Find The Lines That Are (a) Tangent And (b) Normal To The Curve Y=2x^(3) At The Point (1,2).

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Understanding the properties of curves and their associated lines is a fundamental aspect of calculus and analytical geometry. Among these, tangent and normal lines play vital roles in analyzing the behavior and characteristics of a curve at a specific point. In this comprehensive guide, we will delve into the methods of finding the tangent and normal lines to the curve $y=2x^3$ at the point $(1, 2)$.

This topic is crucial for students and professionals involved in mathematics, physics, engineering, and related fields, as it provides insights into the local geometry of curves, slopes of tangents, and perpendicular relationships. By mastering these concepts, you can better understand how curves behave at given points, which is essential for applications like motion analysis, optimization problems, and designing curves in computer graphics.

Understanding the Curve $y=2x^3$

Before calculating the tangent and normal lines, it’s important to understand the nature of the given curve:

- The curve is a cubic function, which means it exhibits polynomial behavior with a degree of 3.
- The function $y=2x^3$ is symmetric about the origin, reflecting the odd degree's characteristic.
- The point of interest is $(1, 2)$, which lies on the curve since substituting $x=1$ yields $y=2(1)^3=2$.

Knowing this, we can proceed to analyze the derivatives and the slope of the tangent at the point.

Calculating the Derivative of $y=2x^3$

The derivative of a function provides the slope of the tangent line at any point x. For the function $y=2x^3$, the derivative is:

$$
\frac{dy}{dx} = \frac{d}{dx}(2x^3) = 6x^2
$$

This derivative indicates how steep the curve is at any given x-coordinate.

Finding the Slope of the Tangent Line at $(1, 2)$

To find the tangent line at the specific point $(1, 2)$:

1. Substitute $x=1$ into the derivative:

$$
m_{tangent} = 6(1)^2 = 6
$$

2. The slope of the tangent line at $(1, 2)$ is 6.

Equation of the Tangent Line at $(1, 2)$

Using the point-slope form of a line:

$$
y - y_1 = m(x - x_1)
$$

where:

- $(x_1, y_1) = (1, 2)$
- $m = 6$

Substituting these values:

$$
y - 2 = 6(x - 1)
$$

Simplify:

$$
y - 2 = 6x - 6 \Rightarrow y = 6x - 4
$$

Thus, the equation of the tangent line is:

\[

\boxed{ y = 6x - 4 }
\]

Understanding the Normal Line

The normal line to a curve at a point is the line perpendicular to the tangent line at that point. Its slope is the negative reciprocal of the tangent line's slope.

Calculating the Slope of the Normal Line

Given the tangent slope \(m_{tangent} = 6 \), the slope of the normal line \(m_{normal} \) is:

\[
m_{normal} = -\frac{1}{m_{tangent}} = -\frac{1}{6}
\]

Equation of the Normal Line at \((1, 2) \)

Using the point-slope form again:

\[
y - y_1 = m(x - x_1)
\]

where:

- \((x_1, y_1) = (1, 2) \)
- \(m = -\frac{1}{6} \)

Substitute:

\[
y - 2 = -\frac{1}{6}(x - 1)
\]

Multiply both sides by 6 to clear the denominator:

\[
6(y - 2) = -(x - 1)
\]

Simplify:

$$
6y - 12 = -x + 1
$$

Rearranged:

$$
6y = -x + 13
$$

Divide both sides by 6:

$$
y = -\frac{1}{6}x + \frac{13}{6}
$$

Therefore, the equation of the normal line is:

$$
\boxed{ y = -\frac{1}{6}x + \frac{13}{6} }
$$

Summary of Results

Line Type	Equation	Slope	Point of Contact
Tangent Line	$y = 6x - 4$	6	$(1, 2)$
Normal Line	$y = -\frac{1}{6}x + \frac{13}{6}$	$-\frac{1}{6}$	$(1, 2)$

Visualizing the Tangent and Normal Lines

Graphical representation helps in understanding the relationship between the curve and the lines:

- The tangent line $y=6x-4$ touches the curve $y=2x^3$ exactly at $(1, 2)$, with matching slope.
- The normal line $y=-\frac{1}{6}x + \frac{13}{6}$ passes through the same point and is perpendicular to the tangent.

Plotting these lines along with the curve provides a clear geometric picture of their interaction.

Applications of Tangent and Normal Lines

Understanding tangent and normal lines is fundamental in various real-world scenarios:

- Physics: Analyzing the instantaneous velocity and acceleration of objects moving along curved paths.
- Engineering: Designing curves and surfaces with specific properties, ensuring smooth transitions.
- Optimization: Finding maximum or minimum points on a curve by analyzing tangent lines.
- Computer Graphics: Rendering curves and surfaces with accurate tangent and normal vectors for lighting and shading.

