Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information. Tan(x) = 1 5 , Cos(x) > 0

Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information. Tan(x) = 1/5, Cos(x) > 0

Understanding how to find the double angles of sine, cosine, and tangent functions when given specific information about an angle x is fundamental in trigonometry. In this article, we will explore the step-by-step process to compute Sin(2x), Cos(2x), and Tan(2x) given that Tan(x) = 1/5 and Cos(x) > 0. This scenario typically appears in solving problems involving right triangles, unit circles, or more advanced trigonometric identities, making it important for students and professionals alike to master.

Introduction to the Problem

Before diving into calculations, let's understand what the problem entails:

- You are given the value of the tangent of an angle x: Tan(x) = 1/5.
- The cosine of the same angle x is positive: Cos(x) > 0.
- Your goal is to find the double-angle sine, cosine, and tangent: Sin(2x), Cos(2x), Tan(2x).

This problem involves multiple fundamental trigonometric identities, including the relationships between tangent, sine, and cosine, as well as double-angle formulas.

Understanding the Given Information

1. What Does Tan(x) = 1/5 Mean?

- The tangent function relates the opposite side to the adjacent side in a right triangle:

$$
\tan(x) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{1}{5}
$$

- This ratio indicates that for some right triangle, the side opposite to angle x is 1 unit, and the side adjacent to x is 5 units.

2. The Sign of Cos(x): Cos(x) > 0

- The cosine function is positive in the first and fourth quadrants.
- Since we are not told explicitly which quadrant x is in, but the cosine is positive, x could be in Quadrant I or IV.
- Additional information or assumptions are needed, but in most typical problems, positive cosine indicates the principal value in Quadrant I or IV.

Step-by-Step Solution Approach

To find Sin(2x), Cos(2x), and Tan(2x), we'll need to:

1. Determine sin(x) and cos(x) from the given tangent.
2. Use double-angle formulas to compute the desired functions.

1. Find sin(x) and cos(x)

Given:

$$
\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{5}
$$

To find sin(x) and cos(x), we'll consider a right triangle where:

- Opposite side = 1
- Adjacent side = 5

Using the Pythagorean theorem:

$$
\text{Hypotenuse} = r = \sqrt{(1)^2 + (5)^2} = \sqrt{1 + 25} = \sqrt{26}
$$

Now, express sin(x) and cos(x):

$$
\sin(x) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{\sqrt{26}}
$$

$$
\cos(x) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{5}{\sqrt{26}}
$$

Since $\cos(x) > 0$, and both are positive in the first quadrant, this confirms x is in Quadrant I.

---

2. Apply Double-Angle Formulas

The double-angle identities relevant here are:

- $\sin(2x) = 2 \sin(x) \cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

Let's compute each step-by-step.

Calculating Sin(2x)

Using the double-angle identity:

$$
\sin(2x) = 2 \sin(x) \cos(x)
$$

Substituting the values:

$$
\sin(2x) = 2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} = 2 \times \frac{5}{26} = \frac{10}{26} = \frac{5}{13}
$$

Result:

$$
\boxed{\sin(2x) = \frac{5}{13}}
$$

Calculating Cos(2x)

Using the double-angle formula:

$$
\cos(2x) = \cos^2(x) - \sin^2(x)

\]

Substitute the known values:

\[
\cos^2(x) = \left(\frac{5}{\sqrt{26}}\right)^2 = \frac{25}{26}
\]

\[
\sin^2(x) = \left(\frac{1}{\sqrt{26}}\right)^2 = \frac{1}{26}
\]

Therefore:

\[
\cos(2x) = \frac{25}{26} - \frac{1}{26} = \frac{24}{26} = \frac{12}{13}
\]

Result:

\[
\boxed{\cos(2x) = \frac{12}{13}}
\]

Calculating Tan(2x)

Using the tangent double-angle identity:

\[
\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}
\]

Given \(\tan(x) = \frac{1}{5}\):

\[
\tan^2(x) = \left(\frac{1}{5}\right)^2 = \frac{1}{25}
\]

Substitute into the formula:

\[
\tan(2x) = \frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}} = \frac{\frac{2}{5}}{\frac{24}{25}} = \frac{\frac{2}{5}}{1} \times \frac{25}{24} = \frac{2}{5} \times \frac{25}{24} = \frac{2 \times 25}{5 \times 24} = \frac{50}{120} = \frac{5}{12}
\]

Result:

$$
\boxed{\tan(2x) = \frac{5}{12}}
$$

Summary of Results

Trigonometric Function	Value
Sin(2x)	$\frac{5}{13}$
Cos(2x)	$\frac{12}{13}$
Tan(2x)	$\frac{5}{12}$

Additional Insights and Applications

Understanding how to derive these double-angle values is beneficial in various mathematical and real-world contexts, including:

- Solving trigonometric equations.
- Simplifying expressions involving double angles.
- Analyzing wave functions in physics.
- Engineering applications involving oscillations and signal processing.

