

How To Find The Slope Intercept Form For The Equation Of The Line Through (5,1) And (3,-7)

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Understanding how to find the equation of a line in slope-intercept form is a fundamental skill in algebra that helps students analyze and graph linear relationships efficiently. When given two points, such as (5,1) and (3,-7), the goal is to determine the line's equation expressed as $y = mx + b$, where m is the slope and b is the y-intercept. This guide provides a comprehensive, step-by-step approach to calculating the slope, deriving the line's equation, and understanding the underlying concepts involved in transforming two points into a slope-intercept form.

What Is The Slope-Intercept Form?

Before diving into calculations, it's essential to understand what the slope-intercept form of a line means and why it's useful.

Definition

The slope-intercept form is a straight-line equation expressed as:

$$y = mx + b$$

Where:

- **m** is the slope of the line, indicating its steepness and direction.
- **b** is the y-intercept, the point where the line crosses the y-axis.

Importance of Slope-Intercept Form

- Simplifies graphing because the y-intercept is directly visible.
- Makes it easy to identify the rate of change.
- Serves as a foundation for understanding linear relationships in various contexts.

Step 1: Understanding the Given Points

Given points:

- Point 1: (5, 1)
- Point 2: (3, -7)

These points are coordinates in the Cartesian plane, representing specific locations on the line.

Why Are These Points Important?

- They provide the necessary data to calculate the slope.
- They define the specific line we are analyzing.

Visualizing the Points

- Plotting these points on a graph can help visualize the line.
- (5, 1): Located five units to the right and one unit up.
- (3, -7): Located three units to the right and seven units down.

Step 2: Calculating the Slope (m)

The slope of a line passing through two points is a measure of how much y changes relative to x.

Formula for Slope

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Where:

- (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.

Applying the Formula

Using the points (5, 1) and (3, -7):

- $x_1 = 5, y_1 = 1$
- $x_2 = 3, y_2 = -7$

Calculate numerator:

- $y_2 - y_1 = -7 - 1 = -8$

Calculate denominator:

$$- x_2 - x_1 = 3 - 5 = -2$$

Compute the slope:

$$m = (-8) / (-2) = 4$$

Interpretation of the Slope

- The slope $m = 4$ indicates the line rises 4 units vertically for every 1 unit moved horizontally to the right.
- The positive value signifies an upward slope from left to right.

Step 3: Finding the Y-Intercept (b)

Once the slope is known, the next step is to find the y-intercept, the value of y when $x = 0$.

Using the Slope-Intercept Equation

Recall:

$$y = mx + b$$

Rearranged to solve for b:

$$b = y - mx$$

Substituting Known Values

Choose either point; using (5, 1):

Calculate:

$$- b = y - m x$$

$$- b = 1 - 4 \cdot 5$$

$$- b = 1 - 20$$

$$- b = -19$$

Resulting Equation

Putting it all together:

$$y = 4x - 19$$

This is the slope-intercept form of the line passing through (5,1) and (3,-7).

Step 4: Verifying the Equation

To ensure accuracy, verify the equation with the second point (3, -7).

Substitution Check

Plug $x = 3$ into $y = 4x - 19$:

- $y = 4 \cdot 3 - 19$

- $y = 12 - 19$

- $y = -7$

Since this matches the y-coordinate of the second point, the equation is confirmed.

Additional Insights and Applications

Understanding how to derive the equation in slope-intercept form is practical beyond academic exercises. Here are some insights and real-world applications.

Graphing the Line

- The y-intercept at -19 means the line crosses the y-axis at (0, -19).
- The slope of 4 indicates the line rises 4 units for every 1 unit move to the right.
- Plotting these points and the y-intercept helps sketch the line accurately.

Using the Equation in Word Problems

- The slope can represent rates of change, such as speed, cost per item, or growth rates.
- The equation can be used to predict y-values given x-values or vice versa.

Transformations and Variations

- If the problem states different points, follow similar steps.
- For vertical lines, the slope is undefined, and the equation takes the form $x = \text{constant}$.
- For horizontal lines, the slope is zero, and the equation is $y = \text{constant}$.

