Find The Area Of The Region Between The Curves Y = Cos(x), Y = Sin(2x), X 0, And X = /2

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Understanding the area enclosed between curves is a fundamental problem in integral calculus. When two functions intersect within a specified interval, the region between their graphs can be computed by integrating the difference of the functions over the relevant subintervals. In this article, we will explore how to find the exact area of the region bounded by the curves y = cos(x) and y = sin(2x), between x = 0 and x = π/2. We will analyze the intersection points, determine which curve is on top in each subinterval, and perform the necessary integrations to arrive at the solution.

Understanding the Problem and Setting Up the Framework

Given Curves and Limits of Integration

The problem specifies two curves:

- y = cos(x)
- y = sin(2x)

and the interval:

- x from 0 to π/2

Our goal is to find the area of the region enclosed between these two curves within the specified bounds.

Why Is It Necessary to Find Intersection Points?

Since the curves may cross each other within the interval, the area between them is not simply the integral of their difference over the entire interval. Instead, we need to:

1. Find the points where the curves intersect in [0, π/2].
2. Use these points to partition the interval into subintervals where one curve is consistently on top.
3. Integrate the difference of the functions over each subinterval accordingly.

Finding Intersection Points of the Curves

Set the Equations Equal to Each Other

To find intersection points:

$$
\cos(x) = \sin(2x)
$$

Recall the double-angle identity:

$$
\sin(2x) = 2 \sin x \cos x
$$

Substituting:

$$
\cos x = 2 \sin x \cos x
$$

Dividing both sides by $\cos x$ (bearing in mind $\cos x \neq 0$ at the point of division):

$$
1 = 2 \sin x
$$

Thus:

$$
\sin x = \frac{1}{2}
$$

The solutions for $\sin x = 1/2$ in [0, π/2] are:

$$
x = \frac{\pi}{6}

\]

Now, check if $\cos x = 0$ yields additional intersection points in [0, π/2]:

$$
\cos x=0 \Rightarrow x= \frac{\pi}{2}
$$

At $x = \pi/2$:

$$
\cos(\pi/2) = 0
$$
$$
\sin(2 \times \pi/2) = \sin \pi = 0
$$

They are equal, so $x=\pi/2$ is also an intersection point.

Summary of intersection points:

- $x= \pi/6$
- $x= \pi/2$

Determining Which Curve Is on Top in Each Subinterval

Intervals of Interest

Based on the intersection points, the interval [0, π/2] can be split into:

1. [0, π/6]
2. [π/6, π/2]

We need to analyze the relative positions of the curves over these subintervals.

Evaluating the Curves at Sample Points

- At $x=0$:

$$
\cos 0 = 1
$$
$$
\sin(0) = 0
$$

So, y = cos(x) is on top at x=0.

- At $x=\pi/8$ (~0.3927):

$$
\cos(\pi/8) \approx 0.9239
$$
$$
\sin(2 \times \pi/8) = \sin(\pi/4) \approx 0.7071
$$

cos(x) > sin(2x) in [0, π/6].

- At $x=\pi/4$ (~0.7854):

$$
\cos(\pi/4) = \frac{\sqrt{2}}{2} \approx 0.7071
$$
$$
\sin(2 \times \pi/4) = \sin(\pi/2) = 1
$$

Here, sin(2x) > cos(x).

- At $x= \pi/3$ (~1.0472):

$$
\cos(\pi/3) = 0.5
$$
$$
\sin(2 \times \pi/3) = \sin(2.094) \approx 0.8660
$$

Again, sin(2x) > cos(x).

- At $x=\pi/6$:

$$
\cos(\pi/6)= \frac{\sqrt{3}}{2} \approx 0.8660
$$
$$
\sin(2 \times \pi/6) = \sin(\pi/3)= \frac{\sqrt{3}}{2} \approx 0.8660
$$
They are equal at the intersection point, as expected.

- At $x= \pi/2$:

$$
\cos(\pi/2)=0
$$
$$
\sin(\pi) = 0
$$
Equal at x=π/2.

Conclusion:

- From 0 to $\pi/6$: $\cos(x) \geq \sin(2x)$
- From $\pi/6$ to $\pi/2$: $\sin(2x) \geq \cos(x)$

Calculating the Enclosed Area

Formulating the Integrals

The total area between the curves can be expressed as:

$$
A = \int_{0}^{\pi/6} [\cos(x) - \sin(2x)] \, dx + \int_{\pi/6}^{\pi/2} [\sin(2x) - \cos(x)] \, dx
$$

This accounts for the fact that the top and bottom curves switch roles at $x= \pi/6$.

