Find The Equation Of The Tangent To The Circle X^+y^=25 At Thepoint P(4,-3) On The Circle

Find The Equation Of The Tangent To The Circle X^2 + y^2 = 25 At The Point P(4, -3) On The Circle

Understanding how to find the tangent line to a circle at a given point is a fundamental concept in coordinate geometry. It combines knowledge of the circle's equation, the point of contact, and the principles of differentiation or geometric properties. In this article, we will explore the step-by-step process to determine the equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $P(4, -3)$, which lies on the circle. This example will also serve as a comprehensive guide for solving similar problems involving circles and tangent lines.

Understanding the Equation of a Circle

Before diving into the solution, it is crucial to understand the general form of a circle's equation and the significance of its parameters.

The Standard Equation of a Circle

The standard form of a circle's equation is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:
- (h, k) is the center of the circle,
- r is the radius.

In our case, the circle's equation is:

$$
x^2 + y^2 = 25
$$

which indicates:
- Center at $(0, 0)$,
- Radius $r = 5$.

Verifying the Point $P(4, -3)$ Lies on the Circle

Before finding the tangent, verify that the point $P(4, -3)$ indeed lies on the circle.

Calculate:

$$
x^2 + y^2 = 4^2 + (-3)^2 = 16 + 9 = 25
$$

Since this equals the circle's radius squared, the point $P(4, -3)$ is on the circle.

Methods to Find the Equation of the Tangent Line

There are primarily two methods to find the tangent line at a point on a circle:

1. Geometric Method (Using the Perpendicular Radius)

- The radius drawn from the circle's center to the point of tangency is perpendicular to the tangent line.
- The slope of the radius OP is used to find the slope of the tangent, which is the negative reciprocal.

2. Calculus Method (Using Derivatives)

- Differentiate the circle's equation implicitly to find the slope of the tangent line at the point.
- Use the point-slope form to write the equation of the tangent.

Both methods are valid. In this article, we will explore both for completeness.

Method 1: Geometric Approach

This approach is straightforward when the circle's center and the point are known.

Step 1: Find the slope of the radius OP

Given:
- Center $O(0, 0)$,
- Point $P(4, -3)$.

Calculate the slope of the radius OP:

$$
m_{OP} = \frac{y_P - y_O}{x_P - x_O} = \frac{-3 - 0}{4 - 0} = \frac{-3}{4}
$$

Step 2: Find the slope of the tangent line $m_{tangent}$

Since the radius is perpendicular to the tangent,

$$
m_{tangent} = - \frac{1}{m_{OP}} = - \frac{1}{-\frac{3}{4}} = \frac{4}{3}
$$

Step 3: Write the equation of the tangent line

Using point-slope form:

$$
y - y_P = m_{tangent} (x - x_P)
$$

Plugging in values:

$$
y - (-3) = \frac{4}{3} (x - 4)
$$

Simplify:

$$
y + 3 = \frac{4}{3} (x - 4)
$$

Multiply both sides by 3 to clear denominator:

$$
3(y + 3) = 4(x - 4)
$$

Expand:

$$
3y + 9 = 4x - 16
$$

Rearranged as:

$$
4x - 3y - 25 = 0
$$

Final Equation of the Tangent:

$$
\boxed{4x - 3y = 25}
$$

This is the equation of the tangent line at point $P(4, -3)$.

Method 2: Calculus (Implicit Differentiation)

This method involves differentiating the circle's equation to find the slope of the tangent at the given point.

Step 1: Implicit differentiation of the circle's equation

Given:

$$
x^2 + y^2 = 25
$$

Differentiate both sides with respect to x:

$$
2x + 2y \frac{dy}{dx} = 0
$$

Simplify:

$$
x + y \frac{dy}{dx} = 0
$$

Solve for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = - \frac{x}{y}
$$

Step 2: Find the slope at $P(4, -3)$

Substitute $x=4$, $y=-3$:

$$
\frac{dy}{dx} = - \frac{4}{-3} = \frac{4}{3}
$$

The slope of the tangent line at P is $\frac{4}{3}$, matching the previous method.

