Find The Equation Of A Line Perpendicular To X=6+6y That Passes Through The Point (-8,6)

Find The Equation Of A Line Perpendicular To X=6+6y That Passes Through The Point (-8,6)

Understanding how to find the equation of a line that is perpendicular to a given line and passes through a specific point is a fundamental skill in coordinate geometry. In this guide, we will explore the process step-by-step, focusing on the line given by the equation $X = 6 + 6y$ and the point (-8, 6). By the end of this article, you'll be able to determine the equation of the perpendicular line with confidence.

Analyzing the Given Line: X = 6 + 6y

Before we find the perpendicular line, it is essential to understand the nature of the original line.

Converting the Equation to Slope-Intercept Form

The given line:

```plaintext
X = 6 + 6y
```

can be rearranged to express y in terms of x, which helps in identifying the slope.

Rearranged as:

```plaintext
X - 6 = 6y
```

or

```plaintext
6y = X - 6
```

Dividing both sides by 6:

```plaintext
y = (X - 6)/6
```

which simplifies to:

```plaintext
y = (1/6)X - 1
```

Note: It's common to see the slope-intercept form as $y = mx + b$. Here, the slope (m) is 1/6.

Understanding the Slope of the Given Line

- Since the line's equation in slope-intercept form is $y = (1/6)X - 1$, the slope is $m_1 = 1/6$.
- The slope indicates the steepness and direction of the line.

Finding the Slope of the Perpendicular Line

In coordinate geometry, two lines are perpendicular if the slopes are negative reciprocals of each other.

Negative Reciprocal Relationship

- If the slope of the original line is m_1, then the slope of the line perpendicular to it, m_2, is:

```plaintext
m_2 = -1 / m_1
```

Applying this to our current slope:

```plaintext
m_2 = -1 / (1/6) = -6
```

Therefore, the slope of the perpendicular line is -6.

Determining the Equation of the Perpendicular Line

Now that we know the slope, $m_2 = -6$, and a point through which the line passes, (-8, 6), we can find the equation.

Using the Point-Slope Form

The point-slope form of a line's equation is:

```plaintext
y - y₁ = m(x - x₁)
```

where:

- (x_1, y_1) is a point on the line,
- m is the slope.

Substituting:

```plaintext
x₁ = -8, y₁ = 6, m = -6
```

we get:

```plaintext
y - 6 = -6(x + 8)
```

Converting to Slope-Intercept Form

Expanding:

```plaintext
y - 6 = -6x - 48
```

Adding 6 to both sides:

```plaintext
```

y = -6x - 48 + 6
```

which simplifies to:

```plaintext
y = -6x - 42
```

The equation of the line perpendicular to X = 6 + 6y passing through (-8, 6) is:

```plaintext
y = -6x - 42
```

Verifying the Solution

Ensuring the correctness of the derived line involves checking:

- The point (-8, 6) satisfies the line equation.
- The slope is correctly calculated as the negative reciprocal.

Checking the Point

Substitute x = -8 into y = -6x - 42:

```plaintext
y = -6(-8) - 42 = 48 - 42 = 6
```

which matches the y-coordinate of the point. Thus, the point lies on the line.

Confirming the Slope

- The slope of the perpendicular line is -6.
- The original line's slope is 1/6.
- Their product: (1/6) (-6) = -1, confirming they are perpendicular.

```

Additional Insights and Practical Applications

Understanding how to find perpendicular lines has practical uses in various fields, such as engineering, architecture, and physics. For example:

- Designing structures with perpendicular supports.
- Analyzing trajectories in physics.
- Navigating in coordinate systems.

Knowing the process enhances problem-solving skills in these areas.

Summary of Steps to Find a Perpendicular Line

1. Identify the slope of the given line:
- Convert to slope-intercept form if necessary.
2. Calculate the slope of the perpendicular line:
- Use the negative reciprocal of the original slope.
3. Apply the point-slope form with the known point and perpendicular slope.
4. Simplify to slope-intercept form for clarity and standardization.
5. Verify by substitution and slope relationships.

Conclusion

Finding the equation of a line perpendicular to a given line and passing through a specific point is straightforward once you understand the relationships between slopes. In this case, starting with the line X = 6 + 6y, we identified its slope as 1/6. The perpendicular line then has a slope of -6. Using the point (-8, 6), we derived the equation y = -6x - 42, which passes through the point and is perpendicular to the original line.

Mastering these steps enhances your ability to solve a wide range of geometric problems efficiently. Practice with different lines and points to reinforce your understanding and become proficient in coordinate geometry.

Keywords: Equation of a line, perpendicular line, slope, coordinate geometry, point-slope form, slope-intercept form, negative reciprocal, line passing through a point, geometry problems, algebra.

Frequently Asked Questions

What is the slope of the line given by the equation x = 6 + 6y?

First, rewrite the equation in slope-intercept form. Starting with $x = 6 + 6y$, solve for y: $6y = x - 6$, so $y = (x - 6)/6$. The slope of this line is the coefficient of x, which is 1/6.

How do you find the slope of a line perpendicular to the given line?

The slope of a line perpendicular to another is the negative reciprocal of the original line's slope. Since the original slope is 1/6, the perpendicular slope is -6.

What is the general form of the equation for a line passing through a point with a given slope?

The point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope.

How do you find the specific equation of the perpendicular line passing through (-8, 6)?

Use the point-slope form with point (-8, 6) and slope -6: $y - 6 = -6(x + 8)$. Simplify to get the equation in slope-intercept form.

What is the simplified form of the equation of the line perpendicular to x=6+6y passing through (-8, 6)?

Starting from $y - 6 = -6(x + 8)$, expand: $y - 6 = -6x - 48$, then $y = -6x - 48 + 6$, so $y = -6x - 42$.

Why is the line x=6+6y considered a vertical or horizontal line?

Actually, $x=6+6y$ is neither purely vertical nor horizontal; it's a line with a slope of 1/6 when rewritten in y=form. It’s an oblique line, not vertical or horizontal.

Can the line x=6+6y be classified as a standard

form? How?

Yes. By rearranging, x - 6 = 6y, or 6y = x - 6, which can be written as 6y - x + 6 = 0, a standard form ax + by + c = 0.

What is the importance of understanding the slope when finding perpendicular lines?

Understanding the slope is crucial because perpendicular lines have slopes that are negative reciprocals. This relationship helps in quickly determining the equation of the perpendicular line.

How can I verify that the line y = -6x - 42 passes through the point (-8, 6)?

Substitute x = -8 into the equation: y = -6(-8) - 42 = 48 - 42 = 6. Since y = 6 matches the point's y-coordinate, the line passes through (-8, 6).

Additional Resources

Find The Equation Of A Line Perpendicular To X=6+6y That Passes Through The Point (-8,6)

When approaching the problem of finding a line perpendicular to a given line and passing through a specific point, it is essential to understand the fundamental principles of line equations, slopes, and perpendicularity in coordinate geometry. In this case, the problem involves the equation X = 6 + 6y, and the goal is to determine the equation of a line perpendicular to it passing through the point (-8, 6). This task combines algebraic manipulation, understanding of slopes, and the application of perpendicularity conditions, making it a comprehensive exercise in analytic geometry.

Understanding the Given Equation: X = 6 + 6y

Rearranging the Equation into Slope-Intercept or Standard Form

The initial step involves interpreting the provided line equation:

\[X = 6 + 6y \]

This equation is somewhat unconventional because it expresses x in terms of y rather than the more common y in terms of x. To analyze the slope and the orientation of this line, it's preferable to rewrite it in a more familiar form.

Rearranged:

$$ X - 6 = 6y $$

or

$$ y = \frac{X - 6}{6} $$

which simplifies to:

$$ y = \frac{1}{6}X - 1 $$

This form indicates that the line is expressed with x as the independent variable and y as the dependent variable, with a slope of 1/6.

Key features:

- The slope of the given line is 1/6.
- The line passes through all points satisfying the equation $y = \frac{1}{6}x - 1$.

Determining the Slope of a Perpendicular Line

The Concept of Perpendicular Slopes

In coordinate geometry, the slopes of perpendicular lines are negative reciprocals of each other. If a line has slope m, then a line perpendicular to it has slope m_{perp}, where:

$$ m_{perp} = -\frac{1}{m} $$

Applying this to the given line:

$$ m = \frac{1}{6} $$

the slope of the perpendicular line will be:

$$ m_{perp} = -\frac{1}{\frac{1}{6}} = -6 $$

This is a critical step because it directly determines the slope of the line we are trying to find.

Features:

- The slope of the perpendicular line is -6.
- This relationship holds true in Euclidean geometry for lines intersecting at right angles.

Pros and Cons:

- Pros: Using the negative reciprocal rule simplifies calculations and provides a quick way to find perpendicular lines.
- Cons: It assumes the line is not vertical or horizontal; in cases where the original line is vertical or horizontal, the approach differs slightly.

Finding the Equation of the Perpendicular Line Passing Through (-8, 6)

Using Point-Slope Form

The point-slope form of a line's equation is:

$$ y - y_1 = m(x - x_1) $$

where (x_1, y_1) is a point on the line, and m is the slope.

Given:

- Point: $(-8, 6)$
- Slope: -6

Plugging into the point-slope form:

$$ y - 6 = -6(x + 8) $$

Expanding:

$$ y - 6 = -6x - 48 $$

Adding 6 to both sides:

$$ y = -6x - 48 + 6 $$

$$ y = -6x - 42 $$

This is the slope-intercept form of the perpendicular line passing through the point $(-8, 6)$.

Features:

- The line has a slope of -6.
- It passes through the specified point.
- The y-intercept is -42, meaning the line crosses the y-axis at -42.

Pros and Cons:

- Pros: Straightforward and easy to interpret.
- Cons: Assumes the use of the point-slope formula; other forms could be used, such as the standard form.

Alternative Forms of the Equation

Standard Form

The line's equation in standard form:

$$ y = -6x - 42 $$

can be rewritten as:

$$ 6x + y = -42 $$

This form is particularly useful in certain applications, such as when analyzing the intersection of lines or applying linear systems.

Features:

- Clear coefficients for x and y.
- Useful in systems of equations.

Verifying the Perpendicularity

To confirm that the derived line is indeed perpendicular to the original, we can check the slopes:

- Original line slope: $ \frac{1}{6} $
- Perpendicular line slope: $ -6 $

Their product:

$$ \frac{1}{6} \times -6 = -1 $$

which confirms perpendicularity because the product of slopes of perpendicular lines in Euclidean geometry is -1.

Summary of the Solution

- The original line X = 6 + 6y was rewritten as $y = \frac{1}{6}x - 1$, revealing its slope as 1/6.
- The slope of the perpendicular line is -6.
- Using the point $(-8, 6)$, the line's equation in point-slope form was derived as:

$$ y - 6 = -6(x + 8) $$

which simplifies to:

$$ y = -6x - 42 $$

- The line equation in standard form is:

$$ 6x + y = -42 $$

- Confirmed perpendicularity via slope product.

Additional Considerations and Tips

Handling Vertical and Horizontal Lines

- If the original line had been vertical (e.g., $x = c$), then the perpendicular line would be horizontal ($y = k$).
- Conversely, if the original line had been horizontal ($y = c$), the perpendicular line would be vertical ($x = c$).
- Special cases require careful treatment because the slope of a vertical line is undefined, and the negative reciprocal concept doesn't directly apply.

Using Graphical Methods

- Visualizing the lines on a graph can help verify the perpendicularity.
- Plotting the original line and the perpendicular line passing through the given point can confirm the accuracy.

Practice Problems for Better Understanding

- Find the equation of a line perpendicular to $y = 2x + 3$ passing through (4, -1).
- Determine the perpendicular bisector of a segment with endpoints (1, 2) and (3, 4).

Conclusion

Understanding how to find the equation of a line perpendicular to a given line passing through a specific point is a fundamental skill in analytic geometry. This problem illustrates the importance of converting equations into familiar forms, calculating slopes accurately, and applying the negative reciprocal rule. The step-by-step approach — from rewriting the original line, determining the perpendicular slope, and applying the point-slope formula — ensures clarity and correctness. Mastering these techniques enhances problem-solving skills in coordinate geometry and provides a solid foundation for tackling more complex geometric problems.

In summary:

- The given line X = 6 + 6y is rearranged to find its slope.
- The perpendicular line's slope is the negative reciprocal.
- The line passing through $(-8, 6)$ with this slope is derived in both slope-intercept and standard forms.
- The method can be adapted to handle special cases like vertical or horizontal lines.
- Verifying perpendicularity via the product of slopes confirms the solution.

By mastering these concepts, students and enthusiasts can confidently approach similar problems, enhancing their understanding of the elegant relationships in coordinate geometry.

Find The Equation Of A Line Perpendicular To X 6 6y That Passes Through The Point 8 6

Find The Equation Of A Line Perpendicular To X 6 6y That Passes Through The Point 8 6

Related Articles

- TRUE / FALSE. Fiber Optics Cable Is More Expensive And Harder To Install Thatn Wire Media
- The Most Significant Event Of The Roman Period Was The Birth,life,and Death Of Jesus Christi
- A Shortcut Or Rule Of Thumb Or Guduing Principle In Solving Prbolems Or Making Descions Is?

Back to Home