

Give An Example Of A Quadratic Function That Has No Real Root But Has 2 Complex Solutions.

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Understanding quadratic functions is fundamental in algebra and calculus. These functions, typically written in the form $(ax^2 + bx + c)$, are essential for modeling various real-world phenomena such as projectile motion, profit maximization, and area optimization. A key aspect of quadratic functions is their roots or solutions, which are the values of (x) that satisfy the equation $(ax^2 + bx + c = 0)$. These roots can be real or complex, depending on the discriminant $(\Delta = b^2 - 4ac)$. In this comprehensive guide, we will explore an example of a quadratic function that has no real roots but possesses two complex solutions. We will delve into the discriminant's role, analyze the specific function, and discuss the implications of complex roots in mathematical and real-world contexts.

Understanding Quadratic Functions and Roots

What Is a Quadratic Function?

A quadratic function is a second-degree polynomial function of the form:

$$-(y = ax^2 + bx + c)$$

where:

- $(a \neq 0)$,
- (b) and (c) are real numbers.

The graph of a quadratic function is a parabola, opening upwards if $(a > 0)$ and downwards if $(a < 0)$.

Roots of a Quadratic Function

Roots (or zeros) are the points where the parabola intersects the x-axis. These are solutions to the quadratic equation:

$$-(ax^2 + bx + c = 0)$$

The nature of these roots depends on the discriminant:

- Discriminant $(\Delta > 0)$: Two distinct real roots.
- Discriminant $(\Delta = 0)$: One real root (a repeated root).
- Discriminant $(\Delta < 0)$: No real roots, but two complex conjugate solutions.

The Discriminant and Its Significance

Definition of the Discriminant

The discriminant of a quadratic equation $(ax^2 + bx + c = 0)$ is given by:

$$- (\Delta = b^2 - 4ac)$$

This value determines the nature of the roots:

- If $(\Delta > 0)$, roots are real and distinct.
- If $(\Delta = 0)$, roots are real and equal.
- If $(\Delta < 0)$, roots are complex conjugates.

Implications of a Negative Discriminant

When $(\Delta < 0)$, the quadratic has no real solutions because the square root of a negative number is imaginary. The solutions in this case are complex conjugates:

$$- (x = \frac{-b \pm \sqrt{\Delta}}{2a})$$

Expressed explicitly:

$$- (x = \frac{-b}{2a} \pm \frac{\sqrt{-\Delta}}{2a} i)$$

where (i) is the imaginary unit, satisfying $(i^2 = -1)$.

Example of a Quadratic Function with No Real Roots but Two Complex Solutions

Constructing the Example

Let's consider a specific quadratic function:

$$- (y = x^2 + 4x + 8)$$

We'll analyze this function to determine its roots and demonstrate that it has no real roots but two complex solutions.

Calculating the Discriminant

For the quadratic $(x^2 + 4x + 8)$:

- $(a = 1)$
- $(b = 4)$
- $(c = 8)$

Calculate the discriminant:

$$- \Delta = b^2 - 4ac = (4)^2 - 4(1)(8) = 16 - 32 = -16$$

Since $\Delta = -16 < 0$, the quadratic has no real solutions but two complex conjugate solutions.

Finding the Complex Roots

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Substitute the known values:

$$x = \frac{-4 \pm \sqrt{-16}}{2 \times 1}$$

$$x = \frac{-4 \pm 4i}{2}$$

$$x = -2 \pm 2i$$

Thus, the roots are:

$$x = -2 + 2i$$

$$x = -2 - 2i$$

These roots are complex conjugates, reflecting the symmetry of quadratic solutions in the complex plane.

Graphical Interpretation of the Example Function

Plotting $y = x^2 + 4x + 8$

The quadratic opens upward (since $a = 1 > 0$). The vertex of the parabola can be found at:

$$x_{\text{vertex}} = -\frac{b}{2a} = -\frac{4}{2} = -2$$

$$y_{\text{vertex}} = c - \frac{b^2}{4a} = 8 - \frac{16}{4} = 8 - 4 = 4$$

The vertex point is at:

$$(-2, 4)$$

Since the discriminant is negative, the parabola does not touch or cross the x-axis. The parabola is entirely above the x-axis, with its lowest point at $y = 4$.

Implications of the Graph

- No x-intercepts: The parabola does not intersect the x-axis.

- The roots are complex: The solutions are not visible on the real coordinate plane but exist in the complex plane.

- The symmetry about the vertical line $x = -2$: The conjugate roots reflect symmetric points in the complex plane.

