Find The Fourier Series Expansion Of The Function F(x) With Period P = 2l 1. F(x) = -1 (-2

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Understanding Fourier series is fundamental in mathematical analysis, signal processing, and various engineering applications. The process of deriving the Fourier series for a given periodic function allows us to represent complex signals as a sum of simple sinusoidal components. Today, we will explore how to find the Fourier series expansion of a specific function $F(x)$ with a period $P = 2l$, focusing on the function $F(x) = -1$ over a specified interval and extending periodically. This comprehensive guide will cover the theoretical background, step-by-step calculation, and practical applications.

Introduction to Fourier Series

The Fourier series is a method to express a periodic function $F(x)$ as an infinite sum of sines and cosines. This decomposition is particularly useful because sinusoidal functions are the building blocks of many signals and are easier to analyze and manipulate.

Key Concepts:

- Periodicity: $F(x + P) = F(x)$ for all x.
- Fundamental period: P.
- Fourier series representation:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)
$$

where:

$$
a_0 = \frac{1}{l} \int_{-l}^{l} F(x) \, dx
$$

$$
a_n = \frac{1}{l} \int_{-l}^{l} F(x) \cos \frac{n\pi x}{l} \, dx
$$

$$
b_n = \frac{1}{l} \int_{-l}^{l} F(x) \sin \frac{n\pi x}{l} \, dx
$$

\]

with \(P = 2l \).

Note: The choice of interval \([-l, l] \) aligns with the period \(P = 2l \).

Understanding the Function \(F(x) \)

The problem specifies the function \(F(x) = -1 \) over a particular interval, extending periodically with period \(P = 2l \). To proceed, it is essential to clarify the specific segment over which \(F(x) \) is defined.

Assumption:

For typical Fourier analysis, especially in basic examples, \(F(x) \) is often defined over one period as:

\[
F(x) = \begin{cases}
-1, & \text{for } -l < x < 0 \\
\text{(possibly different expression)}, & \text{for } 0 \leq x < l
\end{cases}
\]

or as a constant over the entire period. Since the problem states \(F(x) = -1 \), it suggests a constant function over the period \(P = 2l \).

Therefore:

\[
F(x) = -1 \quad \text{for all } x
\]

This simplifies the Fourier series expansion, as the function is a constant.

However, if the problem involves a piecewise function, such as a square wave or other periodic functions, the approach would differ. For now, we will proceed with the assumption that \(F(x) \equiv -1 \) over \([-l, l] \).

Calculating Fourier Coefficients for \(F(x) = -1 \)

Given \(F(x) = -1 \) over the period \([-l, l] \), the Fourier coefficients can be computed directly:

1. Constant term a_0:

$$
a_0 = \frac{1}{l} \int_{-l}^{l} F(x) \, dx = \frac{1}{l} \int_{-l}^{l} (-1) \, dx
$$

Calculating the integral:

$$
a_0 = \frac{1}{l} \times [-x]_{-l}^{l} = \frac{1}{l} \times \left(-l - (-(-l)) \right)
$$

Since:

$$
\int_{-l}^{l} (-1) \, dx = -1 \times (l - (-l)) = -1 \times 2l = -2l
$$

Thus,

$$
a_0 = \frac{-2l}{l} = -2
$$

2. Fourier cosine coefficients a_n:

$$
a_n = \frac{1}{l} \int_{-l}^{l} F(x) \cos \frac{n\pi x}{l} \, dx
$$

Substitute $F(x) = -1$:

$$
a_n = \frac{1}{l} \int_{-l}^{l} (-1) \cos \frac{n\pi x}{l} \, dx = -\frac{1}{l} \int_{-l}^{l} \cos \frac{n\pi x}{l} \, dx
$$

The integral:

$$
\int_{-l}^{l} \cos \frac{n\pi x}{l} \, dx = \left[\frac{\sin \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right]_{-l}^{l} = \frac{l}{n\pi} \left(\sin n\pi - \sin (-n\pi) \right)
$$

Since $\sin n\pi = 0$, and $\sin(-n\pi) = - \sin n\pi = 0$, the entire integral becomes zero:

$$
\int_{-l}^{l} \cos \frac{n\pi x}{l} \, dx = 0
$$

Therefore,

$$
a_n = -\frac{1}{l} \times 0 = 0
$$

3. Fourier sine coefficients b_n:

$$
b_n = \frac{1}{l} \int_{-l}^{l} F(x) \sin \frac{n\pi x}{l} \, dx
$$

Substitute $F(x) = -1$:

$$
b_n = -\frac{1}{l} \int_{-l}^{l} \sin \frac{n\pi x}{l} \, dx
$$

The integral:

$$
\int_{-l}^{l} \sin \frac{n\pi x}{l} \, dx = \left[-\frac{\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right]_{-l}^{l} = -\frac{l}{n\pi} \left(\cos n\pi - \cos (-n\pi) \right)
$$

Recall that $\cos n\pi = (-1)^n$, and $\cos(-n\pi) = \cos n\pi = (-1)^n$. So,

$$
\cos n\pi - \cos(-n\pi) = (-1)^n - (-1)^n = 0
$$

Thus,

$$
b_n = -\frac{1}{l} \times 0 = 0
$$

Summary of coefficients:

$$
a_0 = -2, \quad a_n = 0, \quad b_n = 0
$$

Fourier Series for $F(x) = -1$:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right) = -1 + 0 + 0 = -1
$$

Thus, the Fourier series of the constant function $F(x) = -1$ is simply:

$$
\boxed{
F(x) = -1
}
$$

which confirms our expectation: the Fourier series of a constant function is the constant itself.

