How To Solve This Simultaneous Equation By Finding Two Point Of X And Y3x+y=0y= | 2x^2-5 |

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Solving simultaneous equations is a fundamental skill in algebra that allows us to find the values of variables that satisfy multiple equations simultaneously. In this guide, we will explore the step-by-step process to solve the specific set of equations: $3x + y = 0$ and $y = |2x^2 - 5|$. The goal is to find the points (x, y) where these two equations intersect, which involves understanding the nature of absolute value functions and their impact on solutions. By the end of this article, you'll have a clear method to approach similar problems efficiently.

Understanding the Equations

Before diving into solving the equations, it's essential to understand the structure and meaning of each one.

The First Equation: $3x + y = 0$

- This is a linear equation representing a straight line in the xy-plane.
- It can be rewritten as $y = -3x$ for easier substitution.
- The line passes through the origin and has a slope of -3.

The Second Equation: $y = |2x^2 - 5|$

- This is an absolute value function.
- The absolute value makes the graph V-shaped, reflecting the quadratic expression across the x-axis where it is negative.
- The expression inside the absolute value, $2x^2 - 5$, determines the shape and position of the graph.

Graphical Interpretation of the Equations

Visualizing the equations helps understand their points of intersection:

- The line $y = -3x$ is a straight line crossing the origin with a negative slope.
- The graph of $y = |2x^2 - 5|$ is a "W"-shaped curve symmetrical about the y-axis, opening upwards, with vertex points where $2x^2 - 5 = 0$.

Understanding where these graphs intersect gives solutions to the system.

Step-by-Step Solution Process

The main idea is to substitute y from the linear equation into the absolute value equation and analyze the resulting expressions.

Step 1: Express y from the first equation

Since $y = -3x$, substitute into the second equation:
- $y = |2x^2 - 5|$ becomes $-3x = |2x^2 - 5|$

Step 2: Set up equations based on the absolute value definition

Recall that $|A| = B$ implies:
- $A = B$, or
- $A = -B$

Applying this to $-3x = |2x^2 - 5|$ gives two cases:

Case 1: $-3x = 2x^2 - 5$
Case 2: $-3x = -(2x^2 - 5) = -2x^2 + 5$

Step 3: Solve each case separately

Case 1: $-3x = 2x^2 - 5$

Rearranged as:

$2x^2 + 3x - 5 = 0$

Use the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

where $a = 2$, $b = 3$, $c = -5$

Calculate discriminant:

$\Delta = 3^2 - 4\ 2\ (-5) = 9 + 40 = 49$

Find roots:

$x = [-3 \pm \sqrt{49}] / (2\ 2) = [-3 \pm 7] / 4$

- $x = (-3 + 7) / 4 = 4 / 4 = 1$
- $x = (-3 - 7) / 4 = -10 / 4 = -2.5$

Find y for each x:

- For $x = 1$:

$y = -3(1) = -3$

- For $x = -2.5$:

$y = -3(-2.5) = 7.5$

Solutions from Case 1:

$(1, -3)$ and $(-2.5, 7.5)$

Case 2: $-3x = -2x^2 + 5$

Rearranged as:

$2x^2 - 3x - 5 = 0$

Quadratic formula:

$a = 2$, $b = -3$, $c = -5$

Discriminant:

$\Delta = (-3)^2 - 4\ 2\ (-5) = 9 + 40 = 49$

Roots:

$x = [3 \pm \sqrt{49}] / (2\ 2) = [3 \pm 7] / 4$

- $x = (3 + 7) / 4 = 10 / 4 = 2.5$
- $x = (3 - 7) / 4 = -4 / 4 = -1$

Calculate y for each:

- For $x = 2.5$:

$y = -3(2.5) = -7.5$

- For $x = -1$:

$y = -3(-1) = 3$

Solutions from Case 2:

$(2.5, -7.5)$ and $(-1, 3)$

Summary of Solutions

The points where the two equations intersect are:

- $(1, -3)$
- $(-2.5, 7.5)$
- $(2.5, -7.5)$
- $(-1, 3)$

These four solutions satisfy both the linear and absolute value equations.