Step 3: Write the tangent line's equation

Using point-slope form:

$$
y - y_P = m (x - x_P)
$$

$$
y - (-3) = \frac{4}{3} (x - 4)
$$

$$
y + 3 = \frac{4}{3} (x - 4)
$$

Multiply through by 3:

$$
3(y + 3) = 4(x - 4)
$$

$$
3y + 9 = 4x - 16
$$

Rearranged:

$$
4x - 3y = 25
$$

Final Equation of the Tangent:

$$
\boxed{4x - 3y = 25}
$$

Both methods lead to the same equation, confirming its accuracy.

Additional Tips and Common Mistakes

Tips for Solving Similar Problems

- Always verify that the point lies on the circle before proceeding.
- Remember that the radius to the point of tangency is perpendicular to the tangent.
- When using the calculus method, implicit differentiation simplifies the process, especially for more complex circles.
- Convert the final equation into the desired form (standard, slope-intercept, or general) as needed.

Common Mistakes to Avoid

- Confusing the slope of the radius with the slope of the tangent (they are perpendicular, hence negative reciprocals).
- Forgetting to verify the point's position on the circle.
- Errors in arithmetic during algebraic manipulations.
- Not simplifying the tangent line equation fully.

Summary

Finding the equation of the tangent to a circle at a given point involves understanding the geometric relationship between the radius and the tangent, or applying calculus techniques. In our example, with the circle $x^2 + y^2 = 25$ and point $P(4, -3)$, the tangent line is:

$$
\boxed{4x - 3y = 25}
$$

This line touches the circle exactly at the point P and is perpendicular to the radius OP. Both geometric and calculus methods yield the same result, providing a solid understanding of the problem-solving process in coordinate geometry.

Conclusion

Mastering how to find the tangent line to a circle at a specific point is essential for students and professionals working with geometric figures. Whether approached through geometric principles or calculus, the key steps involve verifying the point on the circle, determining the slope of the radius or using implicit differentiation, and then applying the point-slope form of a line. With practice, solving such problems becomes intuitive, enhancing your overall understanding of circle properties and tangent lines.

Remember: Practice with various circle equations and points to build confidence and develop a strong grasp of tangent line concepts in coordinate geometry.

Frequently Asked Questions

How do you find the equation of the tangent to the circle $x^2 + y^2 = 25$ at a given point P(4, -3)?

To find the tangent line at P(4, -3), first verify that P lies on the circle by plugging into the equation. Then, use the fact that the radius vector from the circle's center to P is perpendicular to the tangent. The slope of the radius is $(-3)/(4)$. The tangent's slope is the negative reciprocal, which is $4/3$. Using point-slope form: $y + 3 = (4/3)(x - 4)$. Simplify to get the tangent's equation.

What is the general method to find the tangent line to a circle at a specific point?

The general method involves calculating the slope of the radius at the point, then finding the negative reciprocal of that slope to get the tangent's slope. Using the point and this slope, apply the point-slope form of a line to derive the tangent's equation.

Why is the tangent to a circle perpendicular to the radius at the point of contact?

Because the radius of a circle always points from the center to the point on the circle, and the tangent line touches the circle at that point without crossing the radius, the tangent is perpendicular to the radius at that point.

How do you verify that the point P(4, -3) lies on the circle $x^2 + y^2 = 25$?

Substitute $x=4$ and $y=-3$ into the circle's equation: $4^2 + (-3)^2 = 16 + 9 = 25$. Since the sum equals 25, P is on the circle.

What is the equation of the circle centered at the origin with radius 5?

The equation is $x^2 + y^2 = 25$.

How do you find the slope of the radius at point P(4, -3) for the circle $x^2 + y^2 = 25$?

