Find The Derivatives. Please Do Not Simplify Your Answers. A. Y = Xe^4x B. F(t)= Ln(t1)/ T

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Understanding how to find derivatives is fundamental in calculus, especially when dealing with functions involving exponential and logarithmic expressions. Derivatives measure the rate at which a function changes at any given point, and mastering their calculation is essential for solving problems in physics, engineering, economics, and beyond. In this article, we will explore how to compute derivatives for two specific functions: one involving an exponential function multiplied by a variable and the other involving a logarithmic function with a quotient. We will approach each problem step-by-step, emphasizing the importance of applying the correct differentiation rules without simplifying the final answers prematurely.

Understanding the Derivative of Y = Xe^{4x}

The first function under consideration is:

Y = Xe^{4x}

This function combines two types of functions: a polynomial (the variable x) and an exponential function with a composite argument (e^{4x}). To find its derivative, we need to apply the product rule because Y is the product of two functions:

- u(x) = x
- v(x) = e^{4x}

The product rule states:

d/dx [u(x) v(x)] = u'(x) v(x) + u(x) v'(x)

Let's delve into the steps involved:

Step 1: Identify u(x) and v(x)

- u(x) = x

- $v(x) = e^{4x}$

Step 2: Compute $u'(x)$ and $v'(x)$

- $u'(x) = d/dx [x] = 1$
- $v'(x) = d/dx [e^{4x}]$

To find $v'(x)$, we use the chain rule because the exponent is a function of x:

$$d/dx [e^{g(x)}] = e^{g(x)} g'(x)$$

where $g(x) = 4x$, so $g'(x) = 4$.

Therefore,

$$v'(x) = e^{4x} 4 = 4e^{4x}$$

Step 3: Apply the product rule

Using the derivatives:

$$Y' = u'(x) v(x) + u(x) v'(x)$$

$$Y' = 1 e^{4x} + x 4e^{4x}$$

$$Y' = e^{4x} + 4x e^{4x}$$

This is the derivative of the function Y, expressed without simplifying further.

Summary of the derivative for $Y = Xe^{4x}$

$$Y' = e^{4x} + 4x e^{4x}$$

Understanding the Derivative of $F(t) = Ln(t1)/ T$

The second function is somewhat more complex:

$$F(t) = Ln(t1)/ T$$

At first glance, there's ambiguity in the notation. Assuming "Ln" refers to the natural logarithm, and "t1" is a variable or constant, the expression appears to be:

$F(t) = \ln(t1)$ divided by T

However, since the variable of differentiation is t, and "t1" and "T" may be constants or functions involving t, clarity is essential. Let's consider two possible interpretations:

Interpretation 1: Both t1 and T are constants.

Interpretation 2: T is a function of t, and t1 is a constant.

For the purpose of this article, we'll assume that:

- t1 is a constant
- T is a function of t, i.e., $T = T(t)$

Thus, the function becomes:

$F(t) = \ln(t1) / T(t)$

Since $\ln(t1)$ is the natural logarithm of a constant, it is itself a constant (say, $C = \ln(t1)$). Therefore,

$F(t) = C / T(t)$

The derivative of F(t) with respect to t, using the quotient rule or simply recognizing it's a constant divided by a function, is straightforward if T(t) is differentiable.

Calculating the Derivative of F(t) = C / T(t)

Given:

$F(t) = C / T(t)$

where $C = \ln(t1)$, a constant, and T(t) is differentiable.

Step 1: Recognize the structure

F(t) is a constant divided by a function T(t). The derivative can be obtained using the quotient rule or by rewriting as:

F(t) = C T(t)^{-1}

Applying the chain rule:

d/dt [T(t)^{-1}] = -1 T(t)^{-2} T'(t)

Therefore,

F'(t) = C d/dt [T(t)^{-1}] = C (-1) T(t)^{-2} T'(t) = -C T'(t) / T(t)^2

Step 2: Write the derivative explicitly

F'(t) = -ln(t1) T'(t) / T(t)^2

This derivative expresses the rate of change of F(t) in terms of T(t) and its derivative T'(t).

Summary of Derivatives Without Simplification

In this article, we've demonstrated how to find derivatives of two functions involving different types of mathematical expressions, emphasizing the importance of not simplifying answers prematurely. Here's a quick recap:

- For Y = xe^{4x}, applying the product rule and the chain rule yields:

Y' = e^{4x} + 4x e^{4x}

- For F(t) = ln(t1)/ T(t), assuming T(t) is differentiable, the derivative is:

F'(t) = -ln(t1) T'(t) / T(t)^2

Practical Tips for Finding Derivatives

When approaching derivatives of complex functions, consider the following tips:

1. **Identify the rules needed:** Recognize whether to apply the product rule, quotient rule, chain rule, or a combination.

2. **Keep answers in their unsimplified form:** This aids in understanding the structure of the derivative and can be useful for further calculations.

3. **Check the differentiability:** Ensure the functions involved are differentiable over the domain of interest.

4. **Use substitution wisely:** Replacing complicated parts with constants or simpler functions can clarify the differentiation process.

Conclusion

Mastering the art of differentiation involves understanding various rules and knowing how to combine them appropriately. Whether dealing with exponential functions like e^{4x} multiplied by x or with logarithmic functions divided by other functions of t, the key is to approach each problem systematically. Remember to identify the correct rules, apply them carefully, and present your answers without rushing to simplify. This approach not only enhances your understanding but also ensures accuracy in your calculations. Keep practicing with different functions to become proficient in finding derivatives and solving calculus problems confidently.

Frequently Asked Questions

What is the derivative of Y = x e^{4x} using the product rule?

The derivative is dY/dx = e^{4x} + x 4 e^{4x} = e^{4x} (1 + 4x)

How do you differentiate F(t) = □(t^1) / t?

First, rewrite F(t) as □(t) / t, then apply the quotient rule: dF/dt = [(□(t))' t - □(t) 1] / t^2

What is the derivative of Y = x e^{4x} with respect to x?

Using the product rule, dY/dx = e^{4x} + 4x e^{4x}

How do you differentiate the function F(t) = □(t) /

t?

Apply the quotient rule: dF/dt = □'(t) t - □(t) 1 / t^2

What is the derivative of Y = x e^{4x} in terms of x?

dY/dx = e^{4x} + 4x e^{4x}

How can you differentiate F(t) = □(t) / t without simplifying first?

Use the quotient rule directly: dF/dt = (□'(t) t - □(t)) / t^2

What is the derivative of the function Y = x e^{4x}?

dY/dx = e^{4x} + 4x e^{4x}

Can you differentiate F(t) = □(t) / t without simplifying, and if so, how?

Yes, by applying the quotient rule directly: dF/dt = (□'(t) t - □(t)) / t^2

Additional Resources

Find The Derivatives. Please Do Not Simplify Your Answers. A. Y = Xe^{4x} B. F(t)= \frac{\ln(t^1)}{T}

Introduction

In calculus, derivatives serve as the foundational tool for understanding how functions change at any given point. Whether analyzing the rate at which a quantity grows or diminishes, or examining the slope of a curve, derivatives unlock insights that are essential across mathematics, physics, engineering, and economics. This article delves into the process of finding derivatives for two specific functions, emphasizing the importance of precision and adherence to the rules of differentiation without simplifying the final answers. By exploring these functions step-by-step, we aim to provide a comprehensive understanding suitable for students and professionals who seek clarity and rigor in their calculus practice.

Understanding the Basics of Differentiation

Before tackling the specific functions, it’s crucial to revisit core differentiation rules that will be applied:

- Power Rule: For $f(x) = x^n$, $f'(x) = n x^{n-1}$
- Product Rule: For $f(x) = u(x) v(x)$, $f'(x) = u'(x) v(x) + u(x) v'(x)$
- Chain Rule: For a composite function $f(g(x))$, $f'(g(x)) = f'(g(x)) \cdot g'(x)$
- Logarithmic Differentiation: Used for functions involving logs, such as $f(x) = \ln u(x)$, where $f'(x) = \frac{u'(x)}{u(x)}$

Applying these rules carefully ensures accurate derivatives, especially for more complex functions like the ones discussed here.

Part A: Deriving $Y = x e^{4x}$

Understanding the Function

The first function, $Y = x e^{4x}$, is a product of two functions: a linear term x and an exponential function e^{4x}. This structure necessitates the use of the Product Rule, as it is a product of two differentiable functions.

Step-by-Step Differentiation

Applying the Product Rule:

$$
\frac{dY}{dx} = \frac{d}{dx} (x) \times e^{4x} + x \times \frac{d}{dx} (e^{4x})
$$

Breaking down each component:

1. Derivative of x:
$$
\frac{d}{dx} (x) = 1
$$

2. Derivative of e^{4x}:
Using the Chain Rule:
$$
\frac{d}{dx} (e^{4x}) = e^{4x} \times \frac{d}{dx} (4x) = e^{4x} \times 4 = 4 e^{4x}
$$

Putting it all together:

$$
\frac{dY}{dx} = 1 \times e^{4x} + x \times 4 e^{4x}

\]

Expressed explicitly:

\[
\boxed{
\frac{dY}{dx} = e^{4x} + 4x e^{4x}
}
\]

Note: As per instructions, the answer remains in this form; no further simplification like factoring is performed.

Significance of the Derivative

This derivative expresses how \(Y\) changes with respect to \(x\). It captures both the direct rate of change of the exponential component and the contribution from the linear multiplication, which is key in applications such as growth models where combined exponential and linear factors appear.

Part B: Deriving \(F(t) = \frac{\ln(t^1)}{T}\)

Understanding the Function

The second function is \(F(t) = \frac{\ln(t^1)}{T}\). Recognizing that \(t^1 = t\), the function simplifies to:

\[
F(t) = \frac{\ln t}{T}
\]

However, as per instructions, we will treat it in the original form for differentiation, considering \(T\) as a constant.

Differentiation Approach

Given that \(T\) is a constant, \(F(t)\) is essentially a scaled logarithmic function:

\[
F(t) = \frac{1}{T} \times \ln(t^1)
\]

Using properties of logarithms, we know:

\[
\ln(t^1) = 1 \times \ln t
\]

Thus, the function simplifies to:

$$
F(t) = \frac{1}{T} \times \ln t
$$

Step-by-Step Derivation

Applying the Constant Multiple Rule and the derivative of $\ln t$:

$$
\frac{dF}{dt} = \frac{1}{T} \times \frac{d}{dt} (\ln t)
$$

The derivative of $\ln t$ with respect to t is:

$$
\frac{d}{dt} (\ln t) = \frac{1}{t}
$$

Hence,

$$
\boxed{
\frac{dF}{dt} = \frac{1}{T} \times \frac{1}{t}
}
$$

Once again, this derivative is expressed without further simplification, aligning with the instruction to preserve the form.

Deep Dive into Differentiation Rules Applied

Product Rule in Part A

The product rule is fundamental when differentiating functions composed of two multiplicative parts. In the case of $Y = x e^{4x}$, it ensures the derivative accounts for changes in both the linear and exponential components simultaneously.

Key takeaways:

- Always identify the functions involved.
- Differentiate each component separately.
- Combine derivatives according to the product rule formula.

Chain Rule in Part A and B

The chain rule is essential when dealing with composite functions like

\(e^{4x}\) and the logarithm \(\ln t\). It involves differentiating the outer function and multiplying by the derivative of the inner function.

In practice:

- For \(e^{4x}\), differentiate \(e^u\) with \(u=4x\).
- For \(\ln t\), differentiate \(\ln u\) with \(u=t\).

Logarithmic Differentiation and Log Properties in Part B

Expressing \(\ln(t^1)\) emphasizes the core property: \(\ln u^n = n \ln u\). Even if \(n=1\), recognizing this property clarifies the function's structure.

Practical Implications and Applications

Understanding how to find derivatives of functions like these extends beyond academic exercises. Here’s how these derivatives might be employed:

- In physics: Determining the rate of change of a quantity involving exponential growth or decay, such as radioactive decay or population models.
- In engineering: Analyzing signals or systems where exponential functions describe responses, and their rates of change inform stability.
- In economics: Modeling growth rates of investments or markets where logarithmic functions express proportional changes.

Final Remarks

The process of differentiating functions with multiple components requires meticulous application of calculus rules. While the functions examined here are relatively straightforward, they exemplify key principles such as the product rule, chain rule, and properties of logarithms. Maintaining the integrity of the solutions without simplification emphasizes the importance of clarity and correctness in mathematical derivations.

By mastering these techniques, students and professionals can confidently analyze complex functions, derive meaningful insights from their rates of change, and apply these skills across diverse scientific and mathematical fields. Remember, precision in differentiation lays the groundwork for deeper understanding and effective problem-solving in calculus and beyond.

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Xe 4x B F T Ln T1 T

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