Find The Vector Whose Magnitude Is 5 And Which Is In The Direction Of The Vector 4i -3j +k

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Understanding vector concepts is fundamental in physics, engineering, and mathematics. When dealing with vectors, it's common to encounter problems where you need to find a specific vector that meets certain criteria, such as having a particular magnitude and pointing in a specific direction. In this article, we will explore how to find a vector whose magnitude is 5 and which is in the direction of the vector 4i - 3j + k. We will break down the problem step-by-step, explain the necessary concepts, and provide a comprehensive guide to solving such vector problems.

Introduction to Vectors and Their Properties

What Is a Vector?

A vector is a mathematical quantity that has both magnitude and direction. It is usually represented in component form as:

$$ \mathbf{v} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k} $$

where:

- a_1, a_2, a_3 are the components along the x, y, and z axes respectively.
- $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors in the x, y, and z directions.

Magnitude of a Vector

The magnitude (or length) of a vector $\mathbf{v} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ is given by:

$$ |\mathbf{v}| = \sqrt{a_1^2 + a_2^2 + a_3^2} $$

This scalar value indicates how long the vector is, regardless of its direction.

Direction of a Vector

The direction of a vector can be described by its unit vector, which points in the same direction but has a magnitude of 1. The unit vector $\hat{\mathbf{u}}$ in the direction of $\mathbf{v}$ is:

$$ \hat{\mathbf{u}} = \frac{\mathbf{v}}{|\mathbf{v}|} $$

This concept is crucial when we want to find a vector with a specific magnitude in a given direction.

Problem Breakdown and Approach

The Given Vector

The vector provided is:

$$ \mathbf{A} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k} $$

Our goal is to find a vector $ \mathbf{v} $ such that:

1. It has a magnitude of 5.
2. It is in the direction of $ \mathbf{A} $, i.e., it points in the same or exactly opposite direction.

Step-by-Step Solution Plan

To find such a vector, we will:

- Find the unit vector in the direction of $ \mathbf{A} $.
- Multiply this unit vector by the desired magnitude (5) to get the required vector.

This process involves:

1. Computing the magnitude of $ \mathbf{A} $.
2. Calculating the unit vector $ \hat{\mathbf{u}} $ in the direction of $ \mathbf{A} $.
3. Scaling the unit vector by 5 to get the vector $ \mathbf{v} $.

Let's proceed through each step in detail.

Calculating the Magnitude of the Given Vector

Given $ \mathbf{A} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k} $:

$$ |\mathbf{A}| = \sqrt{(4)^2 + (-3)^2 + (1)^2} = \sqrt{16 + 9 + 1} = \sqrt{26} $$

Therefore,

$$ |\mathbf{A}| = \sqrt{26} \approx 5.099 $$

Finding the Unit Vector in the Direction of $ \mathbf{A} $

The unit vector $ \hat{\mathbf{u}} $ in the direction of $ \mathbf{A} $ is:

$$ \hat{\mathbf{u}} = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{4\mathbf{i} - 3\mathbf{j} + \mathbf{k}}{\sqrt{26}} $$

Expressed component-wise:

$$ \hat{\mathbf{u}} = \left(\frac{4}{\sqrt{26}} \right) \mathbf{i} + \left(\frac{-3}{\sqrt{26}} \right) \mathbf{j} + \left(\frac{1}{\sqrt{26}} \right) \mathbf{k} $$

This unit vector points in the same direction as $\mathbf{A}$ and has a magnitude of 1.

Scaling the Unit Vector to the Desired Magnitude

Since the target vector $\mathbf{v}$ must have a magnitude of 5, and must be in the direction of $\mathbf{A}$, we scale the unit vector by 5:

$$\mathbf{v} = 5 \times \hat{\mathbf{u}}$$

Therefore,

$$\mathbf{v} = 5 \times \left(\frac{4}{\sqrt{26}} \mathbf{i} - \frac{3}{\sqrt{26}} \mathbf{j} + \frac{1}{\sqrt{26}} \mathbf{k} \right)$$

Simplify:

$$\mathbf{v} = \left(\frac{20}{\sqrt{26}} \right) \mathbf{i} - \left(\frac{15}{\sqrt{26}} \right) \mathbf{j} + \left(\frac{5}{\sqrt{26}} \right) \mathbf{k}$$

This vector has the specified magnitude and points in the same direction as $4i - 3j + k$.

Final Expression of the Required Vector

The vector with magnitude 5 in the direction of $4i - 3j + k$ is:

$$\boxed{
\mathbf{v} = \frac{20}{\sqrt{26}} \mathbf{i} - \frac{15}{\sqrt{26}} \mathbf{j} + \frac{5}{\sqrt{26}} \mathbf{k}
}$$

Alternatively, written in component form:

$$\mathbf{v} = \left(\frac{20}{\sqrt{26}}, \; -\frac{15}{\sqrt{26}}, \; \frac{5}{\sqrt{26}} \right)$$

Additional Insights and Variations

Direction Opposite to $\mathbf{A}$

If instead, the problem asked for a vector of magnitude 5 opposite to the direction of $\mathbf{A}$, we would multiply the unit vector by -5:

$$\mathbf{v}_{\text{opposite}} = - \frac{20}{\sqrt{26}} \mathbf{i} + \frac{15}{\sqrt{26}} \mathbf{j} - \frac{5}{\sqrt{26}} \mathbf{k}$$

Generalization to Any Vector and Magnitude

This method applies universally:

- Find the magnitude of the original vector.
- Compute the unit vector.