Generalization: Piecewise Function and Non-Constant Periodic Functions

In many practical scenarios, functions are not constant but piecewise or possess discontinuities. For example, consider a square wave or a sawtooth wave. The methodology to find their Fourier series involves defining the function over one period, calculating the coefficients as shown, and summing the series.

Example: Square Wave Function

Suppose $F(x)$ is a square wave with amplitude 1 over half the period and -1 over the other half, defined as:

$$
F(x) = \begin{cases}
1, & 0 < x < l \\
-1, & -l < x < 0
\end{cases}
$$

and extended periodically with period $2l$.

- The Fourier coefficients are computed by integrating over one period.
- The series converges to the square wave,

Frequently Asked Questions

What is the general approach to find the Fourier series expansion of a periodic function like F(x) with period P = 2l?

To find the Fourier series of F(x), we express it as a sum of sine and cosine terms with coefficients determined by integrating F(x) over one period. Specifically, the coefficients are calculated using integrals of F(x) multiplied by sine and cosine functions over the interval [-l, l], then summed accordingly.

Given the function F(x) = -1 for -2 < x < 0, how do we determine its Fourier coefficients considering the periodic extension with period P = 2l?

We compute the Fourier coefficients by integrating F(x) over one period, considering its piecewise definition. For the given F(x), you'd set up integrals over the interval [-l, l], noting the segment where F(x) = -1 and where it might differ, then calculate the coefficients a0, an, and bn accordingly.

How does the period P = 2l influence the Fourier series expansion of F(x)?

The period P = 2l determines the interval over which the Fourier series is computed and repeated. It affects the limits of integration for the Fourier coefficients, with integrals typically performed over [-l, l], and the frequency components are multiples of π/l.

What is the significance of the function's discontinuities in finding its Fourier series, especially for a piecewise function like F(x) = -1 on part of the period?

Discontinuities in F(x) cause the Fourier series to converge to the average of the left and right limits at the discontinuity points (Gibbs phenomenon). For piecewise functions like F(x) = -1 on part of the period, these jumps influence the sine and cosine coefficients and the convergence behavior of the series.

Can you provide the Fourier series expansion formula for F(x) = -1 on a certain interval with period 2l,

and how are the coefficients explicitly calculated?

Yes. The Fourier series of F(x) with period 2l is given by:

$F(x) = a0/2 + \Sigma [an \cos(n\pi x/l) + bn \sin(n\pi x/l)]$ for n=1 to ∞,

where
$a0 = (1/l) \int_{-l}^{l} F(x) dx$,

$an = (1/l) \int_{-l}^{l} F(x) \cos(n\pi x/l) dx$,

$bn = (1/l) \int_{-l}^{l} F(x) \sin(n\pi x/l) dx$.

The explicit calculation involves integrating the piecewise definition of F(x) over [-l, l] and substituting into these formulas.

Additional Resources

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Introduction

Fourier series serve as a foundational tool in mathematical analysis, enabling the decomposition of periodic functions into sums of sines and cosines. Their applications permeate various scientific and engineering disciplines, including signal processing, heat transfer, and quantum mechanics. The problem of finding the Fourier series expansion of a given function is both classic and nuanced, often revealing deeper insights into the function's behavior and symmetries.

In this investigation, we focus on a particular periodic function characterized by a period $P = 2l$, with an explicitly provided form $F(x) = -1$ over a specified interval. Our goal is to derive its Fourier series expansion systematically, exploring the underlying principles, calculations, and implications of the series representation. This exploration not only demonstrates the procedural aspects of Fourier analysis but also highlights its elegance in representing complex periodic functions.

Clarifying the Function and Its Periodicity

Understanding the Function $F(x)$

The notation provided appears somewhat ambiguous:

- The function is denoted as $F(x) = -1$ over the interval -2

(presumably $-2 \leq x \leq 2$).
- The period $P = 2l$ suggests the function repeats every $2l$, where l is a positive constant.

Given these clues, a reasonable interpretation is:

- $F(x)$ is a piecewise constant function, possibly taking the value -1 within a specific interval and extending periodically.
- The period $P = 2l$ indicates that the function's pattern repeats every $2l$.

Assuming a Specific Function Form

For clarity and to proceed with calculations, we assume the function:

$$
F(x) =
\begin{cases}
-1, & -l \leq x \leq l \\
\text{periodic extension}, & \text{for all } x
\end{cases}
$$

with period $P = 2l$. This form represents a rectangular wave centered at zero, with amplitude -1 over the interval $[-l, l]$, repeating every $2l$.

