

Find The Number Of Unique Permutations Of The Word ALGEBRA.a) 5,040b) 1,260c) 720d) 2,520

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Permutations are fundamental concepts in combinatorics, representing the different ways in which a set of items can be arranged or ordered. When dealing with words or sequences that contain repeated letters, calculating the number of unique permutations becomes a slightly more complex task. The word "ALGEBRA" presents an interesting case because it contains both unique and repeated characters. Understanding how to determine the number of distinct arrangements of such words is a vital skill in mathematics, especially in fields like probability, statistics, and computer science. In this article, we will explore how to find the number of unique permutations of the word "ALGEBRA," analyze the options provided, and learn the step-by-step process involved in such calculations.

Understanding Permutations and Repetitions

What Are Permutations?

Permutations refer to the different arrangements or orderings of a set of objects. For example, given the set {A, B, C}, the permutations include ABC, ACB, BAC, BCA, CAB, and CBA – a total of 6 arrangements.

Permutations with Repeated Elements

When some elements are repeated within the set, the total number of permutations decreases because swapping identical elements does not produce a new unique arrangement. To account for this, we use a modified formula that divides the total permutations by the factorials of the counts of each repeated element.

Analyzing the Word "ALGEBRA"

Letter Composition

Let's examine the composition of the word "ALGEBRA":

- Total letters: 7
- Letters and their counts:
 - A: 2 times
 - L: 1 time
 - G: 1 time
 - E: 1 time
 - B: 1 time
 - R: 1 time

This composition shows that only the letter "A" is repeated, appearing twice, while all other letters are unique.

Implication for Permutation Calculation

Since most of the letters are unique, the total number of permutations will mainly be influenced by the repetition of "A."

Calculating the Number of Unique Permutations

Applying the Permutation Formula

The general formula for the number of permutations of a multiset is:

$$\text{Number of permutations} = \frac{n!}{n_1! \times n_2! \times \dots \times n_k!}$$

Where:

- n is the total number of items
- $n_1, n_2, \dots, n_k$ are the counts of each repeated item

For "ALGEBRA," the calculation becomes:

$$\frac{7!}{2!}$$

since only "A" repeats twice, and all other letters are unique.

Calculating Factorials

Recall that:

- $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$
- $2! = 2 \times 1 = 2$

Therefore:

$$\text{Number of unique permutations} = \frac{5,040}{2} = 2,520$$

This means there are 2,520 unique arrangements of the letters in "ALGEBRA," accounting for the repeated "A."

Matching the Result with Given Options

The options provided are:

- a) 5,040
- b) 1,260
- c) 720
- d) 2,520

Our calculation yields 2,520, which corresponds to option d).

Additional Insights and Related Concepts

Permutations of Words with Multiple Repeated Letters

In cases where more than one letter repeats, the denominator in the permutation formula includes the factorials of each repeated count. For example, if a word has two letters each repeated twice, the total permutations would involve dividing by $(2! \times 2!)$.

Examples of Similar Problems

- Permutations of the word "BALLOON" (which has repeated "L" and "O")
- Arrangements of "MISSISSIPPI" with multiple repeated letters
- Permutations of a set with distinct elements versus repeated elements

Importance in Real-World Applications

Understanding permutations with repetitions has applications in:

- Cryptography
- Probability calculations
- Arranging seating or scheduling
- Generating possible combinations in games and puzzles

Summary and Key Takeaways

- To find the number of unique permutations of a word with repeated letters, identify the total number of letters and the counts of each repeated letter.
- Use the formula: $\frac{n!}{n_1! \times n_2! \times \dots}$
- For "ALGEBRA," with 7 letters and "A" repeated twice, the total unique permutations are $\frac{7!}{2!} = 2,520$.
- The correct answer from the options is d) 2,520.

Conclusion

Calculating the number of unique permutations of a word like "ALGEBRA" involves understanding the principles of permutations and accounting for repeated elements. By applying factorial calculations and the permutation formula, we determine that there are 2,520 distinct arrangements. This process not only helps in solving specific problems but also reinforces foundational concepts in combinatorics, essential for various mathematical and real-world scenarios.

References:

- Combinatorics and Permutations (Mathematics textbooks)
- Permutation formulas and factorial concepts
- Practice problems on permutations with repetitions

Frequently Asked Questions

What is the total number of unique permutations of the word 'ALGEBRA'?

The total number of unique permutations is 5,040.

Why are the permutations of 'ALGEBRA' considered unique?

Because the word contains the letter 'A' twice and the other letters are distinct, so permutations are counted based on letter arrangements considering identical letters.

How do we calculate the number of unique permutations for words with repeated letters?

We use the formula: total permutations = $n!$ divided by the factorials of the counts of each repeated letter. For 'ALGEBRA', it's $7! / 2!$ because 'A' appears twice.

What is the factorial of 7 (7!) used in the calculation?

$7!$ equals 5,040.

Which option correctly represents the number of unique permutations for 'ALGEBRA'?

Option a) 5,040 is correct.

Could the number of permutations be 720 for 'ALGEBRA'?

No, 720 would be the number for permutations of a different set of letters with more repetitions, but for 'ALGEBRA', it is 5,040.

How does the presence of two 'A's affect the total permutations of 'ALGEBRA'?

It reduces the total number of permutations from $7!$ to $7! / 2! = 5,040$ because the two 'A's are indistinguishable.

What is the significance of understanding permutations in words like 'ALGEBRA'?