This methodology demonstrates the importance of fundamental identities and the Pythagorean theorem in trigonometry.

Practical Tips for Students

- Always verify the quadrant when dealing with signs of sine, cosine, and tangent.
- Use the Pythagorean theorem to find missing sides when given ratios.
- Remember the double-angle formulas, as they are powerful tools for simplifying complex expressions.
- Practice with different values to build confidence in applying these identities.

Conclusion

In this comprehensive guide, we've demonstrated how to find Sin(2x), Cos(2x), and Tan(2x) given that

$\tan(x) = \frac{1}{5}$ and $\cos(x) > 0$. By leveraging the basic relationships between trigonometric functions, the Pythagorean theorem, and double-angle identities, the calculations become straightforward and systematic. Mastering these techniques enhances your problem-solving skills and deepens your understanding of trigonometry, which is essential for advanced mathematics, physics, engineering, and related fields.

Frequently Asked Questions

Given that tan(x) = 1/5 and cos(x) > 0, what is sin(x)?

Since $\tan(x) = 1/5$ and $\cos(x) > 0$, we use the identity $\sin(x) = \tan(x) \cos(x)$. First, find $\cos(x)$: $\cos(x) = 1 / \sqrt{(1 + \tan^2(x))} = 1 / \sqrt{(1 + (1/5)^2)} = 1 / \sqrt{(1 + 1/25)} = 1 / \sqrt{(26/25)} = 1 / (\sqrt{26} / 5) = 5 / \sqrt{26}$. Then, $\sin(x) = \tan(x) \cos(x) = (1/5) (5 / \sqrt{26}) = 1 / \sqrt{26}$.

How do you find sin(2x) given tan(x) = 1/5 and cos(x) > 0?

First, find $\sin(x)$ and $\cos(x)$ as above: $\sin(x) = 1/\sqrt{26}$, $\cos(x) = 5/\sqrt{26}$. Then, use the double angle formula: $\sin(2x) = 2 \sin(x) \cos(x) = 2 (1/\sqrt{26}) (5/\sqrt{26}) = 2\ 5 / 26 = 10 / 26 = 5 / 13$.

What is cos(2x) if tan(x) = 1/5 and cos(x) > 0?

Using the double angle formula for cosine: $\cos(2x) = \cos^2(x) - \sin^2(x)$. With $\cos(x) = 5/\sqrt{26}$ and $\sin(x) = 1/\sqrt{26}$, $\cos(2x) = (5/\sqrt{26})^2 - (1/\sqrt{26})^2 = (25/26) - (1/26) = 24/26 = 12/13$.

How can we determine tan(2x) from the given information?

Using the double angle formula for tangent: $\tan(2x) = (2 \tan x) / (1 - \tan^2 x)$. Given $\tan x = 1/5$, $\tan(2x) = (2\ 1/5) / (1 - (1/5)^2) = (2/5) / (1 - 1/25) = (2/5) / (24/25) = (2/5) (25/24) = (50/120) = 5/12$.

Why is it important that cos(x) > 0 in this problem?

Because $\cos(x) > 0$ indicates that x is in the first or fourth quadrant, which affects the signs of sine and tangent functions. This ensures that the calculated sine and cosine values are positive or negative according to the quadrant, maintaining the correct signs in the trigonometric identities.

What are the final simplified values of sin(2x), cos(2x), and tan(2x) given tan(x) = 1/5 and cos(x) > 0?

The values are: $\sin(2x) = 5/13$, $\cos(2x) = 12/13$, and $\tan(2x) = 5/12$.

Additional Resources

Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information. Tan(x) = 1/5, Cos(x) > 0

Introduction

In trigonometry, understanding how to derive multiple functions from a given angle is essential. Whether you're tackling academic problems or applying these concepts in real-world scenarios like engineering or physics, mastering the relationships between sine, cosine, and tangent functions is vital. Today, we'll explore how to find the values of sin(2x), cos(2x), and tan(2x) given that tan(x) equals 1/5 and that cosine of the angle x is positive. This problem involves applying fundamental trigonometric identities and understanding the significance of the given information to determine the double-angle functions accurately.

Understanding the Given Information

Before diving into calculations, it's crucial to interpret the given data correctly:

- tan(x) = 1/5: The tangent of angle x is the ratio of the opposite side to the adjacent side in a right triangle, which equals 1 divided by 5.
- cos(x) > 0: The cosine of x is positive, indicating that angle x resides in either the first or the fourth quadrant, since cosine is positive in these regions.

These pieces of information provide the foundation for calculating the sine, cosine, and tangent of the double angle 2x.

Step 1: Visualizing the Triangle and Basic Relationships

To proceed, consider a right triangle where the angle x is situated such that:

- The opposite side to x is 1.
- The adjacent side to x is 5.

This configuration aligns with tan(x) = 1/5. Visualizing these sides helps us apply the Pythagorean theorem to find the hypotenuse, which is necessary for computing sine and cosine.