The radius vector from the center (0,0) to point P is (4, -3), so its slope is (−3)/4.

What is the equation of the tangent line to the circle at P(4, -3)?

Using the slope of the radius as −3/4, the tangent's slope is 4/3. Applying point-slope form: $y + 3 = (4/3)(x - 4)$. Simplifying gives the tangent line equation: $3(y + 3) = 4(x - 4)$.

Can the tangent line at P(4, -3) be written in slope-intercept form? If so, what is it?

Yes. Starting from $3(y + 3) = 4(x - 4)$: $3y + 9 = 4x - 16$, so $3y = 4x - 25$, and $y = (4/3)x - 25/3$.

What is the significance of the point P(4, -3) in the context of the circle $x^2 + y^2 = 25$?

Point P(4, -3) lies on the circle, serving as the point of tangency where the tangent line touches the circle.

Additional Resources

Circle Geometry and Tangent Line Equations: An Expert Breakdown of Finding the Tangent at a Specific Point

---

Introduction

In the fascinating world of coordinate geometry, the ability to determine the equation of a tangent to a circle at a given point is a fundamental skill. It combines understanding of the circle's equation, the concept of derivatives (or geometric properties), and algebraic manipulation. This article provides an in-depth, comprehensive analysis of how to find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $P(4, -3)$. Whether you're a student preparing for exams or an enthusiast eager to deepen your understanding, this step-by-step guide will clarify each concept and method involved.

Understanding the Problem

Before diving into calculations, let's clarify what is being asked:

- Given: A circle with the equation $x^2 + y^2 = 25$, which is a circle centered at the origin with radius 5.
- Point: $P(4, -3)$ lies on this circle.
- Objective: Find the equation of the tangent line to the circle at point P.

The Significance of the Circle Equation

The circle's standard form:

$$
x^2 + y^2 = r^2
$$

Here, $r = 5$, indicating a circle centered at the origin $(0, 0)$ with radius 5.

Key properties:

- Any point (x, y) on the circle satisfies the equation.
- The tangent line at a point P on the circle is perpendicular to the radius OP, where O is the circle's center.

Confirming that $P(4, -3)$ Lies on the Circle

Before proceeding, verify that P is indeed on the circle:

$$
x^2 + y^2 = 4^2 + (-3)^2 = 16 + 9 = 25
$$

Since the sum equals 25, P lies on the circle, confirming the point of tangency.

Methods to Find the Equation of the Tangent

There are multiple approaches to find the tangent line:

1. Geometric Method (using the perpendicularity of radius and tangent).
2. Using the point-slope form with the slope derived from the radius.
3. Implicit differentiation to find the slope of the tangent at the point.

The most elegant method here is implicit differentiation, which leverages calculus to find the slope at the point, then uses point-slope form to write the tangent's equation.

Method 1: Geometric Approach - The Perpendicular Radius

Step 1: Find the slope of the radius OP.

Since the center is at $O(0, 0)$, the slope of the radius OP connecting $(0,0)$ to $(4, -3)$:

$$
m_{OP} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{4 - 0} = \frac{-3}{4}
$$

Step 2: Find the slope of the tangent line.

The tangent line at P is perpendicular to OP, so:

$$
m_{tangent} = - \frac{1}{m_{OP}} = - \frac{1}{-\frac{3}{4}} = \frac{4}{3}
$$

Step 3: Write the tangent line equation using point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Plugging in $P(4, -3)$ and $m = \frac{4}{3}$:

$$
y + 3 = \frac{4}{3}(x - 4)
$$

Multiply both sides by 3 to clear denominator:

$$
3(y + 3) = 4(x - 4)
$$

$$
3y + 9 = 4x - 16
$$

Rearranged:

$$
4x - 3y = 25
$$

Thus, the equation of the tangent line is:

$$
\boxed{4x - 3y = 25}
$$

This method provides a straightforward geometric solution.

Method 2: Implicit Differentiation

This calculus-based approach is powerful and generalizes to more complex curves.

