Find The Area And Perimeter Of The Rectangle Below. Put Answers In Simplest Radical Form

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Understanding how to calculate the area and perimeter of a rectangle is fundamental in geometry. These measurements help us determine the size and boundary length of rectangular shapes, which are common in everyday life—from designing rooms and furniture to understanding various geometric problems. In this article, we will walk through the process of finding the area and perimeter of a specific rectangle, ensuring all answers are expressed in their simplest radical form for clarity and precision.

Understanding the Basics of Rectangles

Before delving into calculations, it’s essential to review the fundamental properties of rectangles:

- **Definition:** A rectangle is a quadrilateral with four right angles.
- **Opposite sides:** Equal in length and parallel.
- **Diagonals:** They are equal in length and bisect each other.
- **Area:** The total space enclosed within the rectangle.
- **Perimeter:** The total length around the rectangle.

The formulas for the area and perimeter of a rectangle are straightforward:

- **Area (A):** $A = \text{length} \times \text{width}$
- **Perimeter (P):** $P = 2 \times (\text{length} + \text{width})$

However, when the dimensions involve square roots or radicals, expressing the answers in simplest radical form ensures clarity, especially in cases where dimensions are derived from the lengths of diagonals or other measurements involving radicals.

Given Dimensions and Problem Setup

Suppose the rectangle in question has sides that are not immediately obvious but are derived from known measurements such as the length of the diagonals or relationships involving radicals. For example, consider the following scenario:

- The diagonal of the rectangle measures $\sqrt{50}$ units.
- The length of one side (say, the length) is $5\sqrt{2}$ units.
- The other side (the width) is unknown and needs to be determined.

Our goal is to find:

1. The length and width of the rectangle.
2. Its area and perimeter, both expressed in simplest radical form.

Step-by-Step Solution Approach

The process involves:

- Using the Pythagorean theorem to relate the sides and the diagonal.
- Simplifying radical expressions.
- Calculating area and perimeter with simplified radical expressions.

Let's proceed step by step.

Step 1: Express Known Values

- Diagonal $d = \sqrt{50}$
- Known side $L = 5\sqrt{2}$

Note: $\sqrt{50}$ can be simplified as:

$$
\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5 \sqrt{2}
$$

So, the diagonal length equals $5 \sqrt{2}$ units, which interestingly matches the known side length. This suggests the rectangle might be a square, but we need to confirm by checking the other side.

Step 2: Apply the Pythagorean Theorem

In a rectangle, the diagonal forms a right triangle with the length and width as legs:

$$
\text{diagonal}^2 = \text{length}^2 + \text{width}^2
$$

Plugging in known values:

$$
(5 \sqrt{2})^2 = (5 \sqrt{2})^2 + \text{width}^2
$$

Calculate $(5 \sqrt{2})^2$:

$$
(5 \sqrt{2})^2 = 5^2 \times (\sqrt{2})^2 = 25 \times 2 = 50
$$

So,

$$
50 = 50 + \text{width}^2
$$

Subtract 50 from both sides:

$$
50 - 50 = \text{width}^2 \Rightarrow 0 = \text{width}^2
$$

Therefore,

$$
\text{width} = 0
$$

which indicates a degenerate rectangle (a line), not a standard rectangle. To avoid this contradiction, let's modify the problem slightly for clarity.

Alternative Scenario:

Suppose:

- The diagonal $d = \sqrt{50}$,
- The length $L = 5 \sqrt{2}$,
- The width W is unknown.

Applying the Pythagorean theorem:

$$
d^2 = L^2 + W^2
$$

Substitute known values:

$$
(\sqrt{50})^2 = (5 \sqrt{2})^2 + W^2
$$

Calculate:

$$
50 = 25 \times 2 + W^2
$$

$$
50 = 50 + W^2
$$

Subtract 50 from both sides:

$$
0 = W^2
$$

$$
W = 0
$$

Again, this suggests a degenerate rectangle, which isn't meaningful.

Clarification and Corrected Example

To create a meaningful problem, suppose the following:

- The rectangle has sides a and b,
- The diagonal length is $\sqrt{72}$,
- One side, $a = 6$,
- Find the other side b.

Now, proceed with this corrected scenario.

Corrected Example: Calculating Dimensions with

Radical Lengths

Given:

- Diagonal $d = \sqrt{72}$,
- Side $a = 6$,
- Find b.

Step 1: Simplify the diagonal

$$
\sqrt{72} = \sqrt{36 \times 2} = 6 \sqrt{2}
$$

Step 2: Use Pythagoras theorem to find b

$$
d^2 = a^2 + b^2
$$

$$
(6 \sqrt{2})^2 = 6^2 + b^2
$$

Calculate:

$$
(6 \sqrt{2})^2 = 36 \times 2 = 72
$$
$$
6^2 = 36
$$

Set up the equation:

$$
72 = 36 + b^2
$$

Solve for b^2:

$$
b^2 = 72 - 36 = 36
$$

So,

$$
b = \pm \sqrt{36} = \pm 6
$$

Since length cannot be negative, $b = 6$.

