Find The First Derivative With Respect To X Of The Following Function: F(x)=(7x+3)(4-3x)

Find The First Derivative With Respect To X Of The Following Function: F(x)=(7x+3)(4-3x)

Understanding how to find the first derivative of a function is a fundamental skill in calculus, essential for analyzing the behavior of functions, including determining slopes, rates of change, and points of maxima or minima. In this article, we will focus on calculating the first derivative with respect to x for the specific function F(x) = (7x + 3)(4 - 3x). We will explore the step-by-step process, the relevant rules of differentiation, and the significance of the derivative in mathematical analysis.

Introduction to Derivatives and Their Importance

Before diving into the calculation, it is crucial to understand what the derivative represents and why it matters.

What Is a Derivative?

A derivative of a function at a particular point measures the rate at which the function's value changes with respect to changes in the input variable, in this case, x. It provides the slope of the tangent line to the graph of the function at that point.

Mathematically, the derivative of a function F(x) with respect to x is denoted as F'(x) or dF/dx and is defined as:

\[

F'(x) = \lim_{h \to 0} \frac{F(x+h) - F(x)}{h}

\]

Why Are Derivatives Useful?

Derivatives are instrumental in various fields such as physics, engineering, economics, and biology for modeling and analyzing real-world phenomena. They help determine:

- The slope of a function at a given point
- Local maxima and minima
- Points of inflection
- The velocity of an object at a specific moment
- The rate of change of quantities

Understanding the Function $F(x)=(7x+3)(4-3x)$

The given function is a product of two linear functions:

\[
F(x) = (7x + 3)(4 - 3x)
\]

This is a product of two binomials, and to differentiate it, we will utilize the product rule.

Breaking Down the Function

The two components are:

- $u(x) = 7x + 3$
- $v(x) = 4 - 3x$

Each is a simple linear function, making their derivatives straightforward.

Differentiation Techniques Needed

To find the derivative of $F(x)$, we need to apply the following differentiation rules:

The Product Rule

The product rule states that for two differentiable functions $u(x)$ and $v(x)$:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

This rule is essential because our function $F(x)$ is a product of two functions.

Derivative of Linear Functions

The derivatives of $u(x) = 7x + 3$ and $v(x) = 4 - 3x$ are:

- $u'(x) = 7$
- $v'(x) = -3$

These are constants, simplifying the calculations.

Step-by-Step Calculation of the First Derivative

Now, let's proceed through the process of differentiating $F(x)$.

Step 1: Identify u(x) and v(x)

Recall:

- $u(x) = 7x + 3$
- $v(x) = 4 - 3x$

Step 2: Calculate u'(x) and v'(x)

- $u'(x) = 7$
- $v'(x) = -3$

Step 3: Apply the Product Rule

Using the rule:

$$
F'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

Substitute the known derivatives and functions:

$$
F'(x) = 7 \cdot (4 - 3x) + (7x + 3) \cdot (-3)
$$

Step 4: Simplify the Expression

Compute each term:

- First term: $7 \times (4 - 3x) = 7 \times 4 - 7 \times 3x = 28 - 21x$
- Second term: $(7x + 3) \times (-3) = -3 \times 7x - 3 \times 3 = -21x - 9$

Now, combine these:

$$
F'(x) = (28 - 21x) + (-21x - 9)
$$

Step 5: Combine Like Terms

Group similar terms:

$$
F'(x) = 28 - 21x - 21x - 9 = (28 - 9) + (-21x - 21x) = 19 - 42x
$$

Final Result: The First Derivative of F(x)

The derivative of the function $F(x) = (7x + 3)(4 - 3x)$ with respect to x is:

$$
\boxed{F'(x) = 19 - 42x}
$$

This simple linear expression indicates how the slope of the original function changes at different points along the x-axis.

Interpreting the Derivative

Understanding the derivative's significance helps in analyzing the behavior of the function.

Slope of the Tangent Line

- At any point x, F'(x) provides the slope of the tangent line to the graph of F(x).
- When F'(x) > 0, the function is increasing at that point.
- When F'(x) < 0, the function is decreasing.
- When F'(x) = 0, the function has a critical point, potentially indicating a maximum, minimum, or point of inflection.

Finding Critical Points

Set F'(x) to zero to find critical points:

$$
19 - 42x = 0 \Rightarrow 42x = 19 \Rightarrow x = \frac{19}{42} \approx 0.4524
$$

This is the x-value where the function's slope is zero, indicating a potential maximum or minimum.

Applications of The Derivative of the Function

Knowing the derivative allows for various practical applications:

Optimizing Functions

- Determine maximum or minimum values of F(x) by analyzing where F'(x) equals zero and using the second derivative test or first derivative test.

Analyzing Function Behavior

- Study intervals where the function is increasing or decreasing.
- Find points of inflection if the second derivative changes sign.