Practice Problems to Reinforce Concepts

1. Find the equation of the tangent and normal lines to the curve $y=x^2 + 3x$ at the point $(2, 10)$.
2. Determine the tangent and normal lines to $y=4x^3 - 2x$ at $x=1$.
3. For the curve $y=\sin x$, find the tangent and normal lines at $x=\pi/4$.

Practicing these problems will deepen your understanding of derivatives, slopes, and line equations related to curves.

Conclusion

In this detailed exploration, we have successfully determined the tangent and normal lines to the curve $y=2x^3$ at the point $(1, 2)$. The tangent line, which touches the curve at a single point and shares the slope, is given by $y=6x-4$. The normal line, perpendicular to the tangent, is described by $y=-\frac{1}{6}x + \frac{13}{6}$.

Mastering the calculation of these lines enhances your understanding of the local behavior of functions and their geometric interpretations. Whether you're studying calculus fundamentals or applying these concepts in advanced fields, these skills are essential tools in your mathematical toolkit.

Keywords for SEO Optimization:

- Tangent line to curve
- Normal line to curve
- Derivative of $y=2x^3$
- Slope of tangent and normal

- Equation of tangent and normal lines
- Calculus tangent line example
- Geometry of curves
- Differential calculus applications
- Analytical geometry
- Curve analysis at a point

Frequently Asked Questions

How do you find the equation of the tangent line to the curve y = 2x^3 at the point (1, 2)?

First, compute the derivative y' = 6x^2. At x=1, y' = 6(1)^2 = 6. The slope of the tangent line is 6. Using point-slope form: y - 2 = 6(x - 1), so the tangent line is y = 6x - 4.

What is the slope of the normal line to the curve y=2x^3 at the point (1, 2)?

The slope of the tangent line at x=1 is 6. The normal line's slope is the negative reciprocal, which is -1/6.

How do you derive the equation of the normal line to y=2x^3 at (1, 2)?

Using the point-slope form with the normal slope -1/6: y - 2 = -1/6(x - 1). Simplifying, the normal line is y = -1/6 x + 13/6.

What is the equation of the tangent line to y=2x^3 at the point (1, 2)?

The tangent line is y = 6x - 4.

What is the equation of the normal line to y=2x^3 at (1, 2)?

The normal line is y = -1/6 x + 13/6.

Why is the derivative y' = 6x^2 important for finding tangent and normal lines?

Because the derivative gives the slope of the tangent line at any point x, which is essential for writing the tangent line's equation. The normal line's slope is the negative reciprocal of this derivative.

Can the tangent and normal lines intersect at points other

than (1, 2)?

No, tangent and normal lines are specific to a point on the curve. They intersect the curve only at their point of tangency (here, at (1, 2)).

Additional Resources

Find The Lines That Are (a) Tangent And (b) Normal To The Curve $y = 2x^3$ At The Point (1, 2)

Introduction

In calculus, understanding the relationships between curves and lines is fundamental. Among these relationships, tangent and normal lines provide critical insights into the behavior and properties of functions at specific points. Specifically, for the curve $y = 2x^3$, finding the tangent and normal lines at a given point reveals the slope of the curve at that point and the perpendicular direction to the curve’s tangent, respectively.

This comprehensive review explores the process of calculating these lines step-by-step, covering the necessary concepts, methods, and interpretations. We will delve into the derivatives, the point of interest, and the equations of the tangent and normal lines, providing a detailed understanding suitable for students, educators, and enthusiasts eager to master this aspect of differential calculus.

Conceptual Foundations

The Significance of Tangent and Normal Lines

- Tangent Line: A line that "touches" a curve at a particular point, having the same slope as the curve at that point. It represents the instantaneous direction of the curve, giving insights into the function's behavior locally.

- Normal Line: A line perpendicular to the tangent at a given point on the curve. It often plays a role in geometrical interpretations, such as reflections, shortest distances, and curvature analysis.

Mathematical Foundations

- The slope of the tangent line at a point on a curve $y = f(x)$ is given by the derivative $f'(x)$.

- The equation of the tangent line at a point (x_0, y_0) is:

$$
y - y_0 = m_{tangent}(x - x_0)
$$

where $m_{tangent} = f'(x_0)$.

- The slope of the normal line is the negative reciprocal of the tangent slope:

$$
m_{normal} = -\frac{1}{m_{tangent}}
$$

- The equation of the normal line at (x_0, y_0):

$$
y - y_0 = m_{normal}(x - x_0)
$$

Step-by-Step Solution Approach

Step 1: Confirm the Point on the Curve

Given Point:

$$
(1, 2)
$$

Check if this point lies on the curve $y = 2x^3$:

$$
y = 2(1)^3 = 2 \times 1 = 2
$$

Yes, the point $(1, 2)$ is on the curve.