Performing the Integrations

First Integral: $\int_{0}^{\pi/6} [\cos(x) - \sin(2x)] \, dx$

Break into two integrals:

$$
\int_{0}^{\pi/6} \cos(x) \, dx - \int_{0}^{\pi/6} \sin(2x) \, dx
$$

Calculate each:

- $\int \cos(x) dx = \sin(x)$
- $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$

Evaluate:

$$
\left[\sin(x) \right]_0^{\pi/6} - \left[-\frac{1}{2} \cos(2x) \right]_0^{\pi/6}
$$

Plugging in:

$$
\sin(\pi/6) - \sin(0) + \frac{1}{2} [\cos(0) - \cos(2 \times \pi/6)]
$$

Simplify:

$$
\frac{1}{2} - 0 + \frac{1}{2} [1 - \cos(\pi/3)]
$$

$$
= \frac{1}{2} + \frac{1}{2} \left(1 - \frac{1}{2} \right) = \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}
$$

Second Integral: $\int_{\pi/6}^{\pi/2} [\sin(2x) - \cos(x)] \, dx$

Break into two:

$$
\int_{\pi/6}^{\pi/2} \sin(2x) dx - \int_{\pi/6}^{\pi/2} \cos(x) dx
$$

Calculate each:

- $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$
- $\int \cos(x) dx = \sin(x)$

Evaluate:

$$
\left[-\frac{1}{2} \cos(2x) \right]_{\pi/6}^{\pi/2} - [\sin(x)
$$

]_{\pi/6}^{\pi/2}
\]

Plug in bounds:

\[
\left(-\frac{1}{2} \cos(\pi) + \frac{1}{2} \cos(\pi/3) \right) - (
\sin(\pi/2) - \sin(\pi/6))
\]

Simplify:

\[
-\

Frequently Asked Questions

What is the first step in finding the area between the curves $y = \cos(x)$ and $y = \sin(2x)$ from $x = 0$ to $x = \pi/2$?

The first step is to determine the points of intersection of the two curves within the interval, then set up the integral(s) to find the area between them accordingly.

How do you find the points of intersection between $y = \cos(x)$ and $y = \sin(2x)$ in the interval $[0, \pi/2]$?

Solve $\cos(x) = \sin(2x)$ within the interval. Using the identity $\sin(2x) = 2\sin(x)\cos(x)$, the equation becomes $\cos(x) = 2\sin(x)\cos(x)$. Dividing both sides by $\cos(x)$ (where $\cos(x) \neq 0$), gives $1 = 2\sin(x)$, so $\sin(x) = 1/2$. Therefore, $x = \pi/6$ within $[0, \pi/2]$.

How are the curves arranged relative to each other in the interval $[0, \pi/2]$, and how does this affect the integral setup?

In $[0, \pi/6]$, $\cos(x) > \sin(2x)$, and in $[\pi/6, \pi/2]$, $\sin(2x) > \cos(x)$. This means the area between the curves is split into two parts, requiring two integrals with the appropriate upper and lower functions in each interval.

What is the integral expression for the total area between $y = \cos(x)$ and $y = \sin(2x)$ from $x=0$ to

$x=\pi/2$?

The total area A is given by:
$A = \int_0^{\pi/6} [\cos(x) - \sin(2x)]\, dx + \int_{\pi/6}^{\pi/2} [\sin(2x) - \cos(x)]\, dx$.

How do you evaluate the integrals to find the exact area between the curves?

Evaluate each integral separately:
- For $\int [\cos(x) - \sin(2x)]\, dx$, find the antiderivatives of cos(x) and sin(2x).
- For $\int [\sin(2x) - \cos(x)]\, dx$, do the same. Then substitute the limits and sum the results to obtain the total area.

Additional Resources

Find The Area Of The Region Between The Curves Y = Cos(x), Y = Sin(2x), X ≥ 0, And X = π/2

Introduction

Mathematics, especially calculus, provides powerful tools to analyze and understand the geometric properties of curves and regions. One such intriguing problem involves finding the area of the region bounded by two trigonometric functions—specifically, the curves y = cos(x) and y = sin(2x)—along with the vertical boundary at x = 0 and the line x = π/2. This task not only exemplifies the application of integral calculus but also offers insights into how curves intersect and how their areas are computed through definite integrals.

In this article, we delve into a detailed, step-by-step exploration of this problem, illustrating the analytical process, discussing the underlying concepts, and providing interpretations of the results. Our goal is to present a comprehensive understanding suitable for students, educators, and enthusiasts interested in the geometric aspects of calculus.

Understanding the Problem and Its Context

The Geometric Region

The problem involves determining the area of a specific region in the xy-plane bounded by:

- The curve y = cos(x)
- The curve y = sin(2x)

- The vertical line $x = 0$
- The vertical line $x = \pi/2$

This region is confined within the first quadrant, where both functions are well-behaved and their intersection points are within the specified limits.

The Significance of the Curves

- $y = \cos(x)$: A standard trigonometric function with a period of 2π, starting at $y=1$ when $x=0$ and decreasing to 0 at $x=\pi/2$.
- $y = \sin(2x)$: A sinusoid with a period of π, oscillating more rapidly than $\sin(x)$. It equals zero at $x=0$ and $x=\pi/2$, with its maximum at $x=\pi/4$.

Understanding where these curves intersect within $x \in [0, \pi/2]$ is crucial, as it determines how the bounded region is partitioned and which function lies above the other in each subinterval.