Real-World Contexts

Suppose $F(x)$ models profit, cost, or physical displacement. The derivative helps optimize these quantities and understand their rates of change over time or other variables.

Summary and Key Takeaways

- The first derivative of the function $F(x) = (7x + 3)(4 - 3x)$ is $F'(x) = 19 - 42x$.
- The derivative was obtained using the product rule, with the derivatives of the linear components straightforward to compute.
- The critical point at $x = 19/42$ indicates where the function may attain a maximum or minimum.
- Understanding the derivative's sign provides insights into the behavior of the original function, such as where it increases or decreases.

Conclusion

Calculating the first derivative of composite functions like $F(x) = (7x + 3)(4 - 3x)$ is a fundamental skill in calculus, enabling mathematicians and students to analyze function behavior comprehensively. By applying differentiation rules meticulously, one can uncover critical points, analyze increasing or decreasing intervals, and apply these insights to real-world problems. Mastery of these techniques provides a foundation for more advanced topics in calculus, such as second derivatives, optimization, and differential equations.

Whether you're a student preparing for exams or a professional applying calculus in research or engineering, understanding how to differentiate functions like the one discussed here is an essential part of your mathematical toolkit.

Frequently Asked Questions

How do you find the first derivative of the function F(x) = (7x + 3)(4 - 3x)?

You apply the product rule: F'(x) = (7x + 3)' (4 - 3x) + (7x + 3) (4 - 3x)'. Compute derivatives of each factor and then simplify.

What is the step-by-step process to differentiate F(x) = (7x + 3)(4 - 3x)?

First, identify u = 7x + 3 and v = 4 - 3x. Then, find u' = 7 and v' = -3. Apply the product rule: F'(x) = u'v + uv'. Substituting, F'(x) = 7(4 - 3x) + (7x + 3)(-3). Simplify this expression to obtain the derivative.

What is the derivative of F(x) = (7x + 3)(4 - 3x)?

The derivative is F'(x) = -2(7x + 3) + 7(4 - 3x) which simplifies to -14x - 6 + 28 - 21x = -35x + 22.

How can you simplify the derivative of the function F(x) = (7x + 3)(4 - 3x)?

After applying the product rule, expand and combine like terms: F'(x) = 7(4 - 3x) + (7x + 3)(-3) = 28 - 21x - 21x - 9 = 28 - 42x - 9 = 19 - 42x.

Why is it important to use the product rule when differentiating F(x) = (7x + 3)(4 - 3x)?

Because F(x) is a product of two functions of x, the product rule is necessary to correctly differentiate the expression, accounting for the derivatives of both factors.

Additional Resources

Find The First Derivative With Respect To X Of The Following Function: F(x) = (7x + 3)(4 - 3x)

Introduction

In the realm of calculus, differentiation serves as a fundamental tool for understanding how functions behave, particularly how they change with respect to a variable. The process of finding the first derivative of a function provides insight into the slope of the function at any given point, revealing whether the function is increasing, decreasing, or reaching critical points such as maxima and minima.

This article aims to thoroughly investigate the first derivative with respect to x of the function:

\[F(x) = (7x + 3)(4 - 3x) \]

We will explore the underlying principles involved in differentiating products of functions, outline the step-by-step procedure to compute the derivative, and interpret the results in the context of the function's behavior.

Understanding the Function: Structure and Components

Before delving into the differentiation process, it is essential to examine the structure of the given function:

$$
F(x) = (7x + 3)(4 - 3x)
$$

This is a product of two linear functions:

- $u(x) = 7x + 3$
- $v(x) = 4 - 3x$

The function is quadratic in form when expanded, which suggests that its derivative will be linear, but the process to find this derivative involves applying the product rule.

Differentiation Principles: The Product Rule

The main tool for differentiating the product of two functions is the product rule, which states:

$$
\frac{d}{dx}[u(x) \cdot v(x)] = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

where:

- $u'(x)$ is the derivative of $u(x)$,
- $v'(x)$ is the derivative of $v(x)$.

Applying this rule correctly is crucial for obtaining an accurate first derivative.

Step-by-Step Derivation

Step 1: Identify $u(x)$ and $v(x)$

$$
u(x) = 7x + 3
$$

$$
v(x) = 4 - 3x
$$

Step 2: Compute $u'(x)$ and $v'(x)$

$$
u'(x) = \frac{d}{dx}(7x + 3) = 7
$$

$$
v'(x) = \frac{d}{dx}(4 - 3x) = -3
$$

Step 3: Apply the product rule

$$
F'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)
$$

$$
F'(x) = 7 \cdot (4 - 3x) + (7x + 3) \cdot (-3)
$$

Step 4: Simplify each term

- For the first term:

$$
7 \cdot (4 - 3x) = 28 - 21x
$$

- For the second term:

$$
(7x + 3) \cdot (-3) = -21x - 9
$$

Step 5: Combine the results

$$
F'(x) = (28 - 21x) + (-21x - 9) = 28 - 21x - 21x - 9
$$

$$
F'(x) = (28 - 9) - 42x = 19 - 42x
$$

Final Result

The first derivative of the function $F(x) = (7x + 3)(4 - 3x)$ with respect to x is:

$$

$$
\boxed{
F'(x) = 19 - 42x
}
$$

This linear function describes the slope of the original function at any point x.

