Find The Line Integral Of F(x,y)=ye X 2along The Curve R(t)=4ti3tj,1t1. The Integral Of F Is

Find The Line Integral Of F(x,y)=ye^{x^2}along The Curve R(t)=4ti+3tj,1t1. The Integral Of F Is

Calculating line integrals is a fundamental concept in vector calculus, with applications spanning physics, engineering, and mathematics. In this article, we will explore how to evaluate the line integral of the vector field $\mathbf{F}(x, y) = y e^{x^2}$ along a specified parametric curve $\mathbf{R}(t)$. The process involves understanding the nature of the vector field, parametrizing the curve, computing derivatives, and applying the line integral formulas systematically. Let's dive into the detailed steps to obtain the value of the integral.

Understanding the Problem

Before solving, it's essential to interpret the problem correctly:

- Vector Field: $\mathbf{F}(x, y) = y e^{x^2}$
- Curve: $\mathbf{R}(t) = 4t \mathbf{i} + 3t \mathbf{j}$, with t in the interval [1, 1].

Note: The description of the curve's parameter interval appears inconsistent (from 1 to 1), which suggests that perhaps the curve is evaluated over $t \in [a, b]$. For a meaningful integral, assume the parameter interval is from $t=0$ to $t=1$. If the original problem states $t \in [1,1]$, that would imply a zero-length integral. Therefore, for the purpose of this explanation, we'll assume the curve runs from $t=0$ to $t=1$.

Parametric Representation of the Curve

The curve $\mathbf{R}(t)$ is given as:

$$
\mathbf{R}(t) = 4t \mathbf{i} + 3t \mathbf{j}
$$

which explicitly states:

$$
x(t) = 4t, \quad y(t) = 3t
$$

\]

for \(t \in [0,1] \).

Understanding this parametric form allows us to express the vector field \(\mathbf{F} \) along the curve as a composition:

\[
\mathbf{F}(\mathbf{R}(t)) = y(t) e^{x(t)^2}
\]

Calculating the Derivative \(\mathbf{R}'(t) \)

The derivative of \(\mathbf{R}(t) \) with respect to \(t \) is:

\[
\mathbf{R}'(t) = \frac{d}{dt} (4t \mathbf{i} + 3t \mathbf{j}) = 4 \mathbf{i} + 3 \mathbf{j}
\]

which is a constant vector:

\[
\mathbf{R}'(t) = (4, 3)
\]

Formulating the Line Integral

The line integral of a scalar field \(f(x, y) \) along a curve \(\mathbf{R}(t) \) from \(t=a \) to \(t=b \) is:

\[
\int_{C} f(x, y) \, ds = \int_{a}^{b} f(\mathbf{R}(t)) \|\mathbf{R}'(t)\| dt
\]

Here, since the vector field \(\mathbf{F} \) appears to be scalar-valued (assuming it's a scalar field), the integral simplifies to:

\[
\int_{a}^{b} \mathbf{F}(x(t), y(t)) \| \mathbf{R}'(t) \| dt
\]

Alternatively, if the problem considers the line integral of the vector field \(\mathbf{F} \) along \(

\mathbf{R}(t) \), the standard form is:

\[
\int_{C} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} \mathbf{F}(\mathbf{R}(t)) \cdot \mathbf{R}'(t) dt
\]

Given the context, it appears the latter is intended.

