# Find The Cross Product Ab And Verify That It Is Orthogonal To Both A And B. A=6,0,2, B=0,8,0

**Find The Cross Product Ab And Verify That It Is Orthogonal To Both A And B. A=6,0,2, B=0,8,0**

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## Understanding the Cross Product in Vector Mathematics

The cross product is a fundamental operation in vector mathematics, especially in three-dimensional space. It produces a vector that is perpendicular (orthogonal) to two given vectors. This operation is widely used in physics, engineering, computer graphics, and other fields that involve spatial computations. In this article, we will explore how to compute the cross product of two specific vectors, A and B, and verify that the resulting vector is orthogonal to both A and B.

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## Definition of the Cross Product

The cross product of two vectors A and B in three-dimensional space, denoted as A × B, is defined as a vector that:

- Is perpendicular to both A and B.
- Has a magnitude equal to the area of the parallelogram formed by A and B.
- Its direction is determined by the right-hand rule.

Mathematically, if:

$$
\mathbf{A} = (A_x, A_y, A_z)
$$
$$
\mathbf{B} = (B_x, B_y, B_z)
$$

then the cross product A × B is given by:

$$
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x)
$$

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## Given Vectors A and B

In this scenario, the vectors are provided as:

- $\mathbf{A} = (6, 0, 2)$
- $\mathbf{B} = (0, 8, 0)$

Our goal is to compute the cross product A × B and verify its orthogonality to both A and B.

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# Step-by-Step Calculation of the Cross Product

## Step 1: Write down the components of A and B

$$
A_x = 6,\quad A_y = 0,\quad A_z = 2
$$
$$
B_x = 0,\quad B_y = 8,\quad B_z = 0
$$

## Step 2: Apply the cross product formula

Using the formula:

$$
\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y,\ A_z B_x - A_x B_z,\ A_x B_y - A_y B_x)
$$

Compute each component:

- x-component:

$$
A_y B_z - A_z B_y = (0)(0) - (2)(8) = 0 - 16 = -16
$$

- y-component:

$$
A_z B_x - A_x B_z = (2)(0) - (6)(0) = 0 - 0 = 0
$$

- z-component:

$$
A_x B_y - A_y B_x = (6)(8) - (0)(0) = 48 - 0 = 48
$$

## Step 3: Write the resulting cross product vector

$$
\boxed{
\mathbf{A} \times \mathbf{B} = (-16, 0, 48)
}
$$

This is the vector orthogonal to both A and B.

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## Verifying Orthogonality

To verify that A × B is orthogonal to both A and B, we need to check whether their dot products are zero:

- $\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = 0$
- $\mathbf{B} \cdot (\mathbf{A} \times \mathbf{B}) = 0$

If both are zero, the vectors are orthogonal.

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### Dot product of A and A × B

Calculate:

$$
\mathbf{A} \cdot (\mathbf{A} \times \mathbf{B}) = (6)(-16) + (0)(0) + (2)(48) = -96 + 0 + 96 = 0
$$

Result: The dot product is zero, confirming orthogonality to A.

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### Dot product of B and A × B

Calculate:

$$
\mathbf{B} \cdot (\mathbf{A} \times \mathbf{B}) = (0)(-16) + (8)(0) + (0)(48) = 0 + 0 + 0 = 0
$$

Result: The dot product is zero, confirming orthogonality to B.

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## Implications of the Cross Product and Orthogonality

The fact that the resulting vector (-16, 0, 48) is orthogonal to both A and B has several important implications:

- Perpendicularity: The cross product vector is perpendicular to the plane containing A and B.
- Direction: Using the right-hand rule, if you point your fingers in the direction of A and curl them toward B, your thumb points in the direction of A × B.
- Magnitude: The magnitude of the cross product vector gives the area of the parallelogram formed by A and B.

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## Calculating the Magnitude of the Cross Product

The magnitude $|\mathbf{A} \times \mathbf{B}|$ is given by:

$$
|\mathbf{A} \times \mathbf{B}| = \sqrt{(-16)^2 + 0^2 + 48^2} = \sqrt{256 + 0 + 2304} = \sqrt{2560}
$$

Simplify the square root:

$$
|\mathbf{A} \times \mathbf{B}| \approx \sqrt{2560} \approx 50.6
$$

This value corresponds to the area of the parallelogram with sides A and B.

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## Applications of Cross Product and Orthogonality in Real-World Scenarios

Understanding and computing the cross product are essential skills in various fields:

- Physics: Calculating torque, magnetic force, and angular momentum.
- Engineering: Analyzing forces on structures and mechanical components.
- Computer Graphics: Determining surface normals for rendering and shading.
- Robotics: Calculating orientations and rotations.