Conclusion: The rectangle has sides of length 6 units each, which indicates it is a square with diagonal $6 \sqrt{2}$.

Calculating Area and Perimeter in Simplest Radical Form

Given the sides are both 6 units, the calculations are straightforward. However, for a more complex case where sides involve radicals, let’s explore a scenario where the sides are expressed in radical form, and calculations involve radicals.

Example:

Suppose the sides of the rectangle are:

- $a = 3 \sqrt{2}$,
- $b = 2 \sqrt{3}$.

Calculate:

- The area,
- The perimeter,

and express the answers in simplest radical form.

Step 1: Calculate Area

$$
A = a \times b = (3 \sqrt{2}) \times (2 \sqrt{3})
$$

Multiply coefficients:

\[

3 \times 2 = 6
\]

Multiply radicals:

\[
\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}
\]

So,

\[
A = 6 \sqrt{6}
\]

Answer for Area:

\[
\boxed{6 \sqrt{6}}
\]

Step 2: Calculate Perimeter

\[
P = 2(a + b) = 2(3 \sqrt{2} + 2 \sqrt{3})
\]

Expressed as:

\[
P = 2 \times 3 \sqrt{2} + 2 \times 2 \sqrt{3} = 6 \sqrt{2} + 4 \sqrt{3}
\]

Answer for Perimeter:

\[
\boxed{6 \sqrt{2} + 4 \sqrt{3}}
\]

Summary of Key Steps for Finding Area and Perimeter with Radicals

To effectively find the area and perimeter of a rectangle involving radicals, follow these steps:

1. **Simplify radical expressions** for the sides, diagonals, or any given measurements.
2. **Apply the Pythagorean theorem** to find unknown side lengths when diagonals are involved.
3. **Express the product or sum** of radical expressions carefully, simplifying radicals where possible.
4. **Calculate the area** by multiplying the length and width

Frequently Asked Questions

Given the length of the rectangle is 5√2 units and the width is 3 units, what is its area in simplest radical form?

Area = length × width = $5\sqrt{2} \times 3 = 15\sqrt{2}$ square units.

If the rectangle has a length of 8 units and a width of 3√3 units, what is its perimeter in simplest radical form?

Perimeter = 2 × (length + width) = $2 \times (8 + 3\sqrt{3}) = 16 + 6\sqrt{3}$ units.

A rectangle has a length of 4√5 units and a width of 2√5 units. What is its area in simplest radical form?

Area = $4\sqrt{5} \times 2\sqrt{5} = (4 \times 2) \times (\sqrt{5} \times \sqrt{5}) = 8 \times 5 = 40$ square units.

The length of a rectangle is 7 units, and its

width is $2\sqrt{3}$ units. Find the perimeter in simplest radical form.

Perimeter = $2 \times (7 + 2\sqrt{3}) = 14 + 4\sqrt{3}$ units.

A rectangle's length is $6\sqrt{2}$ units and width is $3\sqrt{2}$ units. What is its area in simplest radical form?

Area = $6\sqrt{2} \times 3\sqrt{2} = (6 \times 3) \times (\sqrt{2} \times \sqrt{2}) = 18 \times 2 = 36$ square units.

If the perimeter of a rectangle is 24 units and its length is $3\sqrt{2}$ units, what is its width in simplest radical form?

Perimeter = 2 × (length + width) → $24 = 2 \times (3\sqrt{2} + \text{width})$ → $12 = 3\sqrt{2} + \text{width}$ → width = $12 - 3\sqrt{2}$ units.

Additional Resources

Find The Area And Perimeter Of The Rectangle Below. Put Answers In Simplest Radical Form

Understanding how to calculate the area and perimeter of rectangles is fundamental in geometry, whether you're a student working on homework or a professional designing a blueprint. When the dimensions involve square roots, or radicals, the process requires a clear grasp of algebraic manipulation to simplify answers effectively. This article explores the methods for determining the area and perimeter of a rectangle, especially when given side lengths expressed in radical form, ensuring that your answers are presented in the simplest radical form possible.

Understanding the Basics: What Is a

Rectangle?

Before delving into calculations, it’s essential to review what defines a rectangle. A rectangle is a four-sided polygon (quadrilateral) with:

- Opposite sides that are equal in length.
- Four right angles (each measuring 90 degrees).

In geometry, the key measurements for a rectangle are:

- Length (l): The measure of one side.
- Width (w): The measure of the adjacent side.

The properties of rectangles make calculating area and perimeter straightforward once dimensions are known. The formulas are:

- Area (A): $A = l \times w$
- Perimeter (P): $P = 2(l + w)$

However, when side lengths involve radicals, the process requires simplifying these radical expressions to their simplest form.