Evaluating \(\mathbf{F} \cdot \mathbf{R}'(t) \)

First, compute \(\mathbf{F} \) along the curve:

\[
x(t) = 4t, \quad y(t) = 3t
\]

so

\[
\mathbf{F}(x(t), y(t)) = y(t) e^{x(t)^2} = 3t \, e^{(4t)^2} = 3t \, e^{16t^2}
\]

The differential \(d\mathbf{r} \) is:

\[
d\mathbf{r} = \mathbf{R}'(t) dt = (4, 3) dt
\]

Thus, the dot product:

\[
\mathbf{F}(\mathbf{R}(t)) \cdot \mathbf{R}'(t) = \left(y(t) e^{x(t)^2} \right) (4, 3) \cdot (1, 0) + \left(y(t) e^{x(t)^2} \right) (0, 1) \cdot (0, 1)
\]

However, since \(\mathbf{F} \) is scalar-valued, and the line integral of a scalar field along a curve is:

\[
\int_{a}^{b} f(\mathbf{R}(t)) \|\mathbf{R}'(t)\| dt
\]

But more standard in vector calculus is the line integral of a vector field \(\mathbf{F} \):

\[
\int_{C} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} \mathbf{F}(\mathbf{R}(t)) \cdot \mathbf{R}'(t) dt

$$

Assuming the latter, and that $\mathbf{F}$ is a scalar field, the integral reduces to:

$$
\int_{a}^{b} f(\mathbf{R}(t)) \, \| \mathbf{R}'(t) \| dt
$$

For clarity, let's proceed with the vector field interpretation, assuming $\mathbf{F}$ is scalar and the line integral is of the form:

$$
\int_{C} f(x, y) \, ds
$$

which involves integrating $f(x, y)$ times the differential arc length ds.

The differential arc length:

$$
ds = \| \mathbf{R}'(t) \| dt
$$

where:

$$
\| \mathbf{R}'(t) \| = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5
$$

Computing the Final Integral

Putting it all together, the line integral becomes:

$$
\int_{0}^{1} f(\mathbf{R}(t)) \| \mathbf{R}'(t) \| dt = \int_{0}^{1} 3t e^{16 t^2} \times 5 \, dt = 5 \int_{0}^{1} 3t e^{16 t^2} dt
$$

Simplify:

$$
= 15 \int_{0}^{1} t e^{16 t^2} dt
$$

Evaluating the Integral $\int_{0}^{1} t e^{16 t^2} dt$

This integral is approachable via substitution:

Let:

$$
u = 16 t^2 \Rightarrow du = 32 t dt \Rightarrow t dt = \frac{du}{32}
$$

When $t = 0$, $u = 0$; when $t = 1$, $u = 16$.

Therefore:

$$
\int_{0}^{1} t e^{16 t^2} dt = \int_{u=0}^{u=16} e^{u} \frac{du}{32} = \frac{1}{32} \int_{0}^{16} e^{u} du
$$

Evaluating:

$$
\frac{1}{32} [e^{u}]_{0}^{16} = \frac{1}{32} (e^{16} - 1)
$$

Multiplying back by 15:

$$
15 \times \frac{1}{32} (e^{16} - 1) = \frac{15}{32} (e^{16} - 1)
$$

Final Result

The value of the line integral is:

$$
\boxed{
\frac{15}{32} (e^{16} - 1)
}
$$

This expression provides the exact value of the line integral of $\mathbf{F}(x, y) = y e^{x^2}$ along the curve $\mathbf{R}(t) = 4t \mathbf{i} + 3t \mathbf{j}$

Frequently Asked Questions

What is the vector field F(x, y) given in the problem?

The vector field is F(x, y) = y e^x i + 2 y j.

What is the parameterization of the curve R(t)?

The curve is given by R(t) = (4t) i + (3t) j, with t ranging from 1 to 1.

How do you compute the line integral of F along the curve R(t)?

To compute the line integral, substitute R(t) into F, find R'(t), then evaluate the integral of F(R(t)) · R'(t) dt over the given limits.

What are the limits of integration for t in the curve R(t)?

The problem states t varies from 1 to 1, which suggests the integral may be over a degenerate interval. Clarify the limits to proceed correctly.

How do you evaluate F(R(t)) for the given curve?

Substitute x = 4t and y = 3t into F(x, y) = y e^x i + 2 y j to get F(R(t)).

What is the importance of R'(t) in computing the line integral?

R'(t) provides the differential element along the curve, necessary for evaluating the line integral as an integral of F(R(t)) · R'(t) dt.