Step 1: Differentiate the circle equation $x^2 + y^2 = 25$ with respect to x:

$$
\frac{d}{dx} (x^2 + y^2) = \frac{d}{dx}(25)
$$

$$
2x + 2y \frac{dy}{dx} = 0
$$

Step 2: Solve for $\frac{dy}{dx}$:

$$
2y \frac{dy}{dx} = -2x
$$

\[

$$
\frac{dy}{dx} = - \frac{x}{y}
$$

Step 3: Find the slope at $P(4, -3)$:

$$
\left. \frac{dy}{dx} \right|_{(4, -3)} = - \frac{4}{-3} = \frac{4}{3}
$$

The slope of the tangent line at P matches the previous result.

Step 4: Write the tangent line's equation using point-slope form:

$$
y - y_1 = m (x - x_1)
$$

$$
y + 3 = \frac{4}{3}(x - 4)
$$

This leads again to:

$$
4x - 3y = 25
$$

Summary of Results

Both methods converge to the same tangent line equation:

$$
\boxed{4x - 3y = 25}
$$

which confirms the correctness and robustness of the approach.

Additional Considerations

1. Alternate Forms of the Equation

- Standard form: $4x - 3y = 25$
- Slope-intercept form:

$$
-3y = -4x + 25
$$
$$
y = \frac{4}{3}x - \frac{25}{3}
$$

- Graphical interpretation: The tangent line passes through $P(4, -3)$ with a slope of $\frac{4}{3}$, touching the circle at exactly one point.

2. Verifying the Tangent Line

To verify that the line is indeed tangent:

- The line should intersect the circle at exactly one point.
- Substituting $y = \frac{4}{3}x - \frac{25}{3}$ into the circle's equation:

$$
x^2 + \left(\frac{4}{3}x - \frac{25}{3} \right)^2 = 25
$$

Calculating:

$$
x^2 + \left(\frac{4x - 25}{3} \right)^2 = 25
$$

$$
x^2 + \frac{(4x - 25)^2}{9} = 25
$$

Multiply through by 9:

$$
9x^2 + (4x - 25)^2 = 225
$$

Expand:

$$
9x^2 + 16x^2 - 200x + 625 = 225
$$

Combine like terms:

$$
25x^2 - 200x + 625 = 225
$$

Bring all to one side:

$$
25x^2 - 200x + 400 = 0
$$

Divide entire equation by 25:

$$
x^2 - 8x + 16 = 0
$$

Discriminant:

$$
\Delta = (-8)^2 - 4 \times 1 \times 16 = 64 - 64 = 0
$$

A discriminant of zero indicates exactly one solution, confirming the line is tangent at P.

Final Remarks

Finding the equation of the tangent line to a circle at a specific point is a classic problem that combines geometric intuition with algebraic and calculus-based techniques. The key observations are:

- The tangent at a point on a circle is perpendicular to the radius at that point.
- The slope of the radius can be used to find the slope of the tangent.
- Implicit differentiation provides a calculus-based method to find the slope directly.

In this case, the tangent line to the circle $x^2 + y^2 = 25$ at the point $P(4, -3)$ is $4x - 3y = 25$, a line that just grazes the circle at that point, perfectly illustrating the elegance of coordinate geometry.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Anton, H., Bivens, I., & Davis, S. (2013). Calculus: Early Transcendentals. John Wiley & Sons.
- Khan

Find The Equation Of The Tangent To The Circle X Y 25 At Thepoint P 4 3 On The Circle

Find The Equation Of The Tangent To The Circle X Y 25 At Thepoint P 4 3 On The Circle

Related Articles

- The Night Before Alfred Adler Made His First Trip To The United States, He Dreamed That:
- The High Surface Temperatures Of ________ Have Been Attributed To The Greenhouse Effect.
- True/false. European Societies In The Middle Ages Were Highly Buerecratic And Centralized

Back to Home