Find An Equation In Slope-intercept Form For The Line. Through \((1,4) \) And \((1,5) \)

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Understanding the equation of a line is a fundamental concept in coordinate geometry and algebra. When working with points on a coordinate plane, being able to determine the equation that passes through specific points is essential for solving various mathematical problems, analyzing graphs, and applying these concepts to real-world situations such as engineering, physics, and computer graphics.

In this article, we will explore how to find the equation of a line in slope-intercept form, specifically when given two points: \((1,4) \) and \((1,5) \). We will walk through the process step-by-step, discuss the significance of the slope-intercept form, and highlight key concepts such as slope calculation, recognizing special cases like vertical lines, and interpreting the resulting equations.

Understanding the Slope-Intercept Form of a Line

The slope-intercept form of a linear equation is one of the most commonly used representations of a line. It is expressed as:

\[y = mx + b \]

where:
- \(y \) is the dependent variable (vertical coordinate),
- \(x \) is the independent variable (horizontal coordinate),
- \(m \) is the slope of the line,
- \(b \) is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful because it directly reveals the slope and the y-intercept, making it straightforward to graph the line and understand its behavior.

Step 1: Identifying the Given Points

The problem provides two points through which the line passes:

- \((1,4) \)
- \((1,5) \)

It's crucial to understand what these points represent and how they influence the equation of the line.

Step 2: Calculating the Slope (m)

The slope m describes the steepness and direction of the line. It is calculated using the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Applying this to our points:

- $(x_1, y_1) = (1, 4)$
- $(x_2, y_2) = (1, 5)$

Calculate the differences:

$$\Delta y = 5 - 4 = 1$$
$$\Delta x = 1 - 1 = 0$$

Thus,

$$m = \frac{1}{0}$$

This indicates that the slope is undefined because division by zero is undefined in mathematics.

Step 3: Recognizing the Special Case — Vertical Line

When the change in x (Δx) is zero, as in this case, it means the line is vertical. Vertical lines have a constant x-coordinate for all points on the line, and their equations are of the form:

$$x = c$$

where c is the constant value of x for all points on the line.

Since both points have the same x-coordinate, which is 1, the equation of the line passing through these points is:

$$x = 1$$

Step 4: Interpreting the Results

The initial task was to find the equation in slope-intercept form; however, because the line is vertical, it cannot be expressed in the form $y = mx + b$. Instead, vertical lines are represented by equations where x is constant.

Key Takeaways:

- When two points have different x-coordinates, the slope can be calculated, and the line can be expressed in slope-intercept form.
- When two points share the same x-coordinate, the line is vertical, and its equation cannot be written in slope-intercept form.
- Vertical lines have an undefined slope, and their equations are of the form $x = c$.

Additional Context: How to Find the Equation in Slope-Intercept Form When Points Are Different

While our specific points lead to a vertical line, it's instructive to understand how to find the slope-intercept form when the points are different and not aligned vertically.

Suppose the points were (x_1, y_1) and (x_2, y_2), with $x_1 \neq x_2$. The steps would be:

1. Calculate the slope m:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

2. Find the y-intercept b:

Use the point-slope form:

$$ y - y_1 = m (x - x_1) $$

Rearranged to slope-intercept form:

$$ y = m x + (b) $$

where:

$$ b = y_1 - m x_1 $$

3. Write the equation:

$$ y = m x + b $$

Understanding the Limitations and Special Cases

It's essential to recognize that not all lines can be expressed in the slope-intercept form.

Vertical Lines

- As in our problem, vertical lines occur when x is constant.
- The slope is undefined.
- The equation is $x = c$.

Horizontal Lines

- When y is constant for all points, the slope $m = 0$.
- The equation takes the form:

$$y = k$$

where k is the constant y-value.

Summary of Key Concepts

- The slope-intercept form is $y = mx + b$, ideal for lines with a defined slope.
- To find the line's equation, compute the slope from two points.
- When the two points share the same x-coordinate, the line is vertical, and the equation is $x = c$.
- Vertical lines cannot be expressed in slope-intercept form.
- For non-vertical lines, substitute one point into the point-slope form to find b.

Practical Applications of Finding Line Equations

Understanding how to find the equation of a line through given points is fundamental in many fields:

- Engineering: Designing structures and analyzing forces.
- Physics: Modeling motion and trajectories.
- Computer Graphics: Rendering lines and shapes.
- Economics: Analyzing trends and relationships between variables.
- Data Science: Fitting linear models to data points.

Conclusion

In our specific case, the points $(1,4)$ and $(1,5)$ lie on a vertical line because they share the same x-coordinate. Therefore, the equation of this line is:

$$x = 1$$

This example highlights the importance of understanding the different forms of line equations and recognizing special cases such as vertical lines, which cannot be expressed in slope-intercept form.

Knowing how to determine the line's equation through given points empowers students and professionals alike to analyze and interpret linear relationships accurately. Whether dealing with standard lines, vertical lines, or horizontal lines, mastering these concepts is foundational in mathematics and its applications.

Additional Resources:

- [Khan Academy: Equation of a Line](https://www.khanacademy.org/math/geometry/linear-equations)
- [Math is Fun: Equation of a Line](https://www.mathsisfun.com/geometry/line-equations.html)
- [Purplemath: Slope-Intercept Form](https://www.purplemath.com/modules/lineequ.htm)

Meta description: Learn how to find the equation of a line passing through the points $(1,4)$ and $(1,5)$. Understand why the line is vertical and how to represent it correctly, including key concepts about slope and intercepts.

Frequently Asked Questions

What is the slope of the line passing through the points (1, 4) and (1, 5)?