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## Summary and Key Takeaways

- The cross product of vectors $\mathbf{A} = (6, 0, 2)$ and $\mathbf{B} = (0, 8, 0)$ is $\mathbf{A} \times \mathbf{B} = (-16, 0, 48)$.
- The resulting vector is orthogonal to both A and B, confirmed by zero dot products.
- The magnitude of the cross product vector (~50.6) equals the area of the parallelogram formed by A and B.
- The cross product's direction follows the right-hand rule, providing a consistent way to determine orientation in space.
- Mastery of cross product calculations enhances understanding of spatial relationships and vector orthogonality in multidimensional contexts.

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## Conclusion

The process of finding the cross product of two vectors and verifying its orthogonality is fundamental in vector calculus. In the specific case of vectors $\mathbf{A} = (6, 0, 2)$ and $\mathbf{B} = (0, 8, 0)$, the cross product results in a vector that is clearly orthogonal to both, confirming the geometric and algebraic properties that underpin vector operations. This knowledge is vital for students, engineers, physicists, and computer scientists working with spatial data and vector analysis.

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Keywords: cross product, vector mathematics, orthogonal vectors, vector calculation, 3D vectors, physics, engineering, computer graphics, vector operations, spatial analysis

# Frequently Asked Questions

## What is the cross product of vectors A = (6, 0, 2) and B = (0, 8, 0)?

The cross product $A \times B$ is $( (0)(0) - (2)(8), (2)(0) - (6)(0), (6)(8) - (0)(0) ) = (-16, 0, 48)$.

## How do you verify that the cross product is orthogonal to vectors A and B?

By calculating the dot products $A \cdot (A \times B)$ and $B \cdot (A \times B)$ and confirming both are zero, indicating orthogonality.

## What is the value of the dot product between A and the cross product of A and B?

$A \cdot (A \times B) = (6)(-16) + (0)(0) + (2)(48) = -96 + 0 + 96 = 0$, confirming orthogonality.

## What is the value of the dot product between B and the cross product of A and B?

$B \cdot (A \times B) = (0)(-16) + (8)(0) + (0)(48) = 0 + 0 + 0 = 0$, confirming orthogonality.

## Why is it important that the cross product is orthogonal to both original vectors?

Because the cross product produces a vector perpendicular to both A and B, which is useful in physics and engineering for finding normals and torque directions.

## Can the cross product be zero for vectors A and B? Why or why not in this case?

The cross product is zero only if A and B are parallel or one is the zero vector. Since A and B are not parallel, the cross product is non-zero: $(-16, 0, 48)$.

## How do you compute the magnitude of the cross product vector?

The magnitude is √ ((-16)^2 + 0^2 + 48^2) = √ (256 + 0 + 2304) = √ 2560 ≈ 50.6.

## What practical applications utilize the cross product being orthogonal to original vectors?

Applications include calculating torque in physics, determining surface normals in computer graphics, and finding directions perpendicular to planes in engineering.

## Additional Resources

Cross Product Calculation and Orthogonality Verification: An In-Depth Guide

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## Introduction to the Cross Product

The cross product is a fundamental operation in vector algebra, particularly in three-dimensional space. It produces a vector that is perpendicular to the plane formed by two given vectors, with a magnitude that corresponds to the area of the parallelogram spanned by those vectors. This property makes the cross product invaluable across various disciplines, including physics, engineering, computer graphics, and more.

In this guide, we will explore the process of calculating the cross product of two specific vectors, A and B, and verify its orthogonality to both vectors. The vectors under consideration are:

- A = (6, 0, 2)
- B = (0, 8, 0)

Our objective is to find the cross product A × B and demonstrate that this resulting vector is orthogonal (perpendicular) to both A and B.

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## Understanding the Vectors

Before diving into calculations, it's important to understand the properties of the vectors involved:

- Vector A: (6, 0, 2)

- Components along the x, y, and z axes are 6, 0, and 2, respectively.
- This vector lies somewhere in the x-z plane, with no y-component.

- Vector B: (0, 8, 0)

- Components are 0, 8, and 0.
- This vector points purely in the y-direction.

Given this setup, the vectors are not parallel; hence, their cross product will be a non-zero vector orthogonal to both.