- Scale by the desired magnitude.

This process is fundamental in vector analysis and has applications across physics, computer graphics, and engineering.

Applications of Finding Vectors in a Given Direction and Magnitude

Understanding how to find vectors with specific properties is crucial in various fields:

- Physics: Calculating force vectors, velocity vectors, or displacement vectors in the desired directions.
- Engineering: Designing components with precise directional properties.
- Computer Graphics: Defining directions and magnitudes for motion, lighting, and shading.
- Navigation: Determining course vectors with specific speeds and directions.

Common Mistakes to Avoid

When solving such vector problems, be cautious of:

- Incorrect magnitude calculations: Always double-check the square root computations.
- Sign errors: Remember that the direction can be the same or opposite; ensure the sign is correctly applied.
- Component misplacement: Keep track of which components correspond to which axes.
- Unit vector normalization: Confirm that the unit vector truly has a magnitude of 1 before scaling.

Summary

To find a vector of a specific magnitude in a given direction, follow these steps:

1. Calculate the magnitude of the given vector.
2. Derive the unit vector by dividing each component by the magnitude.
3. Multiply the unit vector by the desired magnitude to get the required vector.

In this case, the vector of magnitude 5 in the same direction as $4i - 3j + k$ is:

$$\boxed{
\mathbf{v} = \frac{20}{\sqrt{26}} i - \frac{15}{\sqrt{26}} j + \frac{5}{\sqrt{26}} k
}$$

This comprehensive approach ensures accuracy and clarity when solving similar vector problems.

SEO Keywords: vector magnitude, find vector in direction, vector problem solution, unit vector, scalar multiplication, vector components, physics vectors, engineering vectors, mathematical vector analysis

Frequently Asked Questions

What is the vector with magnitude 5 in the direction of the vector 4i - 3j + k?

The vector is obtained by first finding the unit vector in the given direction and then multiplying it by 5. The magnitude of the given vector is $\sqrt{4^2 + (-3)^2 + 1^2} = \sqrt{16 + 9 + 1} = \sqrt{26}$. The unit vector is $(4/\sqrt{26})i + (-3/\sqrt{26})j + (1/\sqrt{26})k$. Multiplying by 5 gives the vector: $(20/\sqrt{26})i - (15/\sqrt{26})j + (5/\sqrt{26})k$.

How do you calculate a vector with a specific magnitude in a given direction?

To find a vector with a specific magnitude in a particular direction, first determine the unit vector in that direction by dividing the vector by its magnitude. Then, multiply the unit vector by the desired magnitude to get the required vector.

What is the process to find a vector of magnitude 5 aligned with 4i - 3j + k?

Calculate the magnitude of the original vector, find its unit vector by dividing each component by this magnitude, and then multiply each component of the unit vector by 5 to achieve the desired magnitude.

Can you provide the numeric components of the vector with magnitude 5 in the direction of 4i - 3j + k?

Yes. The vector components are approximately $(4.92)i - (3.69)j + (1.16)k$, derived from multiplying the unit vector $(4/\sqrt{26}, -3/\sqrt{26}, 1/\sqrt{26})$ by 5.

Why is it necessary to normalize the direction vector before scaling to a specific magnitude?

Normalizing the direction vector ensures it has a magnitude of 1, so scaling it by the desired magnitude (5 in this case) produces a vector that points in the same direction but has the exact length required.

What is the significance of finding such a vector in physics or engineering applications?

Finding a vector with a specific magnitude in a given direction is essential in applications like force resolution, motion analysis, and vector projection, where precise magnitude and direction control are required.

Additional Resources

Find The Vector Whose Magnitude Is 5 And Which Is In The Direction Of The Vector 4i - 3j + k

Understanding vector concepts is fundamental in the study of physics, engineering, and mathematics. One common problem students encounter involves finding a specific vector that has a given magnitude and points in a certain direction. In particular, the problem "Find the vector whose magnitude is 5 and which is in the direction of the vector 4i - 3j + k" is a classic example that demonstrates the application of vector normalization and scalar multiplication. This task requires understanding the basic properties of vectors, how to find unit vectors, and how to scale them appropriately. In this comprehensive review, we will explore the step-by-step process to solve this problem, analyze its mathematical concepts, and discuss its broader significance.

Understanding the Problem: Key Concepts and Definitions

Before diving into the solution, it’s essential to comprehend the fundamental concepts involved.

