

Find The Area Of The Convex Polygon With Vertices (0,5), (-1,2), (4,4), (-3,-4) And (2,0).

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Calculating the area of a polygon given its vertices is a fundamental problem in coordinate geometry. When the vertices are known, the most efficient method often employed is the Shoelace Theorem (also known as Gauss's area formula). This approach simplifies the process by converting geometric problems into algebraic calculations based on the coordinates of the vertices. In this article, we will explore how to determine the area of a convex polygon with vertices at (0,5), (-1,2), (4,4), (-3,-4), and (2,0). We will systematically analyze the steps involved, including verifying the polygon's convexity, arranging the vertices in the correct order, and applying the shoelace formula to find the exact area.

Understanding the Given Vertices and the Polygon

List of Vertices

The polygon in question has five vertices:

- A = (0, 5)
- B = (-1, 2)
- C = (4, 4)
- D = (-3, -4)
- E = (2, 0)

Understanding the positions of these points is crucial. The vertices are given as coordinate points in the Cartesian plane, and the polygon formed by connecting these points in order must be convex for the area calculation to be straightforward using the shoelace method.

Verifying Convexity

Before proceeding, it is important to verify whether the polygon formed by these vertices is convex. A convex polygon is one where all interior angles are less than 180° , and no line segment between two points inside the polygon crosses its boundary.

To verify convexity, one approach is to:

- Calculate the cross products of consecutive edges.
- Check the sign of these cross products; consistent signs indicate convexity.

Alternatively, plotting the points or visualizing their arrangement helps, but here, for thoroughness, we will analyze the cross products.

Arranging the Vertices in Proper Order

The vertices must be ordered either clockwise or counterclockwise around the polygon to correctly apply the shoelace formula. Since the vertices are given in arbitrary order, we need to determine their order.

Method to Order Vertices

- Compute the centroid of the points.
- Calculate the angle each point makes with respect to the centroid.
- Sort the vertices based on these angles.

Calculating the centroid:

$$x_c = \frac{0 + (-1) + 4 + (-3) + 2}{5} = \frac{2}{5} = 0.4$$

$$y_c = \frac{5 + 2 + 4 + (-4) + 0}{5} = \frac{7}{5} = 1.4$$

Calculating angles:

For each point $((x,y))$:

$$\theta = \arctan\left(\frac{y - y_c}{x - x_c}\right)$$

Calculating for each:

- A (0,5):

$$\theta_A = \arctan\left(\frac{5 - 1.4}{0 - 0.4}\right) = \arctan\left(\frac{3.6}{-0.4}\right) \approx \arctan(-9) \approx -84.3^\circ$$

- B (-1,2):

$$\theta_B = \arctan\left(\frac{2 - 1.4}{-1 - 0.4}\right) = \arctan\left(\frac{0.6}{-1.4}\right) \approx \arctan(-0.429) \approx -23.1^\circ$$

- C (4,4):

$$\theta_C = \arctan\left(\frac{4 - 1.4}{4 - 0.4}\right) = \arctan\left(\frac{2.6}{3.6}\right) \approx \arctan(0.722) \approx 36.0^\circ$$

- D (-3,-4):

$$\theta_D = \arctan\left(\frac{-4 - 1.4}{-3 - 0.4}\right) = \arctan\left(\frac{-5.4}{-3.4}\right) = \arctan(1.588) \approx 58.0^\circ$$

(Note: Since both numerator and denominator are negative, the point is in the third quadrant; thus, add 180° to the angle.)

$$\theta_D \approx 58^\circ + 180^\circ = 238^\circ$$

- E (2,0):

$$\theta_E = \arctan\left(\frac{0 - 1.4}{2 - 0.4}\right) = \arctan\left(\frac{-1.4}{1.6}\right) \approx \arctan(-0.875) \approx -41.1^\circ$$

Ordering by angle (from smallest to largest):

- $\theta_A \approx -84.3^\circ$
- $\theta_B \approx -23.1^\circ$
- $\theta_E \approx -41.1^\circ$
- $\theta_C \approx 36.0^\circ$
- $\theta_D \approx 238.0^\circ$

Arranged in order:

1. A (0,5)
2. B (-1,2)
3. E (2,0)
4. C (4,4)
5. D (-3,-4)

To ensure the polygon is properly traversed, we will list the vertices in this order:

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(0,5) \rightarrow (-1,2) \rightarrow (2,0) \rightarrow (4,4) \rightarrow (-3,-4) \rightarrow (0,5)
\]
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Applying the Shoelace Theorem

Overview of the Shoelace Formula

The shoelace formula computes the area of a simple polygon when the vertices are ordered sequentially. For vertices $((x_1, y_1), (x_2, y_2), \dots, (x_n, y_n))$, the area (A) is:

$$A = \frac{1}{2} \left| \sum_{i=1}^{n-1} (x_i y_{i+1} - y_i x_{i+1}) + (x_n y_1 - y_n x_1) \right|$$

This formula sums the cross-products of the coordinates in a specific pattern, effectively calculating the polygon's signed area.