The slope is undefined because the x-values are the same, indicating a vertical line.

How do you find the slope of a line passing through two points?

You subtract the y-values and divide by the difference of the x-values: slope = (y2 - y1) / (x2 - x1).

What is the equation of a vertical line passing through (1, 4)

and (1, 5)?

The equation is x = 1, since all points on the line have x-coordinate 1.

Can a vertical line be written in slope-intercept form y = mx + b?

No, vertical lines cannot be expressed in slope-intercept form because their slope is undefined.

How would you write the equation of the line passing through (1, 4) and (1, 5) in standard form?

The equation is x = 1, which is already in standard form.

Why is the slope undefined for the points (1, 4) and (1, 5)?

Because the x-values are the same, dividing the change in y by zero makes the slope undefined.

What type of line passes through the points (1, 4) and (1, 5)?

A vertical line, since both points share the same x-coordinate.

Additional Resources

Find An Equation In Slope-intercept Form For The Line Through (1,4) And (1,5)

When working with linear equations, one of the most fundamental tasks is to find the equation of a line that passes through given points. Specifically, finding an equation in slope-intercept form for the line through (1,4) and (1,5) is a common problem encountered in algebra and analytical geometry. This process not only reinforces understanding of the slope-intercept form but also highlights important concepts like slope calculation, the nature of vertical lines, and their equations. In this guide, we'll explore step-by-step how to determine the equation of such a line, the challenges involved, and key mathematical principles to keep in mind.

Understanding the Slope-Intercept Form

Before diving into the problem, it’s essential to revisit what the slope-intercept form of a line is:

What is the slope-intercept form?

The slope-intercept form is expressed as:

y = mx + b

where:

- m is the slope of the line, representing how steep it is.
- b is the y-intercept, which is the point where the line crosses the y-axis.

This form is particularly convenient because it directly gives the slope and the y-intercept, making graphing and analysis straightforward.

The Given Points: (1,4) and (1,5)

Our goal is to find the equation of the line passing through these two points. Let's analyze what these points tell us.

Key observations:

- Both points share the same x-coordinate: x = 1.
- The y-coordinates are different: 4 and 5.

This indicates that the line passes through the points (1,4) and (1,5). Since both points have the same x-value but different y-values, what does this tell us about the line?

Step 1: Calculating the Slope

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is given by:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying this to our points:

$$
x_1 = 1, \quad y_1 = 4
$$
$$
x_2 = 1, \quad y_2 = 5
$$

Plugging in:

$$
m = \frac{5 - 4}{1 - 1} = \frac{1}{0}
$$

Division by zero indicates that the slope is undefined.

What does an undefined slope mean?

An undefined slope signifies that the line is vertical. Vertical lines are characterized by a constant x-

value for all points on the line, but their y-values can vary freely.

Step 2: Recognizing the Nature of the Line

Since the slope is undefined, the line in question is vertical. Vertical lines have equations of the form:

$$
x = a
$$

where a is the x-coordinate of any point on the line.

In our case, both points have x = 1, so the line passing through (1,4) and (1,5) is:

$$
x = 1
$$

Step 3: Can the Line Be Expressed in Slope-Intercept Form?

This is a crucial question. Recall that the slope-intercept form y = mx + b assumes that the line is not vertical. For vertical lines, the slope is undefined, and thus, the line cannot be expressed as a function of y in the form y = mx + b.

Why?

Because in a vertical line:

- For a given x, there are multiple y-values (in fact, infinitely many).
- The line does not pass the vertical line test for functions: it's not a function of y in the form y = f(x).

Therefore, a vertical line cannot be written in the slope-intercept form.

Summary of Findings

Aspect	Explanation
Slope	Undefined (vertical line)
Equation	x = 1
Can it be in slope-intercept form?	No, because vertical lines are not functions of y, and slope-intercept form requires the line to be a function y = mx + b

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Practical Implications and Alternative Forms

Although the question asks for the equation in slope-intercept form, in this case, it's impossible because the line is vertical. Instead, the most accurate and useful description of the line is:

$x = 1$

This equation precisely describes the vertical line passing through the points (1,4) and (1,5).

Additional Notes: When Does a Line Have a Slope?

To clarify, here are scenarios related to lines passing through two points:

- Non-vertical line: The points have different x-values, and the slope is finite. The line can be written as $y = mx + b$.
- Vertical line: The points have the same x-value, and the slope is undefined. The line is written as $x = a$.
- Horizontal line: The points have the same y-value, and the slope is 0. The line can be written as $y = c$.

Summary and Final Remarks

- To find an equation in slope-intercept form for a line, the line must have a defined slope, and it must be expressible as $y = mx + b$.
- In the specific case of points (1,4) and (1,5), the line is vertical because both points share the same x-coordinate.
- The equation of the line through these points is $x = 1$.
- It is impossible to express this line in the form $y = mx + b$ because vertical lines aren't functions of y.

Key Takeaways:

- Always check whether the points have the same x-value before attempting to find a slope.
- Recognize that vertical lines cannot be expressed in slope-intercept form.
- When dealing with vertical lines, use the equation $x = a$, where a is the common x-coordinate.

Final Thoughts

Understanding the limitations of the slope-intercept form is essential for mastering coordinate geometry. Recognizing when a line is vertical—such as through points with identical x-values—is an important skill that prevents misapplication of formulas and guides you toward the correct form of the line’s equation. In cases like this, conveying the line as $x = 1$ is the most accurate and informative approach.

In summary: The line passing through (1,4) and (1,5) is a vertical line with the equation x = 1. It cannot be expressed in slope-intercept form, but understanding this helps deepen your grasp of the geometry of lines and their equations.

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