 0.735^2)} \approx \sqrt{(1 - 0.540)} \approx \sqrt{0.460} \approx 0.678$

- $\sin^2(2.37) \approx 0.460$

Now, check:

$5 (-0.735) + 8\ 0.460 \approx -3.675 + 3.68 \approx 0.005$

Close to zero, confirming the solution.

Similarly, for $\theta \approx 3.91$ radians:

- $\cos(3.91) \approx -0.735$

- $\sin(3.91) \approx -0.678$

- $\sin^2(3.91) \approx 0.460$

Check:

$5 (-0.735) + 8\ 0.460 \approx -3.675 + 3.68 \approx 0.005$

Again, close to zero, confirming both solutions.

Final Solutions in the Interval [0, 2π)

The solutions for θ in radians are approximately:

- $\theta \approx 2.37$ radians
- $\theta \approx 3.91$ radians

Expressed in degrees (optional):

- 2.37 radians ≈ 135.8°

- 3.91 radians ≈ 224.2°

These are the solutions within the interval [0, 2π).

Conclusion

Solving trigonometric equations like $5\cos\theta = -8\sin^2\theta$ involves converting all terms to a single trigonometric function, forming quadratic equations, and carefully analyzing solutions within the interval. Remember to verify solutions to ensure they satisfy the original equation, especially when dealing with quadratic solutions that may fall outside the permissible range for cosine.

By following these systematic steps—simplification, substitution, solving quadratic equations, and verification—you can confidently find all solutions to similar trigonometric equations within specified intervals, enhancing your problem-solving skills in trigonometry.

Frequently Asked Questions

How do I find all solutions to the equation $5\cos\theta = -2\sin^2\theta$ in the interval [0, 2π)?

Start by expressing everything in terms of a single trigonometric function, such as using the identity $\sin^2 \theta = 1 - \cos^2 \theta$, then rewrite the equation and solve the resulting quadratic for $\cos \theta$. Finally, find θ within $[0, 2\pi)$ that satisfy these solutions.

What steps are involved in solving the equation $5\cos \theta = -2\sin^2 \theta$ for θ in $[0, 2\pi)$?

First, rewrite $\sin^2 \theta$ as $1 - \cos^2 \theta$, substitute into the equation to get a quadratic in $\cos \theta$, solve for $\cos \theta$, then determine the corresponding θ values in the interval that satisfy the solutions.

Can you provide the detailed solution for $5\cos \theta = -2\sin^2 \theta$ in $[0, 2\pi)$?

Yes. Rewrite as $5\cos \theta = -2(1 - \cos^2 \theta)$, which simplifies to $5\cos \theta = -2 + 2\cos^2 \theta$. Rearranged, it becomes $2\cos^2 \theta - 5\cos \theta - 2 = 0$. Solve this quadratic for $\cos \theta$, then find all θ in $[0, 2\pi)$ corresponding to these cosine values.

What are the final solutions for θ in $[0, 2\pi)$ for the equation $5\cos \theta = -2\sin^2 \theta$?

After solving the quadratic, the solutions for $\cos \theta$ are approximately 2.0 and -0.5. Since $\cos \theta = 2.0$ is invalid, only $\cos \theta = -0.5$ applies, giving $\theta = 2\pi/3$ and $4\pi/3$ as solutions within the interval.

Are there any special considerations when solving $5\cos \theta = -2\sin^2 \theta$ in $[0, 2\pi)$?

Yes. When solving for $\cos \theta$, ensure solutions are within the valid range $[-1, 1]$. Also, verify all found θ values lie within the interval $[0, 2\pi)$.

How can I verify the solutions of $5\cos \theta = -2\sin^2 \theta$ in the interval $[0, 2\pi)$?

Substitute each θ back into the original equation to check if both sides are equal. This confirms that the solutions are correct within the given interval.

Additional Resources

Trigonometric Equations

Understanding and solving trigonometric equations is fundamental in mathematics, especially in fields such as engineering, physics, and computer science. One such equation that presents an interesting challenge is:

$5\cos \theta = -2\sin^2 \theta + 4$

In this article, we'll thoroughly analyze this equation, explore methods to find all solutions within the interval $[0, 2\pi)$, and provide a comprehensive step-by-step guide to solving it. Whether you're a student aiming to master trigonometry or a professional brushing up on problem-solving techniques, this deep dive will enhance your understanding.

Breaking Down the Equation

Before jumping into solving, let's dissect the given equation:

$5\cos \theta = -2\sin^2 \theta + 4$

At first glance, the mix of cosine and sine functions suggests that we might benefit from expressing everything in terms of a single trigonometric function to simplify the problem.

Key observations:

- The equation involves both $\cos \theta$ and $\sin^2 \theta$.
- The right side is quadratic in sine ($\sin^2 \theta$).
- The interval of interest is $[0, 2\pi)$, which covers a full cycle of the sine and cosine functions.