Calculating Step-by-Step

Using the ordered vertices:

Point	(x_i)	(y_i)
1 (A)	0	5
2 (B)	-1	2
3 (E)	2	0
4 (C)	4	4
5 (D)	-3	-4

Compute:

$$\text{Sum}_1 = x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_5 + x_5 y_1$$

$$= (0)(2) + (-1)(0) + (2)(4) + (4)(-4) + (-3)(5) = 0 + 0 + 8 - 16 - 15 = -23$$

Similarly:

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$$\text{Sum}_2 = y_1 x_2 + y_2 x_3 + y_3 x_4 + y_4 x_5 + y_5 x_1$$

$$= (5)(-1) + (2)(2) + (0)(4) + (4)(-3) + (-4)(0) = -5 + 4 + 0 - 12 + 0 = -13$$

Now, the area:

$$A = \frac{1}{2} | \text{Sum}_1 - \text{Sum}_2 | = \frac{1}{2} | -23 - (-13) | = \frac{1}{2} | -23 + 13 | = \frac{1}{2} | -10 | = 5$$

This calculation indicates the area of the polygon is 5 square units.

Conclusion and Final Remarks

The process of finding the area of a convex polygon from its vertices involves several steps, including verifying the shape's convexity, correctly ordering the vertices, and applying the shoelace formula. In this

Frequently Asked Questions

How do you find the area of a convex polygon given its vertices?

You can use the Shoelace Theorem (also known as Gauss's area formula) by plugging in the coordinates of the vertices in order and computing the absolute value of half the sum of cross-products.

What are the steps to apply the Shoelace Theorem to the polygon with vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0)?

Arrange the vertices in order, repeat the first vertex at the end, then compute the sum of the products of the x-coordinate of each vertex with the y-coordinate of the next, and vice versa. Subtract these sums, take absolute value, and divide by 2.

What is the ordered list of vertices used in the Shoelace Theorem for this polygon?

The vertices in order are (0,5), (-1,2), (4,4), (-3,-4), (2,0), and then back to (0,5).

Can you demonstrate the calculation of the area step-by-step for these vertices?

Yes. Multiply x of each vertex by y of the next, sum these: $(0 \cdot 2) + (-1 \cdot 4) + (4 \cdot 4) + (-3 \cdot 0) + (2 \cdot 5)$. Then multiply y of each vertex by x of the next and sum: $(5 \cdot -1) + (2 \cdot 4) + (4 \cdot -3) + (0 \cdot -4) + (2 \cdot 0)$. Subtract the second sum from the first, take the absolute value, and divide by 2.

What is the calculated area of the convex polygon with the given vertices?

The area is 27 square units.

Is the polygon with vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0) convex?

Yes, given the order and the shape, the polygon is convex, which makes the Shoelace Theorem applicable.

Why is it important to order the vertices correctly when calculating the area?

Ordering vertices in either clockwise or counterclockwise order ensures the Shoelace Theorem yields the correct area; unordered vertices can lead to incorrect results.

Can the area calculation be verified using an alternative method, such as dividing into triangles?

Yes, dividing the polygon into triangles and summing their areas is an alternative method, which can serve as a verification of the Shoelace Theorem result.

What is the significance of the vertices being in a convex polygon for the area calculation?

Convexity ensures that the vertices are ordered properly without intersecting edges, making the Shoelace Theorem directly applicable and the area calculation straightforward.

Additional Resources

Find The Area Of The Convex Polygon With Vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0)

Understanding how to find the area of a convex polygon when given its vertices is a fundamental skill in coordinate geometry, with applications spanning architecture, engineering, and computer graphics. The problem of finding the area of a convex polygon with vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0) is a classic example that demonstrates the power and elegance of the shoelace formula (also known as Gauss's area formula). This step-by-step guide aims to walk you through the process of calculating the area of this specific polygon, ensuring you grasp both the conceptual and practical aspects of the

method.

