Find Location Of Local Maxima Or Local Minima Over The Interval [0,2]. G(x)=cosx/2+sinx

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Understanding how to identify the local maxima and minima of a function within a specific interval is a fundamental concept in calculus, with numerous applications in fields such as physics, engineering, and economics. In this article, we will explore how to find the locations of local maxima and minima of the function $G(x) = \frac{\cos x}{2} + \sin x$ over the interval $[0, 2]$. We will delve into the process step-by-step, including calculating derivatives, analyzing critical points, and determining the nature of these points to provide a comprehensive understanding of the function's behavior.

Introduction to the Function G(x) = (cos x)/2 + sin x

Before diving into the analysis, it's essential to understand the structure and characteristics of the function $G(x) = \frac{\cos x}{2} + \sin x$.

Key Features of G(x):

- Type of Function: It is a combination of sine and cosine functions, which are periodic and continuous.
- Domain: The domain is all real numbers; however, our focus is on the interval $[0, 2]$.
- Range: Since both sine and cosine functions oscillate between -1 and 1, the combined function's range is limited, but the exact maximum and minimum points within the interval require analysis.

Step 1: Calculating the Derivative G'(x)

To find the local maxima and minima, the first step is to compute the derivative of $G(x)$.

Derivative Calculation:

Given:

$$
G(x) = \frac{\cos x}{2} + \sin x
$$

Differentiate term by term:

\[
G'(x) = \frac{d}{dx} \left(\frac{\cos x}{2} \right) + \frac{d}{dx} (\sin x)
\]

\[
G'(x) = -\frac{\sin x}{2} + \cos x
\]

So, the derivative is:

\[
\boxed{
G'(x) = -\frac{\sin x}{2} + \cos x
}
\]

Step 2: Finding Critical Points where G'(x) = 0

Critical points occur where the derivative is zero or undefined. Since the derivative \(G'(x) \) is continuous everywhere, we set:

\[
-\frac{\sin x}{2} + \cos x = 0
\]

Solving for x:

\[
\cos x = \frac{\sin x}{2}
\]

Rewrite as:

\[
2 \cos x = \sin x
\]

Or:

\[
2 \cos x - \sin x = 0
\]

This is a linear combination of sine and cosine. To solve, we can use the tangent substitution approach:

\[

2 \cos x = \sin x
\]

Divide both sides by \(\cos x \) (assuming \(\cos x \neq 0 \)):

\[
2 = \tan x
\]

Thus:

\[
\boxed{
\tan x = 2
}
\]

Find x in the interval [0, 2]:

- The general solution for \(\tan x = 2 \) is:

\[
x = \arctan(2) + k\pi, \quad k \in \mathbb{Z}
\]

Calculate \(\arctan(2) \):

\[
\arctan(2) \approx 1.107 \text{ radians}
\]

Now, find all solutions within \([0, 2]\):

- For \(k=0 \):

\[
x \approx 1.107
\]

- For \(k=1 \):

\[
x \approx 1.107 + \pi \approx 1.107 + 3.142 \approx 4.249
\]

which is outside the interval \([0, 2]\).

- For \(k=-1 \):

\[
x \approx 1.107 - \pi \approx 1.107 - 3.142 \approx -2.035

\]

which is outside the interval.

Therefore, the only critical point in \([0, 2]\) is approximately:

\[
x \approx 1.107
\]

Step 3: Determining the Nature of Critical Points (Maxima or Minima)

To classify the critical point at \(x \approx 1.107 \), we analyze the second derivative \(G''(x) \) or evaluate \(G'(x) \) around this point.

Calculating the second derivative:

Starting from:

\[
G'(x) = -\frac{\sin x}{2} + \cos x
\]

Differentiate again:

\[
G''(x) = -\frac{\cos x}{2} - \sin x
\]

Evaluate \(G''(x) \) at \(x \approx 1.107 \):

Calculate \(\sin(1.107) \) and \(\cos(1.107) \):

- \(\sin(1.107) \approx 0.894 \)
- \(\cos(1.107) \approx 0.447 \)

Substitute into \(G''(x) \):

\[
G''(1.107) = -\frac{0.447}{2} - 0.894 \approx -0.2235 - 0.894 \approx -1.1175
\]

Since \(G''(1.107) < 0 \), the critical point is a local maximum.

