Find Two Vectors In Opposite Directions That Are Orthogonal To The Vector U.u = 1/4 I - 4/5j

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Introduction

In the realm of vector mathematics, understanding the relationships between vectors—including orthogonality and directionality—is fundamental to numerous applications across physics, engineering, computer graphics, and data science. The problem of finding vectors that are both orthogonal to a given vector and point in opposite directions is a common exercise that tests comprehension of vector operations, dot products, and geometric interpretation.

Specifically, the task at hand involves a complex vector $U.u = \frac{1}{4} I - \frac{4}{5} j$. This vector comprises components along the imaginary unit j, which is often used in engineering and physics to represent complex numbers or signals in the imaginary axis. The challenge is to find two vectors that are:

1. Opposite in direction to each other,
2. Both orthogonal to the given vector $U.u$,
3. And, by extension, understanding their geometric and algebraic properties.

This problem is not only academically stimulating but also practically relevant in fields like signal processing, quantum mechanics, and control systems, where orthogonal vectors and their directions play crucial roles in system design and analysis.

Understanding the Given Vector $U.u = \frac{1}{4} I - \frac{4}{5} j$

Components of the Vector

The provided vector appears to be expressed in a complex form involving real and imaginary components:

- $\frac{1}{4} I$: This term could represent a real component along some basis I, possibly indicating the identity or real axis.
- $- \frac{4}{5} j$: This term indicates an imaginary component along the imaginary axis j.

Clarifying the Vector Components

For clarity, suppose the vector $U.u$ exists in a 2D complex plane (or complex vector space), where:

$$
U.u = \left(\frac{1}{4} \right) I + \left(- \frac{4}{5} \right) j
$$

This can be interpreted as a vector with:

- Real part: $\frac{1}{4}$,
- Imaginary part: $- \frac{4}{5}$.

Expressed in Cartesian coordinates, it becomes:

$$
U.u = \left(\frac{1}{4}, - \frac{4}{5} \right)
$$

which simplifies to:

$$
U.u = (0.25, -0.8)
$$

The Goal: Finding Opposite and Orthogonal Vectors

What Does Opposite Direction Mean?

Two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are in opposite directions if:

$$
\mathbf{v}_2 = -k \mathbf{v}_1,
$$

where $k > 0$. Generally, if two vectors are exact opposites, then:

$$
\mathbf{v}_2 = - \mathbf{v}_1,
$$

meaning they have the same magnitude but point in exactly opposite directions.

Orthogonality Condition

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if their dot product equals zero:

$$
\mathbf{a} \cdot \mathbf{b} = 0
$$

In 2D, the dot product is:

$$
a_x b_x + a_y b_y = 0
$$

for vectors:

$$
\mathbf{a} = (a_x, a_y), \quad \mathbf{b} = (b_x, b_y).
$$

Step-by-Step Solution Approach

Step 1: Represent the Given Vector in Component Form

From the previous section:

$$
U.u = (0.25, -0.8)
$$

Step 2: Set Up the Orthogonality Conditions

Suppose the two vectors we're seeking are $\mathbf{v}_1$ and $\mathbf{v}_2$, both orthogonal to $U.u$ and in opposite directions.

Let:

$$
\mathbf{v}_1 = (x, y)
$$

Since $\mathbf{v}_2$ is in the opposite direction of $\mathbf{v}_1$:

$$
\mathbf{v}_2 = - \mathbf{v}_1 = (-x, -y)
$$

Now, the orthogonality condition for $\mathbf{v}_1$:

$$
U.u \cdot \mathbf{v}_1 = 0
$$

which expands to:

$$
(0.25)(x) + (-0.8)(y) = 0
$$

or:

$$
0.25x - 0.8 y = 0
$$

Step 3: Solve for y in Terms of x

Rearranged:

$$
0.25 x = 0.8 y
$$

$$
y = \frac{0.25}{0.8} x
$$

Simplify:

$$
y = \frac{0.25}{0.8} x = \frac{25/100}{80/100} x = \frac{25}{80} x = \frac{5}{16} x
$$

Thus, the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ can be expressed as:

$$
\mathbf{v}_1 = (x, \frac{5}{16} x)
$$

$$
\mathbf{v}_2 = (-x, - \frac{5}{16} x)
$$

Where $x \neq 0$ (non-zero vectors).

Step 4: Determine Specific Vectors

Choose a convenient value for x to generate specific vectors. For simplicity, set:

$$
x = 16
$$

then:

$$
y = \frac{5}{16} \times 16 = 5
$$

Hence, the vectors are:

$$
\boxed{
\mathbf{v}_1 = (16, 5)
}
$$

$$
\mathbf{v}_2 = (-16, -5)
$$

Additional Considerations: Magnitude and Normalization

Magnitude of the Vectors

Calculate the magnitude of $\mathbf{v}_1$:

$$
|\mathbf{v}_1| = \sqrt{16^2 + 5^2} = \sqrt{256 + 25} = \sqrt{281} \approx 16.76
$$

Similarly, $\mathbf{v}_2$ has the same magnitude.