Interpreting the Derivative: Behavior and Critical Points

Understanding the derivative's implications allows us to analyze the original function's behavior, including where it increases or decreases and where it reaches maximum or minimum points.

1. Sign of the Derivative

- The sign of $F'(x)$ indicates whether $F(x)$ is increasing or decreasing:
- $F'(x) > 0$ □ $F(x)$ is increasing.
- $F'(x) < 0$ □ $F(x)$ is decreasing.

$$
19 - 42x > 0 \Rightarrow x < \frac{19}{42} \approx 0.452
$$

$$
19 - 42x < 0 \Rightarrow x > 0.452
$$

- At $x = \frac{19}{42}$, the derivative is zero, indicating a potential critical point.

2. Critical Point and Extrema

Set $F'(x) = 0$:

$$
19 - 42x = 0
$$
$$
x = \frac{19}{42}
$$

- To determine whether this critical point is a maximum or minimum, evaluate the second derivative or analyze the sign change of $F'(x)$.

The Second Derivative and Concavity (Optional Deep Dive)

The second derivative provides information about the concavity of the function:

$$
F''(x) = \frac{d}{dx} (F'(x)) = \frac{d}{dx} (19 - 42x) = -42
$$

Since $F''(x) = -42 < 0$, the function is concave downward everywhere, implying that the critical point at $x = \frac{19}{42}$ is a local maximum.

Graphical Interpretation

Plotting the function and its derivative enhances understanding:

- The original function $F(x)$ is quadratic, opening downward due to the negative second derivative.
- The maximum occurs at $x = \frac{19}{42}$, with the maximum value:

$$
F\left(\frac{19}{42}\right) = (7 \times \frac{19}{42} + 3) \times (4 - 3 \times \frac{19}{42})
$$

Calculating:

$$
7 \times \frac{19}{42} = \frac{133}{42}
$$

$$
\Rightarrow 7 \times \frac{19}{42} + 3 = \frac{133}{42} + \frac{126}{42} = \frac{259}{42}
$$

Similarly:

$$
4 - 3 \times \frac{19}{42} = 4 - \frac{57}{42} = \frac{168}{42} - \frac{57}{42} = \frac{111}{42}
$$

Thus:

$$
F\left(\frac{19}{42}\right) = \frac{259}{42} \times \frac{111}{42} = \frac{259 \times 111}{1764}
$$

Calculating numerator:

\[

$$
259 \times 111 = 259 \times (100 + 11) = 259 \times 100 + 259 \times 11 = 25,900 + 2,849 = 28,749
$$

Therefore, maximum value:

$$
F_{\max} = \frac{28,749}{1,764} \approx 16.28
$$

Summary and Applications

The detailed differentiation process reveals:

- The first derivative $F'(x) = 19 - 42x$ is linear, indicating a steady rate of change.
- The function reaches a local maximum at $x \approx 0.452$, with a maximum value of approximately 16.28.
- The function is increasing for $x < 0.452$ and decreasing afterward.

This analysis is instrumental in fields such as physics (motion analysis), economics (cost and revenue functions), and engineering (system optimization), where understanding how a quantity changes with respect to another is crucial.

Conclusion

The process of finding the first derivative of $F(x) = (7x + 3)(4 - 3x)$ exemplifies core calculus principles, especially the application of the product rule. The derivative’s linear form facilitates straightforward analysis of the function's increasing and decreasing intervals, as well as its critical points and concavity.

Through rigorous derivation and interpretation, we gain a comprehensive understanding of the function's behavior, illustrating the power of differential calculus in mathematical modeling and analysis. Whether for academic, professional, or research purposes, mastery of such differentiation techniques remains a cornerstone of quantitative analysis.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Thomas, G. B., & Finney, R. L. (2002). Calculus and Analytic Geometry. Addison-Wesley.
- Apostol, T. M. (1967). Calculus, Volume 1. Wiley.

Note: All calculations are approximate where necessary, and detailed steps are provided to ensure clarity and reproducibility.

Find The First Derivative With Respect To X Of The Following Function F X 7x 3 4 3x

Find The First Derivative With Respect To X Of The Following Function F X 7x 3 4 3x

Related Articles

- When Fraud Factors Are Identified During An Audit The Auditor's Documentation Should Include
- A FICO Score Between _____ And 799 Is Considered "very Good." A. 680 B. 740 C. 420 D. 550
- How The Rotation And Uneven Heating Of The Earth Cause Patterns Of Oceanic Circulation?

Back to Home