What steps are involved in solving the integral of F along R(t)?

Steps include parameterizing F, computing R'(t), setting up the integral of F(R(t)) · R'(t), and integrating over the specified t-interval.

What is the final value of the line integral of F along the curve R(t)?

The final value depends on the correct limits of t and the calculations. Clarify the limits and perform the integration to find the result.

Additional Resources

Find The Line Integral Of F(x,y) = y e^{x^2} Along The Curve R(t) = 4t i + 3t j, 1 ≤ t ≤ 1

Introduction

In the realm of vector calculus and multivariable analysis, line integrals serve as fundamental tools for evaluating work done by a force field, calculating flux, and understanding the flow of vector fields along specified paths. The process involves integrating a vector or scalar field along a curve, capturing the cumulative effect of the field as it traverses a particular route.

This article delves into a specific problem: finding the line integral of the scalar function $F(x, y) = y e^{x^2}$ along the curve $R(t) = 4t \mathbf{i} + 3t \mathbf{j}$, where t ranges from 1 to 1. Although the problem statement appears trivial at first glance—since the interval is degenerate at $t=1$—it provides an instructive opportunity to explore the methods of line integral evaluation, parametric substitution, and the importance of proper integral bounds.

Understanding the Components of the Problem

The Scalar Field $F(x, y) = y e^{x^2}$

This function combines a polynomial component y with an exponential component e^{x^2}, creating a scalar field that varies with both x and y. Its behavior is characterized by rapid growth in the exponential term as x increases, and linear variation in y.

The Curve $R(t) = 4t \mathbf{i} + 3t \mathbf{j}$

Expressed parametrically:

$$
x(t) = 4t, \quad y(t) = 3t
$$

for t in the interval $[1, 1]$.

Critical Analysis of the Interval

Key Observation: The original problem states the integral of F along $R(t)$ for $t \in [1, 1]$.

This is a single point interval, which implies the curve is degenerate—no actual traversal occurs, and the line integral over such an interval is zero.

However, in typical scenarios, the interval should be a non-degenerate segment, such as $[a, b]$ with $a < b$, to evaluate a meaningful line integral.

Possible interpretations:

- Typographical Error: The interval might have been intended as $[1, 2]$, $[0, 1]$, or another range.
- Instructional Focus: The problem aims to demonstrate the method at a specific point, emphasizing

the importance of proper bounds.

For educational completeness, we will assume the interval is from $t=1$ to $t=2$. This allows us to illustrate the full process of line integral evaluation.

Step-by-Step Solution Approach

1. Parameterize the Curve

Given:

$$
x(t) = 4t, \quad y(t) = 3t
$$

for $t \in [1, 2]$ (assuming this correction).

The derivatives:

$$
x'(t) = 4, \quad y'(t) = 3
$$

2. Express $F(x, y)$ along the curve

Substitute $x(t)$ and $y(t)$:

$$
F(x(t), y(t)) = y(t) e^{(x(t))^2} = 3t \cdot e^{(4t)^2} = 3t \cdot e^{16 t^2}
$$

---

3. Formulating the Line Integral

The scalar line integral over the curve $R(t)$ is:

$$
\int_{t=1}^{2} F(x(t), y(t)) \sqrt{(x'(t))^2 + (y'(t))^2} \, dt
$$

Alternatively, if integrating a scalar field along a curve, the line integral is:

$$
\int_{C} F(x, y) \, ds
$$

where ds is the differential arc length, given by:

$$
ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt
$$

Calculating ds/dt:

$$
ds/dt = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5
$$

Thus:

$$
ds = 5 dt
$$

The line integral becomes:

$$
\int_{t=1}^{2} F(x(t), y(t)) \cdot 5 \, dt = 5 \int_{1}^{2} 3t e^{16 t^2} \, dt
$$

---

4. Simplify and Compute the Integral

The integral simplifies to:

$$
I = 15 \int_{1}^{2} t e^{16 t^2} \, dt
$$

This integral involves an exponential of a quadratic, which suggests substitution.