Normalized Vectors

To obtain unit vectors in the desired directions:

$$
\hat{\mathbf{v}}_1 = \frac{\mathbf{v}_1}{|\mathbf{v}_1|} = \left(\frac{16}{\sqrt{281}}, \frac{5}{\sqrt{281}} \right)
$$

$$
\hat{\mathbf{v}}_2 = - \hat{\mathbf{v}}_1
$$

Visualizing the Results

Understanding the geometric relationship helps solidify the concepts:

- The original vector $U.u$ points at approximately $(0.25, -0.8)$.
- The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal to $U.u$, meaning they are perpendicular to the original vector.
- These vectors point in opposite directions, satisfying the problem's conditions.

Applications of Finding Orthogonal Opposite Vectors

Signal Processing

In signal processing, orthogonal vectors represent signals that do not interfere with each other. Finding such vectors is crucial in designing filters, error-correcting codes, and in the analysis of complex signals.

Quantum Mechanics

Orthogonal vectors represent mutually exclusive states. Understanding their relationships helps in quantum state analysis, superposition, and measurement.

Control Systems

Designing controllers often involves orthogonal vectors to system dynamics for stability analysis and feedback design.

Computer Graphics

In 2D and 3D graphics, orthogonal vectors are fundamental in defining coordinate systems and transformations.

Summary of the Process

To solve the problem of finding two vectors in opposite directions that are orthogonal to a given vector:

1. Express the given vector in component form.
2. Set the orthogonality condition $\mathbf{v} \cdot U.u = 0$.
3. Express the vectors in terms of a parameter x to satisfy the orthogonality condition.
4. Choose specific values for the parameter to generate concrete vectors.
5. Verify that the vectors are in opposite directions by taking negatives.
6. Calculate magnitudes and normalize if needed for application-specific requirements.

Conclusion

Finding two vectors in opposite directions that are orthogonal to a given vector involves a clear understanding of vector components, dot products, and geometric relationships. By translating the problem into algebraic equations and solving systematically, we can derive explicit vectors that meet the specified conditions. These concepts are widely applicable in various scientific and engineering disciplines, highlighting the importance of vector analysis in practical problem-solving.

Additional Resources

- Vector Algebra and Geometry – Understanding the fundamentals of vectors and their properties.
- Complex Vectors and

Frequently Asked Questions

How can we find two vectors in opposite directions that are orthogonal to a given vector U?

To find two vectors in opposite directions orthogonal to U, first determine a vector orthogonal to U, then take its negative to get the opposite direction. Both will be orthogonal to U and point in opposite directions.

Given U = (1/4, -4/5), how do we verify if a vector V is orthogonal to U?

Calculate the dot product U · V; if the result is zero, then V is orthogonal to U. For example, if U = (1/4, -4/5), find V = (x, y) such that (1/4)x + (-4/5)y = 0.

What is the process to find a vector orthogonal to U = (1/4, -4/5)?

Set up the dot product equation: (1/4)x + (-4/5)y = 0. Solve for y in terms of x, for example, y = (5/4)x. Choosing x = 1 gives V = (1, 5/4); its negative, V' = (-1, -5/4), points in opposite directions.

Are the vectors orthogonal to U = (1/4, -4/5) unique?

No, there are infinitely many vectors orthogonal to U. They form a line in the vector space. Any scalar multiple of a base orthogonal vector is also orthogonal to U.

How do the vectors in opposite directions relate to each other in terms of orthogonality to U?

If one vector is orthogonal to U, its negative is also orthogonal, and they point in opposite directions. Both satisfy the orthogonality condition, maintaining zero dot product with U.

Additional Resources

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Introduction: Deciphering the Challenge of Orthogonality and Opposite Directions

In the realm of linear algebra and vector calculus, understanding the relationships between vectors is fundamental. The task of finding two vectors in opposite directions that are orthogonal to a given vector is both intriguing and instructive, illustrating core principles of vector spaces, dot products, and geometric interpretation. Today, we focus on a particular vector, denoted as U: u = (1/4)I - (4/5)j, and seek to identify two vectors aligned in opposite directions that satisfy the orthogonality condition relative to this vector.

This problem encapsulates a common theme in applied mathematics, physics, and engineering: how to determine vectors that satisfy specific directional and perpendicularity constraints. The solution involves dissecting the properties of the given vector, understanding what it means for vectors to be orthogonal, and then systematically deriving the vectors in question. While the task may seem straightforward, it provides an excellent exploration into the interplay between vector directions and orthogonality, illustrating how these concepts underpin many real-world applications.

Understanding the Given Vector U

Before venturing into the solution, it's essential to interpret the provided vector U accurately. The notation given is:

U: u = (1/4)I - (4/5)j

At face value, this appears to involve vector components along basis vectors I and j. In standard vector notation, I is often used to denote the identity matrix, but in the context of vectors, it may refer to the unit vector along the x-axis, commonly denoted as i. Similarly, j typically denotes the unit vector along the y-axis.