Vectors and Their Components

A vector is a quantity that has both magnitude and direction. In Cartesian coordinates, vectors are represented as an ordered triplet (or components) along the i, j, and k axes:
- i component (x-axis)
- j component (y-axis)
- k component (z-axis)

For example, the vector A = 4i - 3j + k has components:
- x = 4
- y = -3
- z = 1

Magnitude of a Vector

The magnitude (or length) of a vector A = (a_1, a_2, a_3) is calculated using the Euclidean norm:
\[|\mathbf{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \]

Direction of a Vector

The direction of a vector is indicated by its orientation in space. To find a vector in the same direction but with a different magnitude, we use the concept of a unit vector—a vector with magnitude 1 pointing in the same direction.

Step-by-Step Solution to the Problem

The problem asks us to find a vector, say V, that:
- Has a magnitude of 5.
- Is in the same direction as the vector $A = 4i - 3j + k$.

The solution involves the following steps:

1. Find the magnitude of the given vector A

Calculate the magnitude of A:

$$
|\mathbf{A}| = \sqrt{4^2 + (-3)^2 + 1^2} = \sqrt{16 + 9 + 1} = \sqrt{26}
$$

2. Calculate the unit vector in the direction of A

The unit vector $\mathbf{u}$ in the direction of A is:

$$
\mathbf{u} = \frac{\mathbf{A}}{|\mathbf{A}|} = \left(\frac{4}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right)
$$

This vector has a magnitude of 1 and points in the same direction as A.

3. Scale the unit vector to the desired magnitude

Since we want V to have a magnitude of 5, we multiply the unit vector by 5:

$$
\mathbf{V} = 5 \times \mathbf{u} = \left(5 \times \frac{4}{\sqrt{26}}, 5 \times \frac{-3}{\sqrt{26}}, 5 \times \frac{1}{\sqrt{26}} \right)
$$

$$
\mathbf{V} = \left(\frac{20}{\sqrt{26}}, \frac{-15}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)
$$

This is the required vector with magnitude 5 in the same direction as the original vector.

Mathematical Significance and Broader Applications

Understanding how to find a vector with a specified magnitude in a given direction is fundamental across various fields.

Applications in Physics

- Force vectors: Decomposing forces into components and scaling them to match the required magnitude.
- Velocity vectors: Adjusting direction and speed for trajectory calculations.
- Displacement: Calculating movement in specific directions with precise magnitudes.

Applications in Engineering

- Structural analysis: Applying forces with specific directions and magnitudes.
- Robotics: Programming movements along vectors with controlled magnitude and direction.
- Navigation: Computing course corrections that involve vector scaling.

Mathematical and Computational Significance

- Serves as a basis for vector normalization algorithms.
- Essential for vector calculus operations, including dot and cross products.
- Useful in computer graphics for scaling and orienting objects in 3D space.

Features, Pros, and Cons of the Approach

Understanding the process of finding a scaled vector in a specific direction has several advantages and some limitations.

Features

- Clear methodology: Uses fundamental vector operations—magnitude calculation, normalization, and scalar multiplication.
- Universality: Applicable across multiple disciplines involving vectors.
- Precision: Provides exact scaled vectors with desired magnitudes.

Pros

- Facilitates precise control over vector quantities.
- Enhances understanding of vector directions and magnitudes.
- Serves as a foundation for more complex vector operations.

Cons

- Requires familiarity with vector algebra.
- Can become computationally intensive for complex vectors or higher dimensions.
- Assumes the vector is non-zero (to avoid division by zero during normalization).

Alternate Methods and Variations

While the above approach is straightforward, there are alternative methods and considerations:

Using Trigonometry

If the vector components are known in terms of angles, trigonometric functions can help find the scaled vector directly.

Numerical Approximation

For computational efficiency, approximate values of $\sqrt{26}$ can be used:

$$
\sqrt{26} \approx 5.099
$$

Thus, the vector can be approximated as:

$$
\left(\frac{20}{5.099}, \frac{-15}{5.099}, \frac{5}{5.099} \right) \approx (3.924, -2.942, 0.981)
$$

Generalization to Any Vector

The same process applies to any vector A:
- Find its magnitude.
- Normalize it to get the unit vector.
- Multiply by the desired magnitude.

Conclusion and Final Remarks

The process of finding a vector with a specific magnitude in a given direction exemplifies fundamental vector operations crucial in multiple scientific and engineering applications. The key steps involve calculating the magnitude, normalizing the original vector to find its direction, and then scaling this unit vector to the desired magnitude. The final vector obtained:
\[
\boxed{
\left(\frac{20}{\sqrt{26}}, \frac{-15}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)
}
\]
has the precise magnitude of 5 and points in the same direction as the original vector 4i - 3j + k.

Mastering this technique enhances one’s ability to manipulate vectors effectively, which is essential for problem-solving in physics, engineering, and computer graphics. Whether dealing with forces, velocities, or displacements, understanding how to scale vectors while preserving their directionality is a vital skill. As you continue exploring vector algebra, this foundational concept will serve as a stepping stone toward mastering more complex vector operations and their applications across various domains.

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