Real-World Applications and Significance of Complex Roots

Why Do Complex Roots Matter?

While complex roots may seem abstract, they have practical significance:

- They appear in physical systems, electrical engineering, quantum mechanics, and control systems.
- They help analyze oscillations, stability, and wave phenomena.

Application Examples

- Electrical Engineering: Complex roots describe the impedance of circuits and damping in oscillatory systems.
- Control Theory: The stability of a system depends on the location of roots in the complex plane.
- Physics: Quantum states and wave functions often involve complex solutions.

Mathematical Insights

- Complex roots ensure the quadratic polynomial can be factored over the complex numbers.
- The Fundamental Theorem of Algebra guarantees that every polynomial of degree (n) has exactly (n) roots in the complex plane.

Summary and Key Takeaways

- A quadratic function $(ax^2 + bx + c)$ with a negative discriminant has no real roots but two complex conjugate solutions.
- The example $(y = x^2 + 4x + 8)$ illustrates this, with roots $(-2 \pm 2i)$.
- The discriminant $(\Delta = b^2 - 4ac)$ determines the nature of roots:
- $(\Delta > 0)$: two real roots
- $(\Delta = 0)$: one real root
- $(\Delta < 0)$: two complex roots
- Visualizing the quadratic's graph confirms the absence of x-intercepts.
- Complex roots have practical applications across various scientific and engineering disciplines.

Additional Resources for Further Learning

- Algebra Textbooks: Deepen understanding of quadratic equations and discriminants.
- Online Calculators: Use quadratic equation solvers to explore different functions.

- Mathematical Software: Tools like Desmos, GeoGebra, or WolframAlpha can graph quadratic functions and display roots in the complex plane.
- Educational Videos: Platforms like Khan Academy or YouTube provide visual explanations of complex roots and quadratic functions.

Conclusion

Understanding quadratic functions with no real roots but two complex solutions reveals the depth and richness of algebra. The example $(y = x^2 + 4x + 8)$ demonstrates how the discriminant informs us about the nature of roots and highlights the importance of complex solutions in mathematics and applied sciences. Recognizing and interpreting these roots enhance problem-solving skills and provide a more comprehensive view of polynomial behavior both on and beyond the real number line.

Keywords: quadratic function, complex solutions, no real roots, discriminant, quadratic equation, roots, complex conjugates, quadratic graph, algebra, polynomial solutions

Frequently Asked Questions

What is an example of a quadratic function that has no real roots but has two complex solutions?

An example is $f(x) = x^2 + 4$. Its discriminant is negative ($D = 0^2 - 4 \cdot 1 \cdot 4 = -16$), so it has no real roots but two complex solutions: $x = \pm 2i$.

How can you determine if a quadratic function has no real roots but complex solutions?

By calculating the discriminant ($D = b^2 - 4ac$). If $D < 0$, the quadratic has no real roots but two complex conjugate solutions.

What is the general form of a quadratic function with no real roots?

Any quadratic function where the discriminant is negative, such as $f(x) = ax^2 + bx + c$ with $D < 0$, will have no real roots and two complex solutions.

Can you give a specific quadratic equation with complex solutions but no real roots?

Yes, for example, $f(x) = x^2 + 2x + 5$ has a discriminant of $2^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16$, so it has no real

roots but two complex solutions.

Why do quadratic functions with negative discriminant have complex solutions?

Because the square root of a negative number introduces imaginary components, leading to solutions that are complex conjugates rather than real numbers.

What is the significance of complex solutions in quadratic functions with no real roots?

They indicate that the parabola does not intersect the x-axis, but the solutions still exist in the complex plane, providing a complete understanding of the function's roots.

Additional Resources

Give An Example Of A Quadratic Function That Has No Real Root But Has 2 Complex Solutions

Mathematics often presents us with fascinating phenomena that challenge our intuition and deepen our understanding of the natural world. Among these are quadratic functions, which are fundamental in algebra and calculus, representing parabolas and modeling a wide array of real-world systems. One intriguing aspect of quadratic functions lies in the nature of their roots—specifically, how the solutions to the quadratic equation may be real or complex. In this article, we'll explore an example of a quadratic function that has no real roots but possesses two complex solutions, shedding light on the underlying principles that govern these solutions and their significance in mathematics and beyond.

Understanding Quadratic Functions and Roots

Before delving into specific examples, it's essential to grasp what quadratic functions are and how their roots are determined.

What Is a Quadratic Function?

