

Find An Equation Of The Line That Satisfies The Given Conditions.Through (-1, -2) And (4, 3)

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Understanding how to find the equation of a line passing through two specific points is a fundamental concept in coordinate geometry. This process involves calculating the slope of the line and then using the point-slope form to derive the line's equation. In this article, we will explore step-by-step how to determine the equation of a line passing through the points (-1, -2) and (4, 3), including detailed explanations, formulas, and methods to ensure clarity and thorough understanding.

Understanding the Basics of Line Equations

Before diving into the specific problem, it is essential to review the core concepts related to line equations.

What Is an Equation of a Line?

An equation of a line is an algebraic expression that defines all the points that lie on that line in a coordinate plane. The most common forms of line equations include:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

Each form serves different purposes and is used based on the available information.

Key Components

- Slope (m): A measure of how steep the line is. It indicates the change in y for a unit change in x .
- Coordinates of Points: The specific points through which the line passes, given as (x_1, y_1) and (x_2, y_2) .

Step-by-Step Solution to Find the Equation

Given the points $(-1, -2)$ and $(4, 3)$, our goal is to find the line's equation that passes through both points.

Step 1: Find the Slope (m)

The slope formula is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Applying the points:

- Point 1: $(x_1, y_1) = (-1, -2)$

- Point 2: $(x_2, y_2) = (4, 3)$

Calculating the numerator:

$$y_2 - y_1 = 3 - (-2) = 3 + 2 = 5$$

Calculating the denominator:

$$x_2 - x_1 = 4 - (-1) = 4 + 1 = 5$$

Thus,

$$m = \frac{5}{5} = 1$$

The slope of the line is 1.

Step 2: Use the Point-Slope Form

The point-slope form of a line's equation is:

$$y - y_1 = m(x - x_1)$$

Using one of the points, say $(-1, -2)$, and the slope $m = 1$:

$$y - (-2) = 1(x - (-1))$$

Simplify:

$$y + 2 = 1(x + 1)$$

$$y + 2 = x + 1$$

Step 3: Convert to Slope-Intercept Form

Subtract 2 from both sides:

$$[y = x + 1 - 2]$$

$$[y = x - 1]$$

Therefore, the equation of the line passing through (-1, -2) and (4, 3) is:

$$[y = x - 1]$$

Verification of the Equation

It's good practice to verify the found equation with the second point.

- Substitute $x = 4$:

$$[y = 4 - 1 = 3]$$

- The point is (4, 3), which matches the second point.

This confirms that the equation $y = x - 1$ passes through both points.

Alternative Forms and Additional Insights

While the slope-intercept form $y = x - 1$ is straightforward, other forms can also be useful depending on the context.

Standard Form of the Line

Rearranged to standard form:

$$[y - x = -1]$$

Or,

$$[x - y = 1]$$

Graphical Interpretation

- The line has a slope of 1, indicating a 45-degree angle with the x-axis.

- It crosses the y-axis at (0, -1).
- It passes through the points (-1, -2) and (4, 3), confirming the correctness of the equation.

General Approach to Finding Line Equations From Two Points

The process demonstrated above can be generalized for any two points:

1. Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Use the point-slope form: $y - y_1 = m(x - x_1)$
3. Simplify to the desired form (slope-intercept or standard form)

Common Pitfalls and Tips

- Division by Zero: If $x_2 - x_1 = 0$, the line is vertical, and the equation is simply $x = x_1$.
- Sign Errors: Always double-check signs when plugging in points.
- Verifying: Always substitute the second point into the derived equation to confirm correctness.

Practical Applications of Line Equations

Understanding how to derive and manipulate line equations is invaluable in various fields:

- Engineering: Designing roads, bridges, and structural frameworks.
- Physics: Analyzing motion with constant velocity.
- Economics: Modeling linear relationships between variables.
- Computer Graphics: Drawing lines and shapes on digital screens.

Conclusion

Finding the equation of a line passing through two points is a fundamental skill in algebra and geometry. By calculating the slope and applying the point-slope form, you can derive the equation efficiently. In our specific example, the line passing through (-1, -2) and (4, 3) has the equation:

$$y = x - 1$$

This process reinforces the importance of understanding the relationship between points, slope, and line equations, enabling accurate modeling and problem-solving across numerous disciplines.

Frequently Asked Questions

How do I find the equation of a line passing through the points (-1, -2) and (4, 3)?

First, calculate the slope using $m = (y_2 - y_1) / (x_2 - x_1) = (3 - (-2)) / (4 - (-1)) = 5 / 5 = 1$. Then, use point-slope form with one of the points, for example (-1, -2): $y - (-2) = 1(x - (-1))$, simplifying to $y + 2 = x + 1$, so the equation is $y = x - 1$.

What is the slope of the line passing through (-1, -2) and (4, 3)?

The slope is 1, calculated as $(3 - (-2)) / (4 - (-1)) = 5 / 5 = 1$.

How can I write the equation of the line in slope-intercept form from these points?

Using the slope $m = 1$ and point (-1, -2), plug into $y = mx + b$: $-2 = 1(-1) + b$, so $-2 = -1 + b$, hence $b = -1$. The equation is $y = x - 1$.

Can I use the point-slope form to find the line's equation? How?