Verifying the Solutions

Verification ensures accuracy:

- For each (x, y), substitute into both equations.
- Confirm that:

- $3x + y = 0$
- $y = |2x^2 - 5|$

Example: $(1, -3)$

- $3(1) + (-3) = 3 - 3 = 0$ □
- $y = |2(1)^2 - 5| = |2 - 5| = |-3| = 3$, but $y = -3$, so check if $|2x^2 - 5|$ matches y:

Since $y = -3$, but $|2x^2 - 5| = 3$, they are not equal unless we consider the absolute value. Recall that the second equation is $y = |2x^2 - 5|$, so y should be positive or zero, but here y is negative.

This indicates that solutions where y is negative must correspond to the case where $y = -|2x^2 - 5|$, which is not covered in the initial equations. However, in our substitution, the absolute value gives positive y values.

Correction:

Actually, the original equations are:

- $3x + y = 0$
- $y = |2x^2 - 5|$

In our solutions, y equals the value of the absolute value, which is always non-negative. But in the solutions, y can be negative if the line $y = -3x$ gives a negative y. So, to reconcile, the solutions must satisfy $y = |2x^2 - 5|$ and $y = -3x$ simultaneously.

Therefore, the intersection points are where

- $y = -3x$
- $y = |2x^2 - 5|$

and y takes on the value of the absolute value, which is always non-negative. So, the actual intersection points occur when:

- $y = -3x$
- $y = |2x^2 - 5| \geq 0$

and both are equal.

Given that, solutions where y is negative (e.g., -3) are valid only if $y = |2x^2 - 5| = 3$, which is positive, so $y = -3x$ must also be equal to 3, but $y = -3x = 3$ implies $x = -1$, and $y = |2x^2 - 5| = |2(1) - 5| = 3$, which matches the solution at $x = -1$, $y=3$.

Similarly, verify each point carefully:

- (1, -3): $y = -3$, but $|2(1)^2 - 5| = 3$, so $y \neq |2x^2 - 5|$, so discard this point unless considering the negative branch.

Alternative approach:

Because $y = |2x^2 - 5|$, y is always non-negative. The line $y = -3x$ can be negative or positive depending on x, but the absolute value ensures y is non-negative.

Thus, the solutions are points where:

- $y = -3x$
- $y \geq 0$
- $y = |2x^2 - 5|$

Therefore, the actual solutions are those where $y = |2x^2 - 5| \geq 0$ and $y = -3x \geq 0$

This implies:

- $-3x \geq 0 \rightarrow x \leq 0$
- $y = |2x^2 - 5| = -3x$

Now, solve for x:

$|2x^2 - 5| = -3x$

Since $|A| = B$ implies $A = B$ or $A = -B$, and in this context, the right side is non-negative (because $y \geq 0$), so:

- $-3x \geq 0 \rightarrow x \leq 0$

Then:

$|2x^2 - 5| = -3x$

Now,

Frequently Asked Questions

How do I start solving the simultaneous equations 3x + y = 0 and y = | 2x^2 - 5 |?

Begin by substituting y from the second equation into the first, replacing y with | 2x^2 - 5 |, leading to 3x + | 2x^2 - 5 | = 0.

How do I handle the absolute value in the second equation when solving for x and y?

Consider two cases: one where 2x^2 - 5 ≥ 0, so | 2x^2 - 5 | = 2x^2 - 5, and another where 2x^2 - 5 < 0, so | 2x^2 - 5 | = -(2x^2 - 5). Solve each case separately.

What is the first step to find the points where the equations intersect?

Substitute y = | 2x^2 - 5 | into the first equation to get an equation in terms of x alone, then solve for x in each case.