Summary of Steps to Find the Equation

To summarize, here are the systematic steps to find the line's equation in slope-intercept form given two points:

1. Identify the coordinates of the two points: (x_1, y_1) and (x_2, y_2) .
2. Calculate the slope (m) using $m = (y_2 - y_1) / (x_2 - x_1)$.
3. Choose one point and substitute x and y into $y = mx + b$ to solve for b .
4. Write the final equation as $y = mx + b$, incorporating the calculated slope and y -intercept.
5. Verify the equation by substituting the other point to ensure consistency.

Common Mistakes to Avoid

While calculating the equation of a line, be cautious of these typical errors:

- **Dividing by zero:** If $x_1 = x_2$, the line is vertical, and the slope is undefined.
- **Incorrect order of points:** Always use the same order consistently when calculating the slope.
- **Sign errors:** Pay close attention to signs when subtracting and performing calculations.
- **Forgetting to verify:** Always substitute back into the equation to confirm accuracy.

Conclusion

Finding the equation of a line in slope-intercept form through two points, such as $(5,1)$ and $(3,-7)$, involves a clear process: calculating the slope, determining the y -intercept, and formulating the equation. This method emphasizes understanding the relationships between points, slope, and y -intercept, empowering students and professionals to analyze

linear relationships effectively. By practicing these steps and verifying your results, you'll develop confidence in handling various algebraic and real-world problems involving linear equations.

Additional Resources

- Graphing tools: Use graphing calculators or online graphing software to visualize lines.
- Algebra tutorials: Websites like Khan Academy and Math is Fun offer interactive lessons.
- Practice problems: Engage with exercises involving different points and line types for mastery.

By mastering the process detailed in this guide, you will be well-equipped to find the slope-intercept form of any line given two points, enhancing your algebra skills and understanding of linear functions.

Frequently Asked Questions

How do I find the slope of the line passing through the points (5, 1) and (3, -7)?

To find the slope, use the formula $(y_2 - y_1) / (x_2 - x_1)$. Substituting the points: $(-7 - 1) / (3 - 5) = (-8) / (-2) = 4$.

What is the first step to write the line's equation in slope-intercept form after finding the slope?

The first step is to use the slope and one of the points to find the y-intercept (b) by substituting into the equation $y = mx + b$ and solving for b.

How do I find the y-intercept (b) once I have the slope and a point on the line?

Plug the slope and the coordinates of the point into $y = mx + b$, then solve for b. For example, using (5, 1): $1 = 4x + b$, so $b = 1 - 20 = -19$.

What is the equation of the line passing through (5, 1) and (3, -7) in slope-intercept form?

Using the slope 4 and point (5, 1), the equation is $y = 4x - 19$.

Can I verify the equation of the line by plugging in the other point (3, -7)?

Yes. Substitute $x=3$ into $y=4x-19$: $y=4(3)-19=12-19=-7$, which matches the y-coordinate of the second point, confirming the equation is correct.

Additional Resources

How To Find The Slope-Intercept Form For The Equation Of The Line Through (5,1) And (3,-7)

Understanding how to find the slope-intercept form of a line passing through two specific points is a fundamental skill in algebra and coordinate geometry. The process involves multiple steps, including calculating the slope, then using the point-slope form, and finally converting to the slope-intercept form ($y = mx + b$). This comprehensive guide will walk you through each step with clarity, detailed explanations, and practical tips to master this concept.

Introduction to the Slope-Intercept Form

Before diving into the calculations, it's essential to understand what the slope-intercept form represents and why it's useful.

What is the Slope-Intercept Form?

- The slope-intercept form is a way to write the equation of a straight line.
- It is expressed as $y = mx + b$, where:
- m = the slope of the line, indicating its steepness.
- b = the y-intercept, the point where the line crosses the y-axis.

Why is it Important?

- It provides an easy way to graph the line once you know the slope and y-intercept.
- It helps in understanding how changes in x affect y .
- It's a standard form that simplifies the process of identifying key features of the line.