Mathematical Framework for Fourier Series

Fourier Series of a Periodic Function

Any function $F(x)$ with period $P = 2l$ that is integrable over one period can be expanded as:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \left(\frac{n\pi x}{l} \right) + b_n \sin \left(\frac{n\pi x}{l} \right) \right]
$$

where the Fourier coefficients are given by:

$$
a_0 = \frac{1}{l} \int_{-l}^l F(x) \, dx
$$

$$
a_n = \frac{1}{l} \int_{-l}^l F(x) \cos \left(\frac{n\pi x}{l} \right) dx
$$

$$
b_n = \frac{1}{l} \int_{-l}^l F(x) \sin \left(\frac{n\pi x}{l} \right) dx
$$

Derivation of Fourier Coefficients

Computing a_0

Given $F(x) = -1$ over $[-l, l]$:

$$
a_0 = \frac{1}{l} \int_{-l}^l (-1) \, dx = \frac{1}{l} \times (-1) \times (2l) = -2
$$

Thus,

$$
\boxed{
a_0 = -2
}
$$

The constant term of the Fourier series is $a_0/2 = -1$.

Computing a_n

$$
a_n = \frac{1}{l} \int_{-l}^l (-1) \cos \left(\frac{n \pi x}{l} \right) dx
$$

Pulling out constants:

$$
a_n = - \frac{1}{l} \int_{-l}^l \cos \left(\frac{n \pi x}{l} \right) dx
$$

Evaluate the integral:

$$
\int_{-l}^l \cos \left(\frac{n \pi x}{l} \right) dx = \left[\frac{l}{n\pi} \sin \left(\frac{n \pi x}{l} \right) \right]_{x=-l}^{x=l}
$$

At $x = l$:

\[

\sin (n \pi) = 0
\]

At \(x = -l \):

\[
\sin (-n \pi) = - \sin (n \pi) = 0
\]

Therefore,

\[
a_n = - \frac{1}{l} \times 0 = 0
\]

Result:

\[
\boxed{
a_n = 0 \quad \text{for all } n \geq 1
}
\]

Computing \(b_n \)

\[
b_n = \frac{1}{l} \int_{-l}^l (-1) \sin \left(\frac{n \pi x}{l} \right) dx =
- \frac{1}{l} \int_{-l}^l \sin \left(\frac{n \pi x}{l} \right) dx
\]

Evaluate the integral:

\[
\int_{-l}^l \sin \left(\frac{n \pi x}{l} \right) dx = \left[- \frac{l}{n
\pi} \cos \left(\frac{n \pi x}{l} \right) \right]_{x=-l}^{x=l}
\]

At \(x = l \):

\[
- \frac{l}{n \pi} \cos (n \pi) = - \frac{l}{n \pi} (-1)^n
\]

At \(x = -l \):

\[
- \frac{l}{n \pi} \cos (- n \pi) = - \frac{l}{n \pi} \cos (n \pi) = -
\frac{l}{n \pi} (-1)^n
\]

Subtracting:

$$
\left[- \frac{l}{n \pi} (-1)^n \right] - \left[- \frac{l}{n \pi} (-1)^n \right] = 0
$$

Therefore,

$$
b_n = - \frac{1}{l} \times 0 = 0
$$

Result:

$$
\boxed{
b_n = 0 \quad \text{for all } n \geq 1
}
$$

Final Fourier Series Expression

Combining the coefficients, the Fourier series simplifies to:

$$
F(x) = \frac{a_0}{2} + \sum_{n=1}^\infty (a_n \cos \ldots + b_n \sin \ldots) = -1 + 0 = -1
$$

This indicates that the Fourier series reduces to a constant function, matching the value of $F(x)$ over the interval.

Discussion: The Nature of the Function and Its Fourier Series

Constant Function Case

Under the current assumptions, the function $F(x)$, being a simple constant -1 over each period, has a trivial Fourier series consisting solely of the constant term:

$$
F(x) = -1
$$

This aligns with the Fourier series of any constant function, which is purely that constant.

Considering More Complex Forms

Suppose the initial description aimed at a rectangular wave with values -1 over $[-l, l]$ and $+1$ over $[l, 2l]$, repeating periodically. The Fourier series in such a case would involve non-zero sine and cosine coefficients, capturing the wave's shape.

In that case, the function would be:

$$
F(x) =
\begin{cases}
-1, & -l \leq x < 0 \\
1, & 0 \leq x < l
\end{cases}
$$

with period $2l$. The Fourier coefficients would then be:

$$
a_0 = 0
$$
$$
a_n = 0
$$
$$
b_n = \frac{2}{n \pi} \left[1 - (-1)^n \right]
$$

which would produce the classic Fourier series of a square wave.

Summary and Implications

- If $F(x)$ is constant -1 over the period $2l$, its Fourier series is simply $F(x) = -1$. No sines or cosines are needed, as the function is already in

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