Understanding permutations helps in counting arrangements where order matters, especially when dealing with repeated elements, which is useful in combinatorics and probability problems.

Additional Resources

Find The Number Of Unique Permutations Of The Word ALGEBRA

Introduction

Permutation problems are a cornerstone of combinatorial mathematics, offering insight into how elements can be arranged under various constraints. Among

these, the permutation of words with repeated letters presents a fascinating challenge, as the presence of identical characters reduces the total number of unique arrangements.

The word ALGEBRA is an intriguing example – it is a six-letter word with a mix of unique and repeated characters. In this article, we will explore in detail how to determine the number of unique permutations of ALGEBRA, analyze the problem systematically, and understand the principles that underpin such calculations.

By the end, we aim to establish definitively which of the multiple-choice options provided—5,040, 1,260, 720, or 2,520—is correct, supported by a thorough mathematical investigation.

Understanding the Problem

Before diving into calculations, it's essential to clarify what is meant by the "number of unique permutations" of a word like ALGEBRA.

- Permutation: An arrangement of all the elements (letters) in a specific order.
- Unique permutation: A permutation that differs from others in the sequence of letters, considering identical letters as indistinguishable.

Step 1: Analyzing the Composition of the Word

The word ALGEBRA consists of the following letters:

Letter	Count
A	2
L	1
G	1
E	1
B	1
R	1

Total number of letters: 7

However, in the options provided, the numbers correspond to arrangements of 6, not 7, letters. This suggests a need to verify the spelling carefully.

Is the word "ALGEBRA" a 6-letter word or 7-letter word?

- "ALGEBRA" has 7 letters: A, L, G, E, B, R, A

So, the total length is 7, with the letter A occurring twice.

Step 2: Confirming the Letter Counts

Indeed, ALGEBRA has:

- A: 2 times
- L: 1
- G: 1
- E: 1
- B: 1
- R: 1

Total letters: 7

This raises a question: the options provided (5,040; 1,260; 720; 2,520) are common permutation numbers for 6-letter words. Since the total is 7, we need to reconcile this.

Possible explanation: The question might be considering permutations of the 6-letter word "ALGEBRA" excluding one letter, or perhaps focusing on arrangements of the 6 distinct letters.

Alternatively, the options suggest arrangements of 6 letters, which makes sense because:

- The total number of arrangements of 7 letters with duplicates is:

$$\frac{7!}{2!} = \frac{5040}{2} = 2520$$

which matches option (d).

Therefore, the total arrangements of ALGEBRA are 2520.

Step 3: Formal Calculation of Permutations

Let's formalize the calculation:

Given the word ALGEBRA with:

- 7 total letters
- Letter A repeats twice

The general formula for permutations of n objects with duplicates is:

$$\frac{n!}{n_1! \times n_2! \times \dots}$$

\]

where:

- $(n!)$ is the factorial of total number of items
- $(n_1!, n_2!, \dots)$ are factorials of counts of identical items

Applying this to ALGEBRA:

$$\frac{7!}{2!} = \frac{5040}{2} = 2520$$

This indicates 2520 unique arrangements.

Step 4: Verifying Against the Multiple Choice Options

The options given are:

- a) 5,040
- b) 1,260
- c) 720
- d) 2,520

Our calculation shows 2520, which matches option (d).

Deep Dive: Why Are the Other Options Not Correct?

- 5,040: This is $7!$, the total permutations of 7 distinct elements, which would be correct if all letters were unique. But since A repeats twice, this overcounts arrangements.
- 1,260: This is $7! / (2! 2!)$ – representing permutations with two pairs of duplicate letters, but we only have one duplicate letter, so this is not applicable.
- 720: This equals $6!$, which would be the total permutations of six unique letters, perhaps if one letter is omitted.
- 2,520: As established, matches the calculation for permutations with one duplicate letter.

Additional Considerations: Permutations of Subsets

If the question were about permutations of subsets of ALGEBRA (say,

arrangements of 6 letters), then the calculations would change accordingly.

- For 6-letter arrangements with one duplicate, the total would be:

$$\frac{6!}{2!} = \frac{720}{2} = 360$$

which is not an option provided, reinforcing that the question is about the full 7-letter arrangements.

Final Conclusion

Based on the thorough analysis, the total number of distinct permutations of the word "ALGEBRA"—accounting for the repeated letter A—is:

$$\boxed{2520}$$

which corresponds to option (d).

Summary

- Word analyzed: ALGEBRA
- Letters involved: 7 total, with A repeated twice.
- Permutation formula: $\frac{7!}{2!}$
- Calculated permutations: 2520
- Correct option: (d) 2,520

This investigation underscores the importance of understanding element counts and the impact of duplicates on permutation calculations. Such problems are fundamental in combinatorics, with applications spanning cryptography, algorithm design, and probability theory.

References

- Combinatorics and Permutation Theory, by Richard P. Stanley.
- Discrete Mathematics and Its Applications, by Kenneth H. Rosen.
- Online calculator tools for factorial and permutation calculations.

Final Remarks

The exercise of determining the number of unique permutations of a word like ALGEBRA demonstrates core principles of combinatorics in action. It highlights the necessity of careful analysis of letter counts and the application of factorial formulas. Recognizing the presence of duplicate elements is key to arriving at the correct count, as showcased by the alignment with option (d).

Understanding these concepts not only enhances problem-solving skills but also provides foundational knowledge applicable in various scientific and engineering fields.

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