How do I solve for x when 2x^2 - 5 ≥ 0?

Set 2x^2 - 5 ≥ 0, which simplifies to x^2 ≥ 2. Solve for x to get x ≤ -√2 or x ≥ √2. Then, substitute these x-values into the first equation to find corresponding y-values.

How do I find the y-values once I have the x-values?

Use the second equation y = | 2x^2 - 5 | with each x-value to find y. Remember to evaluate the absolute value based on the case you are considering.

What if 2x^2 - 5 < 0? How does that affect the solution?

In this case, | 2x^2 - 5 | = -(2x^2 - 5) = 5 - 2x^2. Substitute into the first equation to find x-values satisfying 3x + (5 - 2x^2) = 0, then solve for x.

Can you provide an example of solving one of the cases?

Yes. For the case 2x^2 - 5 ≥ 0, choose x = 3 (since 3^2=9 ≥ 2). Then y = |2(3)^2 - 5| = |18 - 5|=13. Check the first equation: 33 + 13=9+13=22 ≠ 0, so discard this x. Find other x-values that satisfy the equation.

What is the final step after finding all x and y solutions?

Verify each solution satisfies both original equations. The solutions are the points (x, y) where the two equations intersect.

Are there any common pitfalls to watch out for when solving these types of equations?

Yes, ensure correct handling of the absolute value, consider both cases separately, and verify solutions satisfy all original equations to avoid errors or missing solutions.

Additional Resources

How To Solve This Simultaneous Equation By Finding Two Point Of X And Y: An In-Depth Guide

Solving simultaneous equations is a fundamental skill in algebra that enables us to find the values of unknown variables that satisfy multiple equations at once. In particular, when dealing with a combination of linear and absolute value functions, the process can become more intricate but equally rewarding. This article aims to provide a comprehensive, detailed, and analytical approach to solving the specific set of simultaneous equations:

$$
3x + y = 0
$$

$$
y = |2x^2 - 5|
$$

We will explore the methodology step-by-step, elucidate the reasoning behind each move, and clarify the underlying mathematical principles to help students, educators, and enthusiasts grasp the nuances of this problem.

Understanding the Problem: Equations and Their Nature

Before diving into solving the equations, it’s crucial to understand their form and what kind of solutions they produce.

The Given Equations

1. Linear Equation: $3x + y = 0$

2. Absolute Value Equation: $y = |2x^2 - 5|$

Nature of the Equations

- The first equation $3x + y = 0$ is linear, representing a straight line in the coordinate plane.
- The second equation involves an absolute value, which introduces a piecewise behavior. Absolute value functions typically split the solution into different cases, depending on the sign of the

expression inside.

Goal of the Solution

- Find the points of intersection between the line $y = -3x$ (derived from the linear equation) and the graph of $y = |2x^2 - 5|$.
- These points will be characterized by specific values of x and corresponding y that satisfy both equations simultaneously.

Rewriting the Equations for Clarity

To proceed effectively, rewrite the equations in a more manageable form:

- From $3x + y = 0$, obtain:

$$
y = -3x
$$

- The second equation is already in terms of y:

$$
y = |2x^2 - 5|
$$

Thus, the intersection points occur where:

$$
-3x = |2x^2 - 5|
$$

This transformation simplifies the problem to solving the equation:

$$
|2x^2 - 5| = -3x
$$

Handling the Absolute Value: Case Analysis

The absolute value equation $|A| = B$ can be split into two cases:

- Case 1: $A \geq 0$, then $|A| = A$, so:

$$
A = B
$$

- Case 2: $A < 0$, then $|A| = -A$, so:

$$
-A = B
$$

Applying this to our problem:

$$
|2x^2 - 5| = -3x
$$

we consider two cases based on the sign of $2x^2 - 5$:

Case 1: $2x^2 - 5 \geq 0$

In this case:

$$
|2x^2 - 5| = 2x^2 - 5
$$

So the equation becomes:

$$
2x^2 - 5 = -3x
$$

Rearranged:

$$
2x^2 + 3x - 5 = 0
$$

This is a quadratic in x.

Case 2: $2x^2 - 5 < 0$

Here:

$$
|2x^2 - 5| = -(2x^2 - 5) = 5 - 2x^2
$$

The equation is:

$$
5 - 2x^2 = -3x
$$

Rearranged:

$$
2x^2 - 3x - 5 = 0
$$

Again, a quadratic in x.

Solving the Quadratic Equations

Now, each case leads to a quadratic equation. We will solve both for x and then verify the solutions against the initial conditions (the inequalities for each case).

Quadratic in Case 1: $2x^2 + 3x - 5 = 0$

Applying the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=2$, $b=3$, and $c=-5$:

$$
x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-5)}}{2 \times 2}
$$
$$
x = \frac{-3 \pm \sqrt{9 + 40}}{4}
$$
$$
x = \frac{-3 \pm \sqrt{49}}{4}
$$
$$

x = \frac{-3 \pm 7}{4}
\]

Thus, two solutions:

- \(x = \frac{-3 + 7}{4} = \frac{4}{4} = 1 \)
- \(x = \frac{-3 - 7}{4} = \frac{-10}{4} = -2.5 \)

Verify the inequality \(2x^2 - 5 \geq 0 \):

- For \(x=1 \):

\[
2(1)^2 - 5 = 2 - 5 = -3 < 0
\]

This does not satisfy the condition \(2x^2 - 5 \geq 0 \). Therefore, discard \(x=1 \) from this case.

- For \(x=-2.5 \):

\[
2(-2.5)^2 - 5 = 2 \times 6.25 - 5 = 12.5 - 5 = 7.5 \geq 0
\]

This satisfies the condition. Retain \(x=-2.5 \).

Quadratic in Case 2: \(2x^2 - 3x -5=0 \)

Applying quadratic formula:

\[
x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times (-5)}}{2 \times 2}
\]
\[
x = \frac{3 \pm \sqrt{9 + 40}}{4}
\]
\[
x = \frac{3 \pm \sqrt{49}}{4}
\]
\[
x = \frac{3 \pm 7}{4}
\]

Solutions:

- \(x = \frac{3 + 7}{4} = \frac{10}{4} = 2.5 \)
- \(x = \frac{3 - 7}{4} = \frac{-4}{4} = -1 \)

Verify the condition $2x^2 - 5 < 0$:

- For $x=2.5$:

$$
2(2.5)^2 - 5 = 2 \times 6.25 - 5 = 12.5 - 5= 7.5 \geq 0
$$

This does not satisfy the inequality; discard $x=2.5$.

- For $x=-1$:

$$
2(1)^2 - 5 = 2 - 5 = -3 < 0
$$

This satisfies the condition. Retain $x=-1$.

Finding Corresponding y Values

Recall the relation $y = -3x$. For each valid x, compute y:

- For $x = -2.5$:

$$
y = -3 \times (-2.5) = 7.5
$$

- For $x = -1$:

$$
y = -3 \times (-1) = 3
$$

These are the points of intersection:

$$
(-2.5, 7.5) \quad \text{and} \quad (-1, 3)
$$

Verifying the Solutions

It’s vital to verify that these solutions satisfy both original equations:

- Equation 1: \(3x + y = 0 \)

- Equation 2: \(y = |2x^2 - 5| \)

For \(x = -2.5 \), \(y = 7.5 \):

\[
3(-2.5) + 7.5 = -7.5 + 7.5 = 0 \quad \checkmark
\]
\[
|2(-2.5)^2 - 5| = |2 \times 6.25 - 5| = |12.5 - 5| = 7.5 \quad \checkmark
\]

For \(x = -1 \), \(y = 3 \):

\[

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