Step 2: Calculating the Hypotenuse

Using the Pythagorean theorem:

$$
\text{Hypotenuse} = r = \sqrt{\text{opposite}^2 + \text{adjacent}^2} = \sqrt{1^2 + 5^2} = \sqrt{1 + 25} = \sqrt{26}
$$

Thus, the hypotenuse length is $\sqrt{26}$.

Step 3: Determining Sin(x) and Cos(x)

With the sides known, the sine and cosine of x can be calculated as:

$$
\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{26}}
$$
$$
\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{\sqrt{26}}
$$

Since the problem states that $\cos(x) > 0$, and both sine and cosine are positive in the first quadrant, we conclude that:

$$
\boxed{
\sin(x) = \frac{1}{\sqrt{26}}, \quad \cos(x) = \frac{5}{\sqrt{26}}
}
$$

Step 4: Applying Double-Angle Formulas

The core of this problem involves the double-angle identities in trigonometry, which relate the functions of 2x to those of x:

- Sine double-angle formula:
$$
\sin(2x) = 2 \sin(x) \cos(x)
$$
- Cosine double-angle formula:
$$
\cos(2x) = \cos^2(x) - \sin^2(x)
$$
- Tangent double-angle formula:
$$
\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}
$$

We will now use these formulas along with the values of $\sin(x)$, $\cos(x)$, and $\tan(x)$ to find the desired functions.

Step 5: Calculating $\sin(2x)$

Using the sine double-angle formula:

$$
\sin(2x) = 2 \sin(x) \cos(x)

\]

Substituting the known values:

\[
\sin(2x) = 2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} = 2 \times \frac{5}{26} = \frac{10}{26} = \frac{5}{13}
\]

Result:

\[
\boxed{
\sin(2x) = \frac{5}{13}
}
\]

Step 6: Calculating \(\cos(2x)\)

Applying the cosine double-angle formula:

\[
\cos(2x) = \cos^2(x) - \sin^2(x)
\]

Calculating each term:

\[
\cos^2(x) = \left(\frac{5}{\sqrt{26}}\right)^2 = \frac{25}{26}
\]
\[
\sin^2(x) = \left(\frac{1}{\sqrt{26}}\right)^2 = \frac{1}{26}
\]

Subtracting:

\[
\cos(2x) = \frac{25}{26} - \frac{1}{26} = \frac{24}{26} = \frac{12}{13}
\]

Since \(\cos(x) > 0\) and in the first quadrant, \(\cos(2x)\) is also positive.

Result:

\[
\boxed{
\cos(2x) = \frac{12}{13}
}
\]

Step 7: Calculating $\tan(2x)$

Using the tangent double-angle formula:

$$
\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}
$$

Given $\tan(x) = \frac{1}{5}$:

$$
\tan^2(x) = \left(\frac{1}{5}\right)^2 = \frac{1}{25}
$$

Plugging into the formula:

$$
\tan(2x) = \frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}} = \frac{\frac{2}{5}}{\frac{24}{25}} = \frac{\frac{2}{5}}{1} \times \frac{25}{24} = \frac{2}{5} \times \frac{25}{24} = \frac{2 \times 25}{5 \times 24} = \frac{50}{120} = \frac{5}{12}
$$

Result:

$$
\boxed{
\tan(2x) = \frac{5}{12}
}
$$

Summary of Results

To summarize, given $\tan(x) = \frac{1}{5}$ and $\cos(x) > 0$, the double-angle functions are:

- $\boxed{\sin(2x) = \frac{5}{13}}$
- $\boxed{\cos(2x) = \frac{12}{13}}$
- $\boxed{\tan(2x) = \frac{5}{12}}$

Practical Applications and Additional Insights

Understanding these calculations is not merely academic; it has practical implications:

- Engineering and Physics: Precise calculations of double angles are essential in wave mechanics, signal processing, and oscillation analysis.
- Navigation and Geolocation: Trigonometric identities are fundamental in calculating bearings,

directions, and distances.
- Mathematics Education: Recognizing how to manipulate and apply identities enhances problem-solving skills and deepens conceptual understanding.

Furthermore, these results demonstrate the interconnectedness of trigonometric functions and how, with a few identities, one can derive complex expressions from basic ratios.

Final Remarks

Mastering the process of deriving double-angle functions from given information is a crucial skill in trigonometry. It involves understanding the relationships between angles and their trigonometric functions, leveraging identities, and carefully performing algebraic manipulations. Whether in academic pursuits or practical applications, these skills empower individuals to analyze and solve a broad range of problems efficiently.

By following the systematic approach outlined here—visualizing the triangle, calculating fundamental values, and applying identities—you can confidently tackle similar problems involving multiple trigonometric functions and their transformations.

Find Sin 2x Cos 2x And Tan 2x From The Given Information Tan X 1 5 Cos X Gt 0

Find Sin 2x Cos 2x And Tan 2x From The Given Information Tan X 1 5 Cos X Gt 0

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