Assuming the standard notation, the vector U can be expressed as:

- Along the x-axis: (1/4) i
- Along the y-axis: - (4/5) j

Thus, U in component form is:

U = (1/4) i - (4/5) j

Expressed as a coordinate vector:

U = (1/4, -4/5)

This interpretation aligns with conventional vector notation, simplifying the problem to a two-dimensional case.

Fundamentals of Orthogonality and Opposite

Directions

To progress, we need a clear understanding of the concepts involved:

Orthogonality

Two vectors are orthogonal if their dot product equals zero. For vectors A and B, this condition is:

$A \cdot B = 0$

In component form:

$A = (a_1, a_2)$, $B = (b_1, b_2)$

then,

$a_1b_1 + a_2b_2 = 0$

Opposite Directions

Vectors are in opposite directions if one is a scalar multiple of the other with a negative scalar, i.e.,

$V = -k W$, where $k > 0$

This means V and W are aligned along the same line but point in opposite ways.

Combining Both Conditions

Our goal is to find two vectors V and -V (opposite directions) that are both orthogonal to U. Since they are in opposite directions, the problem reduces to finding a single vector V that satisfies the orthogonality condition, then considering its negative.

Deriving the Orthogonal Vectors

Given $U = (1/4, -4/5)$, we seek vectors $V = (v_1, v_2)$ such that:

1. $V \cdot U = 0$

2. V and -V are the two vectors in opposite directions.

Step 1: Set Up the Orthogonality Equation

Using the dot product:

$V \cdot U = v_1 (1/4) + v_2 (-4/5) = 0$

Simplify:

$(1/4)\ v_1 - (4/5)\ v_2 = 0$

Step 2: Solve for the Relationship Between Components

Express v_1 in terms of v_2:

$(1/4)\ v_1 = (4/5)\ v_2$

Thus:

$v_1 = (4/5)\ v_2\ (4) = (4/5)\ 4\ v_2 = (16/5)\ v_2$

Alternatively, rearranged:

$v_1 = (16/5)\ v_2$

This linear relationship indicates that any vector V orthogonal to U must satisfy:

$V = (v_1, v_2) = ((16/5)\ v_2, v_2)$

or, factoring out v_2:

$V = v_2\ (16/5, 1)$

Step 3: Define the Two Vectors in Opposite Directions

Since V and -V are in opposite directions and both orthogonal to U, the entire set of solutions can be expressed as scalar multiples of the vector (16/5, 1) and its negative.

Choosing an arbitrary non-zero scalar k, we have:

- First vector: $V_1 = k\ (16/5, 1)$
- Opposite vector: $V_2 = -k\ (16/5, 1)$

These two vectors are in opposite directions, satisfy the orthogonality condition, and are scaled versions of each other.

Practical Examples and Visual Interpretation

To give concrete examples, pick a scalar k. Let's choose k = 1 for simplicity.

- $V_1 = (16/5, 1) \approx (3.2, 1)$
- $V_2 = (-3.2, -1)$

These vectors are:

- Opposite in direction
- Both orthogonal to U

Verification:

Check $V_1 \cdot U$:

$(3.2)(0.25) + (1)(-0.8) = 0.8 - 0.8 = 0$

Similarly, $V_2 \cdot U$:

$(-3.2)(0.25) + (-1)(-0.8) = -0.8 + 0.8 = 0$

Both satisfy the orthogonality condition, confirming the correctness.

Implications and Applications of the Findings

Understanding how to find vectors orthogonal to a given vector, and in opposite directions, has broad implications across various fields:

- Physics: Determining force vectors perpendicular to a velocity vector.
- Engineering: Designing components where orthogonal signals or forces are required.
- Computer Graphics: Calculating normal vectors for surface shading.
- Data Science: Orthogonal vectors in feature spaces to reduce redundancy.

This method of deriving vectors via dot product conditions is fundamental, straightforward, and versatile, illustrating the elegance of linear algebra in solving geometric problems.

Conclusion: Navigating the Vector Space

The process of finding two vectors in opposite directions that are orthogonal to a given vector hinges on understanding the core principles of dot products and vector relationships. Starting with the interpretation of the given vector U, we established the orthogonality condition and derived an explicit form for the vectors satisfying this condition.

Specifically, for $U = (1/4, -4/5)$, the vectors $V = (16/5, 1)$ and $-V = (-16/5, -1)$ are in opposite directions and orthogonal to U. This solution not only demonstrates the practical application of vector algebra but also underscores the importance of geometric intuition in understanding spatial relationships.

Mastering such techniques equips mathematicians, engineers, and scientists with the tools to analyze complex systems, optimize designs, and interpret multidimensional data. As vector spaces underpin much of modern science and technology, developing a deep understanding of their properties continues to be both intellectually rewarding and practically vital.

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