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Understanding how to find the length of segments and the coordinates of points dividing a line segment is fundamental in coordinate geometry. This problem involves calculating the point C that divides the segment AB in a specific ratio and then determining the length OC, which is the distance from the origin to point C. In this comprehensive guide, we will explore the step-by-step process to solve this problem, including the concepts of section formula, coordinate calculations, and distance formulas. Whether you're a student preparing for exams or a math enthusiast looking to deepen your understanding, this article provides detailed explanations and practical methods to approach similar problems.

Understanding the Problem Statement

Before diving into calