Find The Equation Of The Circle, If (4, -2)and (2, 1) Are The Extremities Of Thediameter.

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Understanding how to find the equation of a circle given the endpoints of its diameter is a fundamental concept in coordinate geometry. This problem involves applying the properties of circles, midpoints, and the distance formula to derive the standard form of the circle's equation. Whether you're a student preparing for exams, a teacher creating lesson plans, or a math enthusiast exploring geometry, mastering this process is essential. In this comprehensive guide, we will walk through the steps to find the equation of the circle when provided with the endpoints of its diameter, specifically the points (4, -2) and (2, 1).

Introduction to the Problem

When given two points that are endpoints of a diameter of a circle, the goal is to determine the equation of that circle. The key properties utilized in this problem include:

Key Concepts

1. **Midpoint of the diameter:** The center of the circle is the midpoint of the diameter.
2. **Radius of the circle:** The radius is half the length of the diameter, or the distance from the center to any of the endpoints.
3. **Equation of a circle:** The standard form is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Step-by-Step Solution to Find the Equation of the Circle

This section presents a detailed process to derive the circle's equation from the given points.

Step 1: Find the Midpoint of the Diameter

The midpoint of the diameter will serve as the center of the circle.

Formula for Midpoint:

$$
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
$$

Applying the formula:
Given points $(x_1, y_1) = (4, -2)$ and $(x_2, y_2) = (2, 1)$:

$$
h = \frac{4 + 2}{2} = \frac{6}{2} = 3
$$
$$
k = \frac{-2 + 1}{2} = \frac{-1}{2} = -0.5
$$

Result:
$$
\text{Center, } (h, k) = (3, -0.5)
$$

Step 2: Calculate the Radius

The radius is the distance from the center to one of the endpoints.

Distance formula:
$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Using point (4, -2):
$$
r = \sqrt{(4 - 3)^2 + (-2 + 0.5)^2} = \sqrt{1^2 + (-1.5)^2} = \sqrt{1 + 2.25} = \sqrt{3.25}
$$

Simplified radius:
$$
r = \sqrt{3.25} \approx 1.802
$$

Alternatively, using the other endpoint (2, 1):
$$
r = \sqrt{(2 - 3)^2 + (1 + 0.5)^2} = \sqrt{(-1)^2 + (1.5)^2} = \sqrt{1 + 2.25} = \sqrt{3.25}
$$

Both calculations are consistent, confirming the radius.

Step 3: Write the Equation of the Circle

Using the standard form and the calculated center and radius:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting $h=3$, $k=-0.5$, and $r^2=3.25$:

$$
(x - 3)^2 + \left(y + 0.5 \right)^2 = 3.25
$$

This is the equation of the circle passing through points (4, -2) and (2, 1).

Understanding the Significance of the Solution

Determining the equation of a circle from the diameter endpoints is a common problem in coordinate geometry with various applications:

Applications of the Equation of a Circle

- Designing circular objects in engineering and architecture.
- Analyzing trajectories in physics involving circular motion.
- Graphing and plotting in computer graphics.
- Solving geometric problems involving circles and other shapes.

Why Midpoint and Distance Formulas Matter

Understanding these formulas is crucial because they allow you to:

- Find the center of a circle when given endpoints of the diameter.
- Calculate the radius once the center is known.
- Derive the circle's equation in a straightforward manner.

Additional Tips and Tricks for Finding the Equation of a Circle

To enhance your understanding and efficiency, consider these tips:

Key Points to Remember:

- The midpoint formula is essential for locating the circle's center.
- The distance formula helps determine the radius accurately.
- Always verify your calculations by checking the radius with both endpoints.
- Express the equation in standard form for clarity and consistency.

Common Mistakes to Avoid:

1. Miscalculating the midpoint or radius due to arithmetic errors.
2. Confusing the signs when substituting into the standard form.
3. Using the wrong points; always verify which points are endpoints of the diameter.
4. Forgetting to square the radius when writing the equation.

Summary and Final Remarks

In summary, finding the equation of a circle given the endpoints of its diameter involves three main steps:

1. Calculating the midpoint to determine the circle's center.
2. Using the distance formula to find the radius.
3. Substituting these values into the standard circle equation.

Applying these steps to the points (4, -2) and (2, 1), we derived the circle's equation as:

$$
\boxed{
(x - 3)^2 + \left(y + 0.5 \right)^2 = 3.25
}
$$

This equation precisely describes the circle with the specified diameter endpoints, providing a foundation for further geometric analysis or graphing.

FAQs About Finding the Equation of a Circle

Q1: Can I find the circle's equation if I only know the endpoints of the diameter?
Yes. The endpoints allow you to determine the center (midpoint) and radius (via the distance formula), which together give the complete equation.

Q2: How do I verify my solution?
Substitute the endpoints into the circle's equation. If they satisfy the equation, your solution is correct.

Q3: What if the points are not endpoints of a diameter?
You would need additional information, such as the circle's radius or another point on the circle, to find its equation.

In conclusion, mastering the process of deriving the equation of a circle from diameter endpoints is a key skill in geometry. It combines fundamental concepts like midpoints and distances with algebraic manipulation, culminating in the standard circle equation. With practice, you'll be able to tackle similar problems confidently and accurately, enriching your understanding of coordinate geometry and its practical applications.

Frequently Asked Questions

How do you find the equation of a circle given the endpoints of its diameter?

To find the equation, first determine the center by calculating the midpoint of the diameter endpoints, then find the radius as half the length of the diameter. Use the standard form (x - h)^2 + (y - k)^2 = r^2 with the center (h, k) and radius r.

What are the steps to find the center of the circle when given two points on its diameter?

Calculate the midpoint of the two points using midpoint formula: ((x1 + x2)/2, (y1 + y2)/2). This midpoint is the center of the circle.

How do you calculate the radius of the circle from the given diameter points?

Find the distance between the two points using the distance formula: ⍰ [(x2 - x1)^2 + (y2 - y1)^2], then divide this distance by 2 to get the radius.

Using the points (4, -2) and (2, 1), what is the center of the circle?

The center is at the midpoint: ((4 + 2)/2, (-2 + 1)/2) = (3, -0.5).

What is the length of the diameter given the points (4, -2) and (2, 1)?

The length of the diameter is ⍰ [(4 - 2)^2 + (-2 - 1)^2] = ⍰ [2^2 + (-3)^2] = ⍰ [4 + 9] = ⍰ 13.

What is the equation of the circle with diameter endpoints (4, -2) and (2, 1)?

First, find the center (3, -0.5) and radius r = ⍰ 13 / 2. The equation is (x - 3)^2 + (y + 0.5)^2 = (⍰ 13 / 2)^2 = 13/4.

How do you write the standard form of the circle's equation with the given points?

Substitute the center (3, -0.5) and radius (⍰ 13 / 2) into (x - h)^2 + (y - k)^2 = r^2, resulting in (x - 3)^2 + (y + 0.5)^2 = 13/4.

Additional Resources

Find the Equation of the Circle, If (4, -2) and (2, 1) Are the Extremities of the Diameter

Understanding the fundamental properties of circles is essential in geometry, and one of the most intriguing problems involves determining the equation of a circle given specific points that define its diameter. In this case, the extremities of the diameter are given as (4, -2) and (2, 1). This problem not only tests your knowledge of coordinate geometry but also offers an opportunity to explore the relationships between diameters, centers, radii, and the general equation of a circle. Let us delve into the detailed process of finding the equation of this circle, examining each step with clarity and analytical rigor.

Understanding the Core Concepts

Before tackling the problem, it is crucial to revisit some foundational principles related to circles in coordinate geometry.

The Equation of a Circle

The standard form of the equation of a circle with center (h, k) and radius r is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Here:
- (h, k) is the coordinate of the circle's center.
- r is the radius, the distance from the center to any point on the circle.

Properties of a Diameter

- A diameter is a straight line passing through the center of the circle and touching two points on its circumference.
- The endpoints of the diameter are the extremities, which means they are at a maximum distance from each other.
- The center of the circle is located at the midpoint of the diameter.

Step 1: Identifying the Key Points and Their Significance

Given:
- Extremity 1: $A (4, -2)$
- Extremity 2: $B (2, 1)$

These two points are the endpoints of the diameter of the circle. The goal is to find:
- The center (h, k) of the circle.
- The radius r.
- The complete equation of the circle.

Step 2: Calculating the Center of the Circle

The center (h, k) of the circle lies at the midpoint of the diameter segment connecting points A and B.