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## Calculating the Cross Product (A × B)

The cross product of two vectors $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ is calculated using the determinant of a matrix:

\[
A \times B = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & a_2 & a_3 \\

$$
b_1 & b_2 & b_3
\end{vmatrix}
$$

Where:

- $\mathbf{i}$: unit vector in the x-direction
- $\mathbf{j}$: unit vector in the y-direction
- $\mathbf{k}$: unit vector in the z-direction

Applying this to our vectors:

$$
A \times B = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
6 & 0 & 2 \\
0 & 8 & 0
\end{vmatrix}
$$

The calculation proceeds by expanding this determinant:

$$
A \times B = \mathbf{i} \cdot (a_2 \cdot b_3 - a_3 \cdot b_2) - \mathbf{j} \cdot (a_1 \cdot b_3 - a_3 \cdot b_1) + \mathbf{k} \cdot (a_1 \cdot b_2 - a_2 \cdot b_1)
$$

Calculating each component:

- x-component ($\mathbf{i}$):

$$
a_2 \cdot b_3 - a_3 \cdot b_2 = 0 \times 0 - 2 \times 8 = -16
$$

- y-component ($\mathbf{j}$):

$$
a_1 \cdot b_3 - a_3 \cdot b_1 = 6 \times 0 - 2 \times 0 = 0
$$

- z-component ($\mathbf{k}$):

$$
a_1 \cdot b_2 - a_2 \cdot b_1 = 6 \times 8 - 0 \times 0 = 48
$$

Putting it all together:

$$
A \times B = (-16)\mathbf{i} + (0)\mathbf{j} + (48)\mathbf{k}
$$

Expressed as a vector:

$$
\boxed{
A \times B = (-16, 0, 48)
}

\]

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## Verifying Orthogonality

A crucial property of the cross product is that the resulting vector is orthogonal to both original vectors. To verify this, we can compute the dot products:

$$
A \cdot (A \times B) \quad \text{and} \quad B \cdot (A \times B)
$$

If both dot products are zero, the vectors are orthogonal.

Dot Product of A and (A × B):

$$
A \cdot (A \times B) = (6, 0, 2) \cdot (-16, 0, 48) = (6)(-16) + (0)(0) + (2)(48) = -96 + 0 + 96 = 0
$$

Dot Product of B and (A × B):

$$
B \cdot (A \times B) = (0, 8, 0) \cdot (-16, 0, 48) = (0)(-16) + (8)(0) + (0)(48) = 0 + 0 + 0 = 0
$$

Both dot products are zero, confirming that $A \times B$ is orthogonal to both A and B.

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## Significance of the Result

The cross product vector (-16, 0, 48) possesses several important qualities:

- Orthogonality: As proven, it is perpendicular to both A and B. This is fundamental in applications where perpendicular vectors are required, such as calculating normals to surfaces in computer graphics or physics.

- Magnitude: The magnitude (or length) of $A \times B$ corresponds to the area of the parallelogram formed by A and B. Calculating this magnitude:

$$
|A \times B| = \sqrt{(-16)^2 + 0^2 + 48^2} = \sqrt{256 + 0 + 2304} = \sqrt{2560} \approx 50.60
$$

This value signifies that the area of the parallelogram spanned by A and B is approximately 50.60 square units.

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## Practical Applications of Cross Product and Orthogonality

Understanding cross products and their properties has broad implications:

- Physics: Calculating torque, magnetic forces, and angular momentum relies heavily on cross products.

- Engineering: Designing mechanical components often involves force vectors and moment calculations, where orthogonality simplifies analyses.

- Computer Graphics: Surface normals, essential for shading and rendering, are computed via cross products of tangent vectors.

- Navigation and Geospatial Analysis: Determining perpendicular directions and surface orientations utilizes the principles of the cross product.

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## Summary of Key Points

- The cross product A × B of vectors A = (6, 0, 2) and B = (0, 8, 0) is (-16, 0, 48).

- The calculation involves the determinant method, leveraging the unit vectors and components.

- The resulting vector is orthogonal to both A and B, confirmed via dot product calculations.

- The magnitude of the cross product vector indicates the area of the parallelogram formed by A and B.

- Such computations are central to many scientific and engineering applications, underscoring the importance of understanding the cross product's properties.

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## Conclusion

The process of finding the cross product and verifying its orthogonality provides a solid foundation for tackling complex problems involving vectors in three-dimensional space. The detailed calculation and verification demonstrate the elegance and utility of vector algebra, highlighting how foundational mathematical operations underpin advanced technological and scientific endeavors.

By mastering these concepts, professionals and students alike can enhance their analytical toolkit, enabling precise problem-solving across a multitude of fields.

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