Step 1: Understanding the Points (5,1) and (3,-7)

Given points are:

- Point 1: (5, 1)
- Point 2: (3, -7)

These points are in coordinate form, where the first value is the x-coordinate and the second is the y-coordinate.

Visualizing the Points

- Plotting these points on a Cartesian plane helps visualize the line.
- The point (5,1) is located 5 units along the x-axis and 1 unit along the y-axis.
- The point (3,-7) is located 3 units along the x-axis and 7 units below the x-axis.

Why Visualization Matters

- Visualizing assists in understanding the relationship between points.
- It helps anticipate the slope's sign (positive or negative).

Step 2: Calculating the Slope (m)

The slope measures how much y changes for a unit change in x. It's calculated as:

$$m = (\text{change in } y) / (\text{change in } x) = (y_2 - y_1) / (x_2 - x_1)$$

Applying the Formula

- Assign the points:

- $(x_1, y_1) = (5, 1)$
- $(x_2, y_2) = (3, -7)$

- Compute the numerator (change in y):

$$- y_2 - y_1 = -7 - 1 = -8$$

- Compute the denominator (change in x):

- $x_2 - x_1 = 3 - 5 = -2$

- Calculate the slope:

- $m = (-8) / (-2) = 4$

Interpreting the Slope

- The slope $m = 4$ indicates the line rises 4 units vertically for each 1 unit horizontally.
- The positive slope confirms the line ascends from left to right.

Step 3: Finding the Equation in Point-Slope Form

The point-slope form of a line is:

$$y - y_1 = m(x - x_1)$$

This form is convenient because it uses the slope and one known point on the line.

Choosing the Point

- You can use either of the two points; here, let's use (5, 1).

Constructing the Equation

- Substitute the known point and the slope:

- $y - 1 = 4(x - 5)$

Expanding the Equation

- Distribute the slope:

- $y - 1 = 4x - 20$

- Solve for y:

- $y = 4x - 20 + 1$

- $y = 4x - 19$

This is a preliminary form of the line's equation.

Step 4: Converting to Slope-Intercept Form

The slope-intercept form is $y = mx + b$. From the previous step, we have:

$$y = 4x - 19$$

Verification

- It's good practice to verify this equation by plugging in the other point (3, -7):
- $y = 4(3) - 19 = 12 - 19 = -7$
- Since the calculated y matches the point's y-coordinate, the equation is confirmed.

Final Equation

- The slope-intercept form of the line passing through (5,1) and (3,-7) is:

$$y = 4x - 19$$

Additional Tips and Common Pitfalls

Understanding this process thoroughly involves awareness of common mistakes and tips for accuracy.

Tips for Success

- Always verify your calculations: Plug in both points to confirm the equation is correct.
- Use consistent notation: Clearly label points as (x_1, y_1) and (x_2, y_2) to avoid mistakes.
- Choose the most convenient point: Sometimes, selecting the point with smaller numbers simplifies calculations.
- Remember the signs: Pay attention to negative signs during subtraction and multiplication.

Common Pitfalls to Avoid

- Incorrect slope calculation: Mixing up points or miscalculating the numerator and denominator.
- Forgetting to verify the equation: Always check both points to ensure the line passes through them.
- Mixing up the point-slope and slope-intercept forms: They serve different purposes, so recognize when to use each.

Practice Problems for Mastery

To solidify understanding, try solving similar problems:

1. Find the slope-intercept form of the line passing through (2, 3) and (4, 11).
2. Determine the equation for the line through (-1, -2) and (3, 6).
3. Given points (0, 5) and (2, 1), find the slope-intercept form.

Summary and Key Takeaways

- The process begins with calculating the slope using the two points.
- Use the point-slope form to write an initial equation.
- Convert to slope-intercept form by solving for y.
- Verify the equation by plugging in both points.
- Practice with different points to strengthen understanding.

In conclusion, finding the slope-intercept form of a line passing through two points is a systematic process involving calculation, substitution, and verification. Mastering this process enables you to quickly analyze and graph lines, a foundational skill in algebra and higher mathematics. With practice, identifying the slope and y-intercept becomes intuitive, empowering you to handle more complex mathematical challenges confidently.

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