Step 2: Find the Derivative y'

Given:

$$
y = 2x^3
$$

Differentiate with respect to x:

$$
\frac{dy}{dx} = y' = 6x^2
$$

This derivative provides the slope of the tangent line at any point x.

Step 3: Calculate the Slope of the Tangent Line at $x = 1$

Substitute $x = 1$:

\[

$$
m_{tangent} = y'|_{x=1} = 6(1)^2 = 6
$$

So, the tangent line at $(1, 2)$ has a slope of 6.

Step 4: Write the Equation of the Tangent Line

Using the point-slope form:

$$
y - y_0 = m_{tangent}(x - x_0)
$$

Substitute $(x_0, y_0) = (1, 2)$ and $m_{tangent} = 6$:

$$
y - 2 = 6(x - 1)
$$

Simplify:

$$
y - 2 = 6x - 6
$$
$$
y = 6x - 4
$$

Equation of the tangent line:

$$
\boxed{ y = 6x - 4 }
$$

Detailed Analysis of the Tangent Line

- The slope 6 indicates a steep, positively inclined line, reflecting the rapid increase of the curve $y=2x^3$ at $x=1$.

- The tangent line intersects the curve at $(1, 2)$, sharing the same slope locally, which makes it an excellent approximation to the curve near this point.

- In practical applications, tangent lines are vital for linear approximations, optimization problems, and understanding the instantaneous rate of change.

Step 5: Find the Slope of the Normal Line

Since the normal line is perpendicular to the tangent:

$$
m_{normal} = -\frac{1}{m_{tangent}} = -\frac{1}{6}
$$

Normal slope:

$$
\boxed{ m_{normal} = -\frac{1}{6} }
$$

Step 6: Write the Equation of the Normal Line

Using the point-slope form with $(1, 2)$:

$$
y - 2 = -\frac{1}{6}(x - 1)
$$

Simplify:

$$
y - 2 = -\frac{1}{6}x + \frac{1}{6}
$$

$$
y = -\frac{1}{6}x + \frac{1}{6} + 2
$$

Express 2 as $\frac{12}{6}$:

$$
y = -\frac{1}{6}x + \frac{1}{6} + \frac{12}{6} = -\frac{1}{6}x + \frac{13}{6}
$$

Equation of the normal line:

$$
\boxed{ y = -\frac{1}{6}x + \frac{13}{6} }
$$

Deep Dive: Geometric and Analytical Interpretations

Geometrical Significance of the Tangent and Normal Lines at $(1, 2)$

- The tangent line touches the curve precisely at $(1, 2)$ and has the same slope as the curve at that point, representing the direction in which the curve is heading immediately at $x=1$.

- The normal line, perpendicular to the tangent, indicates the direction orthogonal to the curve's instantaneous direction. It is especially useful in physics (normal force calculations), optimization (perpendicularity constraints), and differential geometry.

Visualizing the Lines and Curve

- Plotting the curve $y=2x^3$, the tangent line $y=6x-4$, and the normal line $y=-\frac{1}{6}x + \frac{13}{6}$ reveals their relative positions and slopes.

- The steep positive slope of the tangent reflects the increasing nature of the cubic function at $x=1$.

- The shallow negative slope of the normal indicates a gentle decrease in y as x increases, orthogonal to the steep tangent.

Curvature and Rate of Change

- The second derivative $y'' = 12x$ indicates the concavity and how the slope of the tangent line changes with x.

- At $x=1$, $y'' = 12$, implying the curve is concave upward and the slope of the tangent is increasing at this point.

Additional Insights and Applications

Use in Approximation and Linearization

- The tangent line serves as the best linear approximation of the function near $x=1$. For small deviations Δx, the change in y can be approximated by:

$$
\Delta y \approx m_{tangent} \Delta x
$$

- This technique is fundamental in differential calculus, simplifying complex functions locally.

Normal Line in Optimization and Geometry

- The normal line's equation is crucial in geometric constructions, such as finding the shortest distance from a point to a curve or constructing normals for reflection problems.

- In physics, normal lines often represent directions perpendicular to surfaces or trajectories, essential in collision mechanics and optical reflections.

Summary of Key Results

Aspect	Result	Equation / Value
Point on curve	$(1, 2)$	Given point
Derivative y'	$6x^2$	General form
Slope of tangent at $x=1$	6	$y'\|_{x=1}$
Equation of tangent line	$y=6x - 4$	Derived using point-slope form
Slope of normal line	$-\frac{1}{6}$	Negative reciprocal
Equation of normal line	$y = -\frac{1}{6}x + \frac{13}{6}$	Derived similarly

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Final Remarks

Understanding how to derive tangent and normal lines to a curve at a specific point is a cornerstone of differential calculus. These lines not only provide local linear approximations but also reveal the geometric and physical properties of the function. In the case of $y=2x^3$, at $(1, 2)$, the tangent and normal lines

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