A quadratic function is a polynomial function of degree two, typically expressed in the standard form:

$$f(x) = ax^2 + bx + c$$

where:

- $a \neq 0$ (the coefficient of x^2)
- b and c are real numbers

The graph of a quadratic function is a parabola, opening upward if $a > 0$ and downward if $a < 0$.

The Roots of a Quadratic Function

The roots (or zeros) of a quadratic function are values of x for which $f(x) = 0$. These roots can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, the expression under the square root, $D = b^2 - 4ac$, known as the discriminant, determines the nature and number of roots:

- If $D > 0$, there are two distinct real roots.
- If $D = 0$, there is exactly one real root (a repeated root).
- If $D < 0$, there are no real roots; instead, there are two complex conjugate roots.

This last case—where $D < 0$ —is especially interesting because, despite lacking real solutions, the quadratic equation still has solutions in the complex number system.

An Example of a Quadratic with No Real Roots but Complex Solutions

Let's consider a specific quadratic function:

$$f(x) = x^2 + 4x + 5$$

This function is straightforward, yet it exemplifies the core concepts we're exploring.

Calculating the Discriminant

Applying the discriminant formula:

$$D = b^2 - 4ac = (4)^2 - 4(1)(5) = 16 - 20 = -4$$

Since $D = -4 < 0$, this quadratic has no real roots. The parabola never touches or crosses the x -axis, meaning the solutions are not real numbers.

Finding the Complex Roots

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Substitute the known values:

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

Recall that $\sqrt{-4} = 2i$, where i is the imaginary unit satisfying $i^2 = -1$.

Thus,

$$x = \frac{-4 \pm 2i}{2}$$

Simplify:

$$x = -2 \pm i$$

These are the two complex conjugate solutions:

$$x = -2 + i \quad \text{and} \quad x = -2 - i$$

Interpreting the Results: The Nature of Complex Roots

The roots $-2 \pm i$ provide more than just solutions to an equation—they reveal the profound connection between algebra and complex analysis.

Complex Roots as Conjugates

When quadratic equations have real coefficients and a negative discriminant, their solutions always come in conjugate pairs. This means:

- If $x = p + qi$ is a root, then $x = p - qi$ is also a root.
- These roots are symmetric with respect to the real axis in the complex plane.

This symmetry has implications in various fields, including signal processing, control systems, and physics, where complex conjugate solutions relate to oscillatory behavior and stability.

Graphical Insight

Since the quadratic's graph is a parabola, and it has no real roots, the parabola lies entirely above or below the x -axis:

- For $f(x) = x^2 + 4x + 5$, the vertex is at:

$$x_v = -\frac{b}{2a} = -\frac{4}{2} = -2$$

Plugging into the function:

$$f(-2) = (-2)^2 + 4(-2) + 5 = 4 - 8 + 5 = 1$$

The vertex is at $(-2, 1)$, which is above the x -axis, confirming that the parabola does not intersect the x -axis.

Broader Significance in Mathematics and Applications

Understanding quadratic functions with complex roots extends beyond pure mathematics, influencing numerous scientific and engineering disciplines.

Why Complex Roots Matter

- Signal Analysis: Complex roots relate to oscillations and damping in electrical circuits and mechanical systems.
- Control Theory: The stability of systems can be determined by the location of roots in the complex plane.
- Quantum Mechanics: Complex solutions often arise in wave functions and probability amplitudes.
- Mathematical Completeness: The Fundamental Theorem of Algebra states that every polynomial has roots in the complex number system, making complex solutions essential for a complete understanding of polynomial equations.

Educational and Theoretical Impacts

Studying quadratic functions with complex roots deepens students' appreciation of:

- The extension of real number concepts to complex numbers
- The importance of the discriminant in classifying solutions
- The geometric interpretation of roots in the complex plane

Conclusion: Embracing the Complex Realm

The quadratic function $f(x) = x^2 + 4x + 5$ exemplifies how equations can defy our initial expectations by lacking real solutions yet possessing a pair of complex conjugate roots. This duality underscores the richness of algebraic structures and the necessity of complex numbers in mathematics. Whether analyzing oscillatory systems, designing stable control mechanisms, or exploring the fundamental properties of polynomials, recognizing and understanding the significance of complex solutions opens pathways to a deeper grasp of the natural and technological worlds. As mathematics continues to evolve, the complex plane remains a vital territory—one that reveals the hidden symmetries and elegant solutions that lie beyond the realm of the real numbers.

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