Step 1: Expressing in a Single Variable

Since the equation contains both sine and cosine, and knowing the fundamental Pythagorean identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

we can substitute $\sin^2 \theta$ with $1 - \cos^2 \theta$. This substitution allows us to rewrite the entire equation in terms of $\cos \theta$:

$$
5\cos \theta = -2(1 - \cos^2 \theta) + 4
$$

Now, expand and simplify:

$$
5\cos \theta = -2 + 2 \cos^2 \theta + 4
$$

\[

5\cos θ = (-2 + 4) + 2 \cos^2 θ
\]

\[
5\cos θ = 2 + 2 \cos^2 θ
\]

This is a quadratic equation in terms of \(\cos θ \):

\[
2 \cos^2 θ - 5 \cos θ + 2 = 0
\]

Step 2: Solving the Quadratic Equation

The quadratic in \(\cos θ \) is:

\[
2x^2 - 5x + 2 = 0
\]

where \(x = \cos θ \).

Applying the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

with \(a=2 \), \(b=-5 \), and \(c=2 \):

\[
x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2}
\]

Calculate discriminant:

\[
(-5)^2 - 4 \times 2 \times 2 = 25 - 16 = 9
\]

Now, find the roots:

\[
x = \frac{5 \pm \sqrt{9}}{4}
\]

\[
x = \frac{5 \pm 3}{4}
\]

This gives two solutions:

- When using \(+3 \):

\[
x = \frac{5 + 3}{4} = \frac{8}{4} = 2
\]

- When using \(-3 \):

\[
x = \frac{5 - 3}{4} = \frac{2}{4} = \frac{1}{2}
\]

Critical note: Since \(x = \cos θ \) and the range of cosine is [-1, 1], the solution \(x=2 \) is invalid because it's outside the permissible range.

Therefore, only:

\[
\cos θ = \frac{1}{2}
\]

is valid within the domain of cosine.

Step 3: Finding θ in [0, 2π) for Valid Solutions

Now, we find all θ such that \(\cos θ = \frac{1}{2} \).

Standard angles:

- \(\cos θ = \frac{1}{2} \) at:

\[
θ = \frac{\pi}{3} \quad \text{(First quadrant)}
\]
\[
θ = 2π - \frac{\pi}{3} = \frac{5π}{3} \quad \text{(Fourth quadrant)}
\]

Thus, the solutions are:

\[

θ = \frac{\pi}{3}, \quad \frac{5π}{3}
\]

Step 4: Verifying the Solutions

Recall that the original equation involved sine squared, which can be derived from cosine solutions. We need to verify these solutions satisfy the original equation:

\[
5 \cos θ = -2 \sin^2 θ + 4
\]

Using \(\sin^2 θ = 1 - \cos^2 θ \):

\[
5 \cos θ = -2 (1 - \cos^2 θ) + 4
\]

which simplifies to the quadratic form we previously solved. Since our derivation is consistent and the only valid cosine solutions are \(\frac{1}{2} \), both \(\theta = \frac{\pi}{3} \) and \(\frac{5π}{3} \) satisfy the equation.

Additional Considerations and Summary of Solutions

- The initial step of converting sine squared to a quadratic in cosine is a common and effective method to handle such equations.
- The invalid solution \(\cos θ = 2 \) was discarded because it lies outside the range of cosine.
- The solutions within [0, 2π) are \(\boxed{\frac{\pi}{3}, \frac{5π}{3}} \).

Final answer:

\[
\boxed{
\theta = \frac{\pi}{3}, \quad \frac{5\pi}{3}
}
\]

Broader Implications and Applications

This problem exemplifies the power of algebraic substitution in trigonometry. By transforming the original mixed sine and cosine equation into a quadratic in a single variable, the problem becomes more manageable. Such techniques are invaluable in:

- Solving complex trigonometric equations in physics, such as wave motion or oscillations.
- Engineering applications involving signal processing, where phase angles are critical.
- Mathematical modeling that requires precise angle solutions within specified intervals.

Furthermore, understanding the domain restrictions of trigonometric functions ensures solutions are valid within the context of the problem.

Conclusion

Solving trigonometric equations like $5 \cos \theta = -2 \sin^2 \theta + 4$ requires a strategic approach—leveraging identities, substituting variables, and carefully analyzing solutions' validity within the domain. By converting the problem into a quadratic, we systematically identified valid solutions within the interval $[0, 2\pi)$, demonstrating both the elegance and practicality of algebraic methods in trigonometry.

Mastering these techniques not only enhances problem-solving skills but also deepens your understanding of the interconnected nature of trigonometric functions, paving the way for tackling more complex mathematical challenges with confidence.

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