Yes. Using point (-1, -2) and slope $m = 1$, the point-slope form is $y - (-2) = 1(x - (-1))$, which simplifies to $y + 2 = x + 1$, leading to $y = x - 1$.

What is the general process to find the equation of a line through two points?

Calculate the slope using the two points, then use either point-slope form or slope-intercept form to write the line's equation. For point-slope form: $y - y_1 = m(x - x_1)$. For slope-intercept form, solve for y after finding the slope and intercept.

Are the two points (-1, -2) and (4, 3) enough to uniquely define the line?

Yes, two distinct points uniquely determine a straight line. In this case, the line passing through these points has the equation $y = x - 1$.

Additional Resources

Find An Equation Of The Line That Satisfies The Given Conditions: Through (-1, -2) And (4, 3)

When working with linear equations in mathematics, one common task is to find the equation of a line that passes through two specified points. This process is fundamental in algebra, coordinate geometry, and many applied fields such as engineering, physics, and computer graphics. In this guide, we will explore how to find an equation of the line that satisfies the conditions of passing through the points (-1, -2) and (4, 3). We will break down the steps involved, explain key concepts, and provide practical examples to ensure a clear understanding of this essential skill.

Understanding the Problem

Given two points, the goal is to determine the equation of the straight line that intersects both. In our specific case, the points are:

- Point 1: (-1, -2)
- Point 2: (4, 3)

The question is: what is the algebraic representation of the line that passes through these points?

Fundamental Concepts to Recall

Before diving into calculations, let's review some key concepts:

The Equation of a Line

The most common form of a line's equation in a two-dimensional Cartesian plane is:

$$y = mx + b$$

where:

- m is the slope of the line
- b is the y-intercept (the point where the line crosses the y-axis)

Alternatively, the point-slope form:

$$y - y_1 = m(x - x_1)$$

which uses a specific point (x_1, y_1) on the line and the slope m .

Calculating the Slope (m)

The slope indicates how steep the line is and is calculated as:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

This ratio measures the change in y relative to the change in x between the two points.

Step-by-Step Guide to Finding the Equation

Step 1: Calculate the Slope (m)

Using the points (-1, -2) and (4, 3):

$$- x_1 = -1, y_1 = -2$$

$$- x_2 = 4, y_2 = 3$$

Plug into the slope formula:

$$m = (3 - (-2)) / (4 - (-1))$$

$$= (3 + 2) / (4 + 1)$$

$$= 5 / 5$$

$$= 1$$

Result: The slope of the line passing through the points is $m = 1$.

Step 2: Use the Point-Slope Form to Find the Equation

Choose either of the two points to substitute into the point-slope form. Let's pick the point (-1, -2):

$$y - y_1 = m(x - x_1)$$

Plugging in the values:

$$y - (-2) = 1(x - (-1))$$

Simplify:

$$y + 2 = 1(x + 1)$$

Distribute the slope:

$$y + 2 = x + 1$$

Step 3: Convert to Slope-Intercept Form (Optional)

Subtract 2 from both sides:

$$y = x + 1 - 2$$

$$y = x - 1$$

Final Equation of the Line:

$$y = x - 1$$

This is the equation of the line passing through the points (-1, -2) and (4, 3).

Validating the Equation

To ensure our equation is correct, let's verify it with the second point (4, 3):

Substitute $x = 4$:

$$y = 4 - 1 = 3$$

which matches the y-coordinate of the second point, confirming our equation is accurate.

Additional Considerations

1. Alternative forms of the line equation

- Standard form: $Ax + By + C = 0$

For $y = x - 1$, rearranged:

$$x - y - 1 = 0$$

- Two-point form: $y - y_1 = [(y_2 - y_1)/(x_2 - x_1)] (x - x_1)$

which we've already used to derive the equation.

2. Graphing the Line

Plotting the points (-1, -2) and (4, 3), then drawing the line $y = x - 1$, helps visualize the solution and verify correctness.

3. Real-world applications

Understanding how to find a line from two points is vital in:

- Data trend analysis
- Navigation and routing

- Computer graphics rendering
- Physics experiments involving linear relationships

Summary of Key Steps

To find an equation of a line passing through two points:

1. Calculate the slope (m): Use the difference in y -values over the difference in x -values.
2. Apply the point-slope form: Choose either point and substitute into $y - y_1 = m(x - x_1)$.
3. Simplify to slope-intercept form: Solve for y to get $y = mx + b$.
4. Verify with the other point: Substitute x and y to confirm the equation holds.

Practice Exercise

Try applying this process to the points (2, 5) and (6, 11):

1. Find the slope.
2. Write the equation in slope-intercept form.
3. Verify with both points.

Conclusion

Finding the equation of a line that passes through specific points is a fundamental skill in algebra that forms the basis for understanding more complex concepts in mathematics and science. By mastering the steps—calculating the slope, using the point-slope form, and converting to slope-intercept form—you can confidently determine the equation of any line given two points. In our example, the line passing through (-1, -2) and (4, 3) is $y = x - 1$, a simple yet powerful illustration of these principles in action.

Remember: Practice makes perfect. Repeating these steps with different data points will strengthen your understanding and ability to quickly find line equations in various contexts.

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