Step 4: Evaluating Endpoints of the Interval

In addition to critical points, the maximum or minimum values over a closed interval might occur at the endpoints $x=0$ and $x=2$. Therefore, evaluate $G(x)$ at these points.

At $x=0$:

$$
G(0) = \frac{\cos 0}{2} + \sin 0 = \frac{1}{2} + 0 = 0.5
$$

At $x=2$:

$$
G(2) = \frac{\cos 2}{2} + \sin 2
$$

Calculate:

- $\cos 2 \approx -0.416$
- $\sin 2 \approx 0.909$

So,

$$
G(2) \approx \frac{-0.416}{2} + 0.909 \approx -0.208 + 0.909 \approx 0.701
$$

At $x \approx 1.107$:

Calculate $G(1.107)$:

$$
G(1.107) = \frac{\cos 1.107}{2} + \sin 1.107 \approx \frac{0.447}{2} + 0.894 \approx 0.2235 + 0.894 \approx 1.1175
$$

Summary of function values:

Point	$G(x)$ value
$x=0$	0.5
$x=2$	0.701
$x \approx 1.107$	1.1175 (max)

Final Analysis: Locating Local Maxima and Minima in [0, 2]

Based on the derivative analysis and endpoint evaluations:

- Local maximum occurs at $x \approx 1.107$, with $G(x) \approx 1.1175$.
- Endpoints are critical to check because maxima or minima can occur there:
- At $x=0$, $G(0)=0.5$
- At $x=2$, $G(2)=0.701$

Since $G(1.107)$ is larger than the function values at the endpoints, the absolute maximum over $[0, 2]$ is approximately 1.1175 at $x \approx 1.107$.

Is there a local minimum?

- The second derivative test indicates the critical point is a maximum, so no local minimum occurs at this point.
- The minimum value in the interval is at $x=0$, with $G(0)=0.5$.

Summary of Key Points for Finding Extrema of G(x) over [0, 2]

- Derivative calculation is crucial for finding critical points.
- Solve $G'(x)=0$ to identify potential maxima and minima.
- Use second derivative test to classify critical points.
- Evaluate endpoints as potential extrema.
- Compare values at critical points and endpoints to determine absolute maxima and minima.

Conclusion: How to Find Local Maxima and Minima of G(x) = (cos x)/2 + sin x

To effectively find the local maxima and minima over a specific interval:

1. Compute the derivative $G'(x)$

Frequently Asked Questions

How do I find the local maxima and minima of G(x) = (cos x)/2 + sin x over the interval [0, 2]?

To find local maxima and minima, first compute the derivative G'(x), set it equal to zero to find critical points, then determine whether each is a maximum or minimum using the second derivative test or the first derivative test within the interval [0, 2].

What is the derivative of G(x) = (cos x)/2 + sin x?

The derivative G'(x) is - (sin x)/2 + cos x.

How do I find critical points of G(x) within [0, 2]?

Set G'(x) = - (sin x)/2 + cos x = 0 and solve for x in the interval [0, 2].

What are the critical points of G(x) = (cos x)/2 + sin x on [0, 2]?

Critical points are solutions to - (sin x)/2 + cos x = 0 within [0, 2], which can be rearranged to tan x = 2.

How do I determine if a critical point is a maximum or minimum?

Use the second derivative test by computing G''(x). If G''(x) > 0 at the critical point, it's a local minimum; if G''(x) < 0, it's a local maximum.

What is the second derivative of G(x) = (cos x)/2 + sin x?

The second derivative G''(x) is - (cos x)/2 - sin x.

Are there any boundary points to consider for maxima or minima on [0, 2]?

Yes, evaluate G(x) at x=0 and x=2 to check for potential absolute maxima or minima at the endpoints.

How do I evaluate G(x) at the critical points and endpoints to find the local maxima and minima?