Given Dimensions in Radical Form: The Challenge

Suppose you are provided with the side lengths of a rectangle expressed as radical expressions. For example:

- Length = $\sqrt{50}$
- Width = $\sqrt{18}$

Your task is to find:

- The area in simplest radical form.
- The perimeter in simplest radical form.

This involves two key steps:

1. Simplify the radical expressions representing sides.
2. Apply the formulas for area and perimeter, simplifying the results further if possible.

Let’s explore each step in detail.

Simplifying Radical Expressions

Radicals can often be simplified by factoring out perfect squares. The process involves:

- Identifying perfect square factors within the radical.
- Rewriting the radical as a product of a simplified radical and a rational number.

Example: Simplify $\sqrt{50}$

1. Find perfect squares that divide 50:

- $50 = 25 \times 2$
- 25 is a perfect square.

2. Rewrite:

- $\sqrt{50} = \sqrt{25 \times 2}$

3. Simplify:

- $\sqrt{25} \times \sqrt{2} = 5 \sqrt{2}$

Similarly, for $\sqrt{18}$:

- $18 = 9 \times 2$

- $\sqrt{18} = \sqrt{9 \times 2} = 3 \sqrt{2}$

Result:

- Length = $5 \sqrt{2}$
- Width = $3 \sqrt{2}$

Calculating the Area in Simplest Radical Form

The area of the rectangle is:

$$ A = l \times w $$

Using the simplified forms:

$$ A = (5 \sqrt{2}) \times (3 \sqrt{2}) $$

Apply the multiplication:

1. Multiply the coefficients:

- $ 5 \times 3 = 15 $

2. Multiply the radicals:

- $ \sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = \sqrt{4} $

3. Simplify the radical:

- $ \sqrt{4} = 2 $

Putting it all together:

$$ A = 15 \times 2 = 30 $$

Answer:

The area is 30 in simplest radical form (since the radical part simplifies to a rational number).

Note: Since the radical part simplified completely, the final answer is a rational number, 30.

Calculating the Perimeter in Simplest Radical Form

The perimeter is given by:

$$ P = 2(l + w) $$

Substitute the simplified side lengths:

$$ P = 2(5 \sqrt{2} + 3 \sqrt{2}) $$

Combine like terms inside the parentheses:

$$ 5 \sqrt{2} + 3 \sqrt{2} = (5 + 3) \sqrt{2} = 8 \sqrt{2} $$

Now multiply by 2:

$$ P = 2 \times 8 \sqrt{2} = 16 \sqrt{2} $$

The perimeter remains in radical form because it cannot be simplified further.

Answer:

The perimeter is $16 \sqrt{2}$ units.

General Approach for Similar Problems

When faced with other problems involving radicals, follow this systematic approach:

1. Identify radical expressions: Write down the given side lengths.
2. Simplify radicals: Factor out perfect squares to reduce radicals to simplest form.
3. Apply formulas: Use the standard area and perimeter formulas.
4. Simplify results: Combine like radicals where applicable, and simplify radicals to their simplest form.
5. Express final answers in simplest radical form: Ensure no further reduction is possible.

Additional Examples and Practice

Example 1:
Side lengths: $\sqrt{72}$ and $\sqrt{98}$

- Simplify:

- $\sqrt{72} = \sqrt{36 \times 2} = 6 \sqrt{2}$
- $\sqrt{98} = \sqrt{49 \times 2} = 7 \sqrt{2}$

- Area:

- $A = 6 \sqrt{2} \times 7 \sqrt{2} = (6 \times 7) \times (\sqrt{2}$

\times \sqrt{2}) = 42 \times 2 = 84 \)

- Perimeter:

- \(P = 2(6 \sqrt{2} + 7 \sqrt{2}) = 2 \times 13 \sqrt{2} = 26 \sqrt{2} \)

Final answers:
- Area: 84
- Perimeter: \(26 \sqrt{2} \)

The Importance of Simplification in Radicals

Simplifying radicals isn't just about neatness; it’s essential for clarity and accuracy in mathematical communication. Expressing answers in simplest radical form:

- Makes it easier to compare results.
- Ensures that calculations are fully reduced.
- Prepares the way for further algebraic manipulations if needed.

In real-world applications, such as construction or engineering, precise measurements expressed clearly can prevent costly errors.

Conclusion

Calculating the area and perimeter of a rectangle when side lengths involve radicals might initially seem daunting. However, by mastering radical simplification and applying fundamental formulas systematically, you can confidently produce accurate, simplified answers. Remember, the key steps involve:

- Recognizing perfect square factors within radicals.
- Simplifying radicals to their simplest radical form.
- Applying the standard formulas for area (\(l \times w \)) and perimeter (\(2(l + w) \)).
- Simplifying the results further whenever possible.

Mastering these techniques enhances your mathematical toolkit, enabling you to tackle more complex problems with confidence and precision.

Whether for academic pursuits or practical applications, a solid understanding of radical simplification and geometric calculations is invaluable.

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