---

5. Substitution Method

Let:

$$
u = 16 t^2 \implies du = 32 t dt \implies t dt = \frac{du}{32}
$$

Expressed in the integral:

$$
I = 15 \int_{t=1}^{2} t e^{16 t^2} dt = 15 \int_{u=16 \cdot 1^2}^{u=16 \cdot 2^2} e^{u} \cdot \frac{du}{32}
$$

Calculate the bounds:

$$
u_{lower} = 16 \times 1 = 16
$$

$$
u_{upper} = 16 \times 4 = 64
$$

Therefore:

$$
I = \frac{15}{32} \int_{16}^{64} e^{u} du
$$

The integral of e^{u} is straightforward:

$$
\int e^{u} du = e^{u}
$$

Thus:

$$
I = \frac{15}{32} [e^{u}]_{16}^{64} = \frac{15}{32} \left(e^{64} - e^{16} \right)
$$

---

6. Final Result

The value of the line integral over the assumed interval $[1, 2]$ is:

$$
\boxed{
\text{Line Integral} = \frac{15}{32} \left(e^{64} - e^{16} \right)
}
$$

This expression captures the cumulative effect of the scalar field $F(x, y) = y e^{x^2}$ along the specified curve.

Discussion and Implications

Significance of Proper Interval Selection

The initial problem statement's interval $[1, 1]$ indicates a degenerate case, leading to an integral value of zero. This underscores the importance of carefully defining the limits of integration in line integrals, especially in applied contexts such as physics or engineering, where the path's length and traversal matter.

Generalization to Arbitrary Intervals

The method demonstrated here applies broadly:

- Parametrize the curve to express $x(t)$ and $y(t)$.
- Compute ds/dt to relate differential arc length to parameter t.
- Substitute into the integral to reduce it to a manageable form.
- Use substitution or integration techniques to evaluate the integral explicitly.

Extensions and Further Considerations

- Vector Field Line Integrals: If the problem involved a vector field $\mathbf{F}(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}$, the line integral would take a different form:

$$
\int_{C} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} \left(P(x(t), y(t)) x'(t) + Q(x(t), y(t)) y'(t) \right) dt
$$

- Work and Physical Interpretations: Such integrals often represent work done by a force field along a path, important in physics and engineering.

- Numerical Approaches: For more complex fields or curves where analytical solutions are infeasible, numerical methods such as Simpson's rule or Gaussian quadrature are employed.

Conclusion

The process of evaluating a line integral, as exemplified in this problem, involves careful parametrization, substitution, and integral computation. Although the initial problem's bounds may have been ambiguous or degenerate, the approach outlined demonstrates the general methodology applicable to a broad class of scalar and vector field integrals along curves.

In applied mathematics and physics, mastering these techniques enables practitioners to analyze complex systems involving fields and paths, with implications across electromagnetism, fluid dynamics, and structural analysis. The precise evaluation of such integrals forms the backbone of many theoretical and practical applications, highlighting their enduring importance in scientific inquiry.

References

- Stewart, James. Calculus: Early Transcendentals. 8th Edition. Cengage Learning, 2015.
- Marsden, J. E., & Tromba, A. J. Vector Calculus. 6th

Find The Line Integral Of F X Y Ye X 2along The Curve R T 4ti3tj 1t1 The Integral Of F Is

Find The Line Integral Of F X Y Ye X 2along The Curve R T 4ti3tj 1t1 The Integral Of F Is

Related Articles

- In Contrast To The Peripheral Route To Persuasion, The "central" Route Depends More On:
- Shells, Sand Dollar's, And Ocean Products Embedded In A Sedimentary Rock Are An Example Of ?
- How Do Claudius' And Polonius' Reasons For Spying On Hamlet Differ? How Are They Similar?

Back to Home