Calculate G(x) at each critical point within [0, 2] and at the endpoints x=0 and x=2. The largest value among these is an absolute maximum, and the smallest is an absolute minimum over the interval.

What is the approximate location of the local maxima and

minima of G(x) on [0, 2]?

Critical points where tan x = 2 occur at x ≈ arctan(2), which is approximately 1.107 radians, within [0, 2]. Evaluate G at these points and the endpoints to identify maxima and minima.

Can I use a graphing calculator to verify the locations of local maxima and minima of G(x) over [0, 2]?

Yes, plotting G(x) over [0, 2] with a graphing calculator or software helps visualize critical points and confirm the locations of local maxima and minima.

Additional Resources

Find Location Of Local Maxima Or Local Minima Over The Interval [0,2]. G(x) = cos(x)/2 + sin(x)

When analyzing the behavior of a function over a specified interval, a fundamental goal is to identify points at which the function attains local maxima or minima. These points are known as local extrema and are critical for understanding the function's overall shape, optimizing solutions, and making informed decisions based on the function's behavior. In this article, we will explore how to find the location of local maxima or local minima over the interval [0, 2] for the function G(x) = cos(x)/2 + sin(x). Through a detailed step-by-step analysis, we will uncover the techniques involved in calculus-based optimization, including derivative testing, critical point identification, and the application of the second derivative test.

Understanding the Function G(x) = cos(x)/2 + sin(x)

Before diving into the calculus techniques, it's essential to understand the nature of the given function:

- G(x) = (1/2) cos(x) + sin(x)

This function combines sine and cosine components, which are periodic and smooth, making calculus an effective tool for analyzing their extrema.

Step 1: Determine the Domain

The problem specifies the interval [0, 2], which is a closed interval on the real line. Since both sine and cosine are continuous and differentiable everywhere, G(x) is continuous and differentiable on this interval, allowing us to apply calculus methods directly.

Step 2: Find the Critical Points by Differentiation

To locate potential local maxima or minima, we need to examine the critical points where the first derivative of $G(x)$ is zero or undefined.

Calculating the First Derivative $G'(x)$

Given:

- $G(x) = (1/2) \cos(x) + \sin(x)$

Derivative:

- $G'(x) = (1/2)(-\sin(x)) + \cos(x) = - (1/2) \sin(x) + \cos(x)$

Thus,

$G'(x) = - (1/2) \sin(x) + \cos(x)$

Step 3: Set the Derivative Equal to Zero and Solve for x

Find critical points by solving:

$G'(x) = 0$

which leads to:

- $(1/2) \sin(x) = \cos(x)$

or equivalently,

- $2 \cos(x) = \sin(x)$

Rearranged as:

- $\sin(x) - 2 \cos(x) = 0$

This is a linear combination of sine and cosine. To solve, we can express it as a single trigonometric function using the tangent substitution or the auxiliary angle method.

Method 1: Auxiliary Angle Technique

Express the linear combination as:

$\sin(x) - 2 \cos(x) = R \sin(x + \alpha)$

where R and α are constants to be determined.

Recall that:

- $R = \sqrt{a^2 + b^2}$ where $a = 1$, $b = -2$

Calculating R:

- $R = \text{sqrt}(1^2 + (-2)^2) = \text{sqrt}(1 + 4) = \text{sqrt}(5)$

Next, find α such that:

- $\sin(\alpha) = b / R = -2 / \text{sqrt}(5)$
- $\cos(\alpha) = a / R = 1 / \text{sqrt}(5)$

Because:

- $\sin(x) - 2 \cos(x) = R \sin(x + \alpha)$

Solve for Critical Points:

Set:

- $R \sin(x + \alpha) = 0$

which implies:

- $\sin(x + \alpha) = 0$

Solutions for this are:

- $x + \alpha = n\pi$, where n is an integer

Therefore:

- $x = n\pi - \alpha$

Recall that:

- $\alpha = \arcsin(-2 / \text{sqrt}(5))$ or equivalently, $\alpha = \arcsin(b / R)$

Calculate α:

- $\alpha \approx \arcsin(-2 / 2.236) \approx \arcsin(-0.8944) \approx -1.095$ radians

(Alternatively, since $\sin(\alpha) = -2/\sqrt{5}$, α is in the fourth or third quadrant; choose the principal value accordingly.)