Midpoint Formula

The midpoint M of a segment connecting points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$
M = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)
$$

Applying this to our points:

$$
h = \frac{4 + 2}{2} = \frac{6}{2} = 3
$$

$$
k = \frac{-2 + 1}{2} = \frac{-1}{2} = -0.5
$$

Therefore, the center of the circle is at $(3, -0.5)$.

Step 3: Calculating the Radius of the Circle

The radius r is the distance from the center (h, k) to either endpoint of the diameter. Since both endpoints are on the circle, calculating the distance from the center to one endpoint suffices.

Distance Formula

The distance d between two points (x_1, y_1) and (x_2, y_2) is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Using point $A (4, -2)$:

$$
r = \sqrt{(4 - 3)^2 + (-2 - (-0.5))^2}
$$

Calculate each component:

$$
(4 - 3)^2 = 1^2 = 1
$$

\[

$$
(-2 - (-0.5)) = -2 + 0.5 = -1.5
$$

$$
(-1.5)^2 = 2.25
$$

Adding these:

$$
r = \sqrt{1 + 2.25} = \sqrt{3.25}
$$

Expressed as a decimal:

$$
r \approx 1.803
$$

For exactness, we often leave it as $\sqrt{13/4}$, since $3.25 = \frac{13}{4}$. So,

$$
r = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}
$$

Step 4: Writing the Equation of the Circle

Now that we have the center $(3, -0.5)$ and radius $r = \frac{\sqrt{13}}{2}$, plug these into the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the values:

$$
(x - 3)^2 + \left(y + 0.5 \right)^2 = \left(\frac{\sqrt{13}}{2} \right)^2
$$

Simplify the right side:

$$
\left(\frac{\sqrt{13}}{2} \right)^2 = \frac{13}{4}
$$

Final Equation:

$$
(x - 3)^2 + \left(y + \frac{1}{2} \right)^2 = \frac{13}{4}
$$

For clarity and to avoid fractions inside the parentheses, multiply through by 4:

\[

$$
4(x - 3)^2 + 4\left(y + \frac{1}{2} \right)^2 = 13
$$

Expanding:

$$
4(x^2 - 6x + 9) + 4(y^2 + y + 0.25) = 13
$$

$$
4x^2 - 24x + 36 + 4y^2 + 4y + 1 = 13
$$

Combine like terms:

$$
4x^2 + 4y^2 - 24x + 4y + 37 = 13
$$

Bring all to one side:

$$
4x^2 + 4y^2 - 24x + 4y + 24 = 0
$$

This is the general form of the circle's equation:

$$
\boxed{
4x^2 + 4y^2 - 24x + 4y + 24 = 0
}
$$

Additional Insights and Verification

To ensure the correctness of the derived equation, consider the following:

- Verification of the center: The midpoint of the diameter points matches the center coordinates derived.
- Verification of the radius: The distance from the center to either diameter endpoint perfectly matches the computed radius.
- Equation consistency: The expanded form is symmetric and aligns with the standard form when simplified.

Practical Applications and Significance

Understanding how to derive the equation of a circle from diameter endpoints is foundational in coordinate geometry, with practical applications in various fields:

- Engineering: Designing circular components and ensuring precise dimensions.
- Computer Graphics: Rendering circles and curves accurately based on coordinate data.
- Navigation and GPS: Calculating coverage areas or zones based on known extremities.

- Physics: Modeling circular motion or fields with specific boundary points.

Furthermore, the problem emphasizes the importance of coordinate midpoints, distances, and algebraic manipulation, skills vital for students and professionals working with geometric data.

Conclusion

Determining the equation of a circle given the extremities of its diameter involves a systematic approach grounded in core geometric principles. By calculating the midpoint of the diameter to find the center and then using the distance formula to find the radius, one can accurately formulate the circle's equation. The process underscores the interconnectedness of various geometric concepts and highlights the elegance of coordinate geometry in solving real-world problems.

This problem not only reinforces theoretical understanding but also demonstrates practical problem-solving skills, essential for students, educators, and professionals alike. Whether for academic pursuits or practical applications, mastering these techniques enhances one's ability to interpret and manipulate geometric data effectively.

In summary:

- The center of the circle is at $(3, -0.5)$.
- The radius of the circle is $\frac{\sqrt{13}}{2}$.
- The equation of the circle in expanded form is:

$$
4x^2 + 4y^2 - 24x + 4y + 24 = 0
$$

This comprehensive analysis not only provides the final answer but also deepens the understanding of the geometric principles involved, ensuring a robust grasp of the topic.

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