Find An Equation Of The Tangent Line To The Curve At The Given Point.y = 5x^2 X^3, (1, 4)

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Understanding how to find the equation of the tangent line to a curve at a specific point is a fundamental concept in calculus. In this article, we will explore the step-by-step process of determining the tangent line to the curve defined by $y = 5x^2 x^3$ at the point $(1, 4)$. The process involves differentiating the function to find the slope of the tangent (the derivative), evaluating this derivative at the given point, and then using the point-slope form of a line to derive the equation of the tangent line.

Understanding the Given Function

Analyzing the Function $y = 5x^2 x^3$

The initial step in our problem is to understand the function:

$$
y = 5x^2 x^3
$$

At first glance, this function appears to be a product of polynomials. To simplify and analyze it properly, we can utilize properties of exponents.

Recall that when multiplying terms with the same base, their exponents add:

$$
x^a \times x^b = x^{a + b}
$$

Applying this to our function:

$$
y = 5x^2 \times x^3 = 5x^{2 + 3} = 5x^5
$$

Thus, the function simplifies to:

$$
y = 5x^5
$$

This simplification makes differentiation straightforward and clarifies the behavior of the function at different points.

Verifying the Given Point (1, 4)

Checking if the Point Lies on the Curve

Before proceeding, it's vital to verify that the point $(1, 4)$ indeed lies on the curve $y = 5x^5$. To do this, substitute $x=1$ into the function:

$$
y = 5(1)^5 = 5 \times 1 = 5
$$

The resulting y-value is 5, but the given point is $(1, 4)$. Since the calculated y-value (5) does not match the given y-coordinate (4), there appears to be a discrepancy.

Possible explanations:

- The given point $(1, 4)$ might be a typo or misprint.
- The original function might be different or written incorrectly.

Assuming the intended function is $y = 5x^5$, then the point on the curve at $x=1$ is $(1, 5)$, not $(1, 4)$.

Alternatively, if the point $(1, 4)$ is to be used as given, then the function should satisfy $y=4$ when $x=1$. To reconcile this, perhaps the original function is:

$$
y = 4 \quad \text{when} \quad x=1
$$

or the function is different.

Clarification and Assumption

For the purpose of this tutorial, we will proceed assuming the function is:

$$
y = 5x^5
$$

and the point of interest is $(1, 5)$, which lies on this curve.

If the original question intended the point $(1, 4)$, then the function would need to be adjusted accordingly.

Finding the Derivative of the Function

Applying Power Rule of Differentiation

The power rule states that:

$$
\frac{d}{dx} [x^n] = n x^{n-1}
$$

Applying this rule to $y = 5x^5$:

$$
\frac{dy}{dx} = 5 \times 5 x^{5-1} = 25 x^4
$$

This derivative $\frac{dy}{dx} = 25 x^4$ gives the slope of the tangent line at any point x.

Calculating the Slope at the Given Point

Evaluate the Derivative at $x=1$

To find the slope of the tangent line at $x=1$, substitute $x=1$ into the derivative:

$$
m = 25 (1)^4 = 25 \times 1 = 25
$$

This means the slope of the tangent line at the point $(1, 5)$ (assuming the corrected point) is 25.

Deriving the Equation of the Tangent Line

Using Point-Slope Form of a Line

The point-slope form of a line's equation is:

$$
y - y_1 = m (x - x_1)
$$

where:

- (x_1, y_1) is a point on the line, here $(1, 5)$,
- m is the slope, here 25.

Plugging in the values:

$$
y - 5 = 25 (x - 1)
$$

Simplifying:

$$
y - 5 = 25x - 25
$$

$$
y = 25x - 20
$$

Final Equation of the Tangent Line

The equation of the tangent line to the curve $y = 5x^5$ at the point $(1, 5)$ is:

$$
\boxed{ y = 25x - 20 }
$$

Summary and Additional Insights

Key Steps Recap

- Simplify the original function to a manageable form (here, $y = 5x^5$).
- Verify the point lies on the curve; if not, clarify the function or the point.
- Differentiate the function to find the slope function $\frac{dy}{dx}$.
- Evaluate the derivative at the specific x-coordinate to find the slope at that point.
- Use the point-slope formula to derive the tangent line's equation.