Analytical Approach: Step-by-Step Methodology

1. Identifying the Intersection Points

Before calculating the area, it is essential to find where the curves intersect within the domain $[0, \pi/2]$. These points partition the region into segments where one curve is above the other.

Set $y = \cos(x)$ equal to $y = \sin(2x)$:

$$
\cos(x) = \sin(2x)
$$

Recall that:

$$
\sin(2x) = 2 \sin(x) \cos(x)
$$

Substituting:

$$
\cos(x) = 2 \sin(x) \cos(x)
$$

Dividing both sides by $\cos(x)$, noting that $\cos(x) \neq 0$ (except at points where $\cos(x)=0$, which we will check separately):

$$
1 = 2 \sin(x)
$$

$$
\sin(x) = \frac{1}{2}
$$

Within [0, π/2], $\sin(x) = 1/2$ at:

$$
x = \frac{\pi}{6}
$$

Now, check the point where $\cos(x)=0$:

$$
\cos(x) = 0 \Rightarrow x = \frac{\pi}{2}
$$

At $x = \pi/2$:

$$
\sin(2 \times \pi/2) = \sin(\pi) = 0
$$

and

$$
\cos(\pi/2) = 0
$$

So, they intersect at $x = \pi/2$ as well.

Summary of intersection points:

- At $x = \pi/6$, $\cos(x) = \sin(2x)$
- At $x = \pi/2$, both functions are zero and intersect

---

2. Determining Which Curve Is Above the Other

To set up the integral correctly, we need to know which function is on top in each subinterval:

- For $x \in [0, \pi/6]$
- For $x \in [\pi/6, \pi/2]$

Testing at a point in each interval:

- At $x=0$:

$$
\cos(0) = 1

$$
\sin(0) = 0
$$

So, $\cos(0) = 1$ is above $\sin(0) = 0$.

- At $x = \pi/4$:

$$
\cos(\pi/4) = \frac{\sqrt{2}}{2} \approx 0.707
$$

$$
\sin(2 \times \pi/4) = \sin(\pi/2) = 1
$$

Here, $\sin(2x) = 1$ is above $\cos(x)$.

- At $x = \pi/3$:

$$
\cos(\pi/3) = 0.5
$$

$$
\sin(2 \times \pi/3) = \sin(2\pi/3) = \sqrt{3}/2 \approx 0.866
$$

Again, $\sin(2x)$ is above $\cos(x)$.

Conclusion:

- From $x=0$ to $x=\pi/6$, $\cos(x)$ is above $\sin(2x)$.
- From $x=\pi/6$ to $x=\pi/2$, $\sin(2x)$ is above $\cos(x)$.

---

3. Setting Up the Integral for the Area

The total area can be partitioned into two parts:

$$
\text{Area} = \int_{0}^{\pi/6} [\cos(x) - \sin(2x)] \, dx +
\int_{\pi/6}^{\pi/2} [\sin(2x) - \cos(x)] \, dx
$$

This accounts for the varying dominance of the functions in each subinterval.

---

4. Computing the Integrals

First Integral: $x \in [0, \pi/6]$

$$
A_1 = \int_{0}^{\pi/6} [\cos(x) - \sin(2x)] \, dx
$$

- $\int \cos(x) dx = \sin(x)$
- $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$

Calculating:

$$
A_1 = \left[\sin(x) + \frac{1}{2} \cos(2x)\right]_0^{\pi/6}
$$

At $x = \pi/6$:

$$
\sin(\pi/6) = 1/2
$$
$$
\cos(2 \times \pi/6) = \cos(\pi/3) = 1/2
$$

At $x=0$:

$$
\sin(0)=0
$$
$$
\cos(0)=1
$$

Plugging in:

$$
A_1 = \left(\frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \right) - \left(0 + \frac{1}{2} \times 1 \right) = \left(\frac{1}{2} + \frac{1}{4} \right) - \frac{1}{2} = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}
$$

Second Integral: $x \in [\pi/6, \pi/2]$

$$
A_2 = \int_{\pi/6}^{\pi/2} [\sin(2x) - \cos(x)] \, dx
$$

- $\int \sin(2x) dx = -\frac{1}{2} \cos(2x)$
- $\int \cos(x) dx= \sin(x)$

```
Calculating:

\[
A_2 = \left[-\frac{1}{2} \cos(2x) - \sin(x) \right]_{\pi/6}^{\pi/2}
\]

At \(x = \pi/2\):

\[
\cos(\pi) = -1
\]
\[
\sin(\pi/2)=1
\]

At \(x = \pi/6\):

\[
\cos(2 \times \pi/6) = \cos(\pi/3) = 1/2
\]
\[
\sin(\pi/6) = 1/2
\]

Plugging in:

\[
A_2 = \left[-\frac{1}{2} \times (-1) - 1 \right] - \left[ -\frac{1}{2} \times
\frac{1}{2} - \frac{1}{2} \
```

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