Understanding the Basics: Coordinates and Convex Polygons

Before diving into calculations, it's essential to understand some key concepts:

- Vertices of a Polygon: These are the corner points that define the shape. In our case, the vertices are given as coordinate pairs: (0,5), (-1,2), (4,4), (-3,-4), and (2,0).
- Convex Polygon: A polygon where all interior angles are less than 180° , and no line segment between any two points inside the polygon crosses its boundary. It's important because the shoelace formula applies straightforwardly to convex polygons, though it can also be used for simple polygons in general.
- Coordinate System: Each vertex is represented as an ordered pair (x, y), indicating its position in the Cartesian plane.

Step 1: Confirming the Polygon is Convex

While the problem states that the polygon is convex, in practice, you may need to verify this by checking the orientation of the vertices or ensuring no internal angles exceed 180° . For this specific set of vertices, plotting or analyzing the cross products of the vectors will confirm convexity.

Plotting the vertices or calculating the cross products between consecutive edges will help confirm convexity. Since the problem states it's convex, we proceed under that assumption.

Step 2: Organizing the Vertices

To apply the shoelace formula, the vertices must be ordered either clockwise or counterclockwise around the polygon. The order in which we list the vertices impacts the calculation, so choosing a consistent order is key.

Let's order the vertices in counterclockwise order:

1. (0, 5)
2. (4, 4)
3. (2, 0)
4. (-3, -4)
5. (-1, 2)

This ordering is based on plotting or analyzing the positions relative to each other, ensuring the edges connect in sequence without crossing.

Step 3: Applying the Shoelace Formula

The shoelace formula is a powerful method for calculating the area of a polygon given its vertices in order:

$$\text{Area} = \frac{1}{2} \left| \sum_{i=1}^n (x_i y_{i+1} - y_i x_{i+1}) \right|$$

where:

- (x_i, y_i) are the coordinates of the vertices,
- (x_{n+1}, y_{n+1}) is taken as the first vertex to close the polygon.

Step-by-step:

1. List the vertices in order, then duplicate the first vertex at the end to facilitate calculations.

Vertex	(x_i)	(y_i)
1	0	5
2	4	4
3	2	0
4	-3	-4
5	-1	2
1 (repeat)	0	5

2. Calculate the sum of $(x_i y_{i+1})$:

$$(0 \times 4) + (4 \times 0) + (2 \times -4) + (-3 \times 2) + (-1 \times 5)$$

$$= 0 + 0 - 8 - 6 - 5 = -19$$

3. Calculate the sum of $(y_i x_{i+1})$:

$$(5 \times 4) + (4 \times 2) + (0 \times -3) + (-4 \times -1) + (2 \times 0)$$

$$= 20 + 8 + 0 + 4 + 0 = 32$$

4. Compute the absolute difference:

$$|(-19) - 32| = |-51| = 51$$

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5. Divide by 2 to find the area:

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$$\text{Area} = \frac{51}{2} = 25.5$$

\]

Final Answer:

The area of the convex polygon with vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0) is 25.5 square units.

Additional Insights and Tips

Verifying the Calculation

- Plotting the vertices on graph paper or using graphing software (like GeoGebra, Desmos, or a calculator) helps visualize the polygon and verify the ordering.
- Double-check the vertex order to ensure the shoelace formula is correctly applied.
- Repeat the calculations or use computational tools for accuracy, especially with more complex polygons.

Why the Shoelace Formula Works

The shoelace formula essentially computes the area by summing the cross-products of adjacent vertices, capturing the overall "signed" area enclosed. The absolute value ensures the area is positive regardless of the order of vertices.

Practical Applications

Calculating the area of polygons based on vertices is useful in numerous fields:

- Land surveying: Determining the size of irregularly shaped plots.
- Computer graphics: Calculating polygon areas for rendering and collision detection.
- Architecture and engineering: Estimating material quantities and space planning.
- Mathematics education: Teaching coordinate geometry and polygon properties.

Conclusion

Finding the area of a convex polygon with given vertices involves understanding the polygon's structure, ordering the vertices correctly, and applying the shoelace formula systematically. The

process not only provides an exact numerical answer but also deepens comprehension of coordinate geometry principles. For our specific case with vertices (0,5), (-1,2), (4,4), (-3,-4), and (2,0), the calculated area of 25.5 square units exemplifies how coordinate-based methods efficiently solve geometric problems that may seem complex at first glance.

Remember, practice with different polygons enhances proficiency, and visualizing the shape can often clarify the calculation process. Happy geometry exploring!

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