Critical points within [0, 2]:

Find all x in [0, 2] satisfying:

- $x = n\pi - \alpha$

Let's compute:

- For n=0:

$x \approx 0 - (-1.095) \approx 1.095$

- For n=1:

$x \approx \pi - (-1.095) \approx 3.1416 + 1.095 \approx 4.2366$ (which exceeds 2, discard)

- For n=-1:

$x \approx -\pi - (-1.095) \approx -3.1416 + 1.095 \approx -2.0466$ (less than 0, discard)

Thus, the only critical point in [0, 2] is approximately $x \approx 1.095$.

Step 4: Determine the Nature of Critical Points

To classify whether the critical point at $x \approx 1.095$ is a maximum or minimum, we can use the second derivative test.

Computing the Second Derivative $G''(x)$

Recall:

- $G'(x) = -(1/2)\sin(x) + \cos(x)$

Differentiate again:

- $G''(x) = -(1/2)\cos(x) - \sin(x)$

Now, evaluate $G''(x)$ at $x \approx 1.095$.

Numerical Evaluation:

- $\sin(1.095) \approx 0.888$
- $\cos(1.095) \approx 0.459$

Calculate:

$G''(1.095) \approx -(1/2)(0.459) - 0.888 \approx -0.2295 - 0.888 \approx -1.1175$

Since $G''(x) < 0$, the function is concave down at this point, indicating a local maximum at $x \approx 1.095$ within the interval.

Step 5: Check Endpoints of the Interval

Because the interval is closed, potential extrema can occur at the critical points and at the endpoints x=0 and x=2.

Evaluate G(x) at x=0:

- $G(0) = (1/2) \cos(0) + \sin(0) = (1/2)1 + 0 = 0.5$

Evaluate $G(x)$ at $x=2$:

- $G(2) = (1/2) \cos(2) + \sin(2)$

Calculate:

- $\cos(2) \approx -0.4161$
- $\sin(2) \approx 0.9093$

So,

$G(2) \approx (1/2)(-0.4161) + 0.9093 \approx -0.2081 + 0.9093 \approx 0.7012$

Evaluate $G(x)$ at critical point $x \approx 1.095$:

- $\cos(1.095) \approx 0.459$
- $\sin(1.095) \approx 0.888$

$G(1.095) \approx (1/2)(0.459) + 0.888 \approx 0.2295 + 0.888 \approx 1.1175$

Summary of Findings

Point	x (approximate)	G(x) value	Nature of extremum
0	0	0.5	Not an extremum (endpoint)
Critical point	1.095	1.1175	Local maximum
2	2	0.7012	Not an extremum (endpoint)

Conclusion: Locating the Local Maxima and Minima

Based on the analysis:

- A local maximum occurs at approximately $x \approx 1.095$, with $G(x) \approx 1.1175$.
- The endpoints do not produce a higher value than the critical point, so the absolute maximum over $[0, 2]$ is approximately 1.1175 at $x \approx 1.095$.
- The local minima can be checked by examining the second derivative at the endpoints or other critical points, but since only one critical point exists within the interval, and the second derivative there indicates a maximum, the endpoints are candidates for minima.

Compare $G(0) \approx 0.5$ and $G(2) \approx 0.7012$:

- The smallest value is at $x=0$, $G(0) \approx 0.5$, which is less than $G(2)$ and $G(1.095)$.

Therefore:

- The local minimum occurs at x=0, with G(0) = 0.5.

Final Remarks

In summary, to find the location of local maxima or minima over the interval [0, 2] for the function G(x) = cos(x)/2 + sin(x):

1. Differentiate to find G'(x).
2. Solve G'(x)=0 for critical points within the interval.
3. Use the second derivative test to classify each critical point.
4. Evaluate G(x) at critical points and endpoints to determine the local and absolute extrema.
5. Identify the points where the function att

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