Important Considerations

- Always verify whether the point lies on the curve before proceeding.
- Be cautious of potential typos or ambiguities in the problem statement.
- Differentiation rules like the power rule simplify the process significantly.
- The tangent line provides a linear approximation of the curve near the point.

Extensions and Practice Problems

To deepen understanding, consider practicing with similar problems:

1. Find the tangent line to $y = 3x^4 - 2x + 1$ at $x=2$.
2. Given $y = \sin x$, find the tangent line at $x=0$.
3. Determine the tangent line to $y = \frac{1}{x}$ at $x=1$.

These exercises reinforce the process of differentiation, evaluation, and line equations.

Conclusion

Finding the equation of a tangent line to a curve at a specific point involves a systematic approach rooted in calculus. By differentiating the function to obtain the slope function, evaluating it at the point of interest, and then employing the point-slope form, one can accurately derive the tangent line. This process not only aids in understanding the local linear approximations of functions but also forms the foundation for more advanced topics such as optimization, curve analysis, and differential equations. Remember to verify the point's position on the curve and ensure clarity on the given function to avoid miscalculations.

Frequently Asked Questions

How do I find the equation of the tangent line to the curve y = 5x^2 + x^3 at the point (1, 4)?

First, find the derivative y' = d/dx (5x^2 + x^3) = 10x + 3x^2. Then, evaluate y' at x=1: y'(1) = 10(1) + 3(1)^2 = 10 + 3 = 13. The slope of the tangent line is 13. Using point-slope form with point (1, 4): y - 4 = 13(x - 1), so the equation is y = 13x - 9.

What is the derivative of y = 5x^2 + x^3?

The derivative y' = 10x + 3x^2.

How do I verify the point (1, 4) lies on the curve y = 5x^2 + x^3?

Plug x=1 into the equation: y = 5(1)^2 + (1)^3 = 5 + 1 = 6. Since the y-coordinate is 6, not 4, there appears to be an inconsistency; the point (1, 4) does not lie on the curve y=5x^2 + x^3.

Is the point (1, 4) on the curve y = 5x^2 + x^3?

No, because substituting x=1 yields y=6, not 4. Therefore, (1, 4) is not on the curve.

Can I find the tangent line at a point not on the curve?

No, the tangent line is defined at a point on the curve. Since (1, 4) is not on y=5x^2 + x^3, you need to find the tangent at the actual point on the curve closest to (1, 4).

What is the correct point on the curve $y=5x^2 + x^3$ where the tangent line has a slope of 13?

At $x=1$, the slope is 13, and the point on the curve is (1, 6). The tangent line at that point is $y = 13x - 9$.

What is the final equation of the tangent line to $y=5x^2 + x^3$ at $x=1$?

The tangent line at $x=1$ is $y = 13x - 9$, passing through the point (1, 4) is inconsistent because the point (1, 4) is not on the curve. The correct point for the tangent line is (1, 6), and the line is $y = 13x - 9$.

Additional Resources

Find An Equation Of The Tangent Line To The Curve At The Given Point.y = 5x^2 X^3, (1, 4)

Understanding how to find the tangent line to a curve at a specific point is a fundamental concept in calculus, vital for analyzing the behavior of functions and their slopes at given points. In this article, we will explore the process of deriving the equation of the tangent line to the curve defined by $y = 5x^2 \cdot x^3$ at the point $(1, 4)$. We will delve into the step-by-step methodology, clarify the underlying principles, and discuss the nuances involved in such calculations.

Understanding the Problem

Before diving into the mathematics, it’s crucial to interpret the problem correctly. The function given is:

$$y = 5x^2 \times x^3$$

which simplifies algebraically to:

$$y = 5x^{2 + 3} = 5x^5$$

The point at which we need the tangent line is $(1, 4)$. However, it's worth verifying whether this point indeed lies on the curve, as this is essential for the problem to be well-posed.

Verifying the Point on the Curve

To verify if $(1, 4)$ lies on the curve:

$$ y = 5x^5 $$

Substitute $x=1$:

$$ y = 5 \times 1^5 = 5 \times 1 = 5 $$

But the given point's y-coordinate is 4, which does not match 5. This indicates a potential inconsistency—either the point is incorrect, or the function is different.

Possible explanations:

- The function might be $y = 5x^2 \times x^3$, which simplifies to $y= 5x^5$, as shown.
- The point $(1, 4)$ does not satisfy $y=5x^5$.

Conclusion:

Given the function $y=5x^5$, the point $(1,4)$ does not lie on it. Therefore, either the original problem contains a typo, or the point is different. For the purpose of this analysis, we will assume the point intended was $(1, 5)$, which correctly lies on the curve, as $y=5 \times 1^5 = 5$.

Alternatively, if the point $(1,4)$ is fixed, then the curve must be different. For clarity, we will proceed with the assumption that the point of interest is $(1, 5)$, which lies on $y=5x^5$. If you prefer, I can also analyze the tangent at the original point $(1,4)$ assuming the curve is different.

Deriving the Equation of the Tangent Line

To find the tangent line to a curve at a specific point, we need two key components:

1. The slope of the tangent line at that point, given by the derivative of the function evaluated at the x-coordinate.
2. The point-slope form of a line, which uses the point and the slope to write the equation.

Step 1: Find the derivative dy/dx

Given $y=5x^5$, differentiate with respect to x:

$$
\frac{dy}{dx} = 5 \times 5x^{4} = 25x^{4}
$$

This derivative represents the slope of the tangent line at any point x.

Step 2: Evaluate the derivative at $x=1$

Evaluate the derivative at $x=1$:

$$
m = 25 \times 1^{4} = 25
$$

Thus, the slope of the tangent line at $x=1$ is 25.

Step 3: Find the corresponding y-coordinate

Calculate y at $x=1$:

$$
y = 5 \times 1^{5} = 5
$$

So, the point of tangency is $(1, 5)$.

Formulating the Equation of the Tangent Line

Using the point-slope form:

$$
y - y_1 = m (x - x_1)
$$

where $(x_1, y_1) = (1, 5)$, and $m=25$, the equation becomes:

$$
y - 5 = 25 (x - 1)
$$

Expanding:

$$
y - 5 = 25x - 25
$$

$$
y = 25x - 20
$$

Final equation of the tangent line:

$$
\boxed{ y = 25x - 20 }
$$

Features and Analysis of the Derived Tangent Line

Key features:

- The tangent line has a steep slope of 25, indicating a rapidly increasing function at $x=1$.
- It intersects the y-axis at -20.

Graphical interpretation:

- At the point $(1, 5)$, the line just touches the curve, sharing the same slope.
- The tangent line visually represents the instantaneous rate of change of the function at that point.

Considerations and Possible Variations

If the point is actually $(1,4)$:

- Since $y=5x^5$, at $x=1$, $y=5$. The point $(1,4)$ does not lie on this curve.
- To find the tangent at $(1,4)$, the curve must be different, or perhaps the point is off the curve.

Alternative approach if the curve is different:

Suppose the curve is $y = 5x^2 x^3 = 5x^5$, and the point is $(1,4)$. Since it doesn't lie on the curve, perhaps the problem is to find the tangent line at $x=1$ for a different function or at a different point.

In case of generality:

- The process remains the same: find the derivative, evaluate at the point, then use point-slope form.
- Always verify that the point lies on the curve before proceeding.

Pros and Cons of the Methodology

Pros:

- The process is systematic and straightforward once the derivative is known.
- It provides the exact slope at the point, enabling precise tangent line equations.
- The derivative can be computed using basic differentiation rules, even for complex functions.

Cons:

- Requires correct identification of the point on the curve, which can sometimes be overlooked.
- If the function is complicated, differentiation can become cumbersome.
- Assumes the function is differentiable at the point; otherwise, the tangent line may not exist.

Additional Features and Tips

- Using technology: Calculators or computer algebra systems (CAS) can simplify differentiation and evaluation.
- Graphing: Plotting the function alongside the tangent provides visual confirmation.
- Extensions: For curves where the derivative isn't straightforward, alternative methods like implicit differentiation or numerical approximation can be used.

Conclusion

Finding the equation of a tangent line to a curve at a given point involves understanding the function's derivative to determine the slope and then applying the point-slope form of a line. For the function $y=5x^5$, the tangent line at $(1,5)$ is $y=25x-20$, illustrating how rapidly the function increases at that point. This process underscores the importance of verifying points on the curve and carefully differentiating functions, especially when dealing with more complex forms. Mastery of this technique is essential for anyone delving into calculus, as it forms the basis for understanding instantaneous rates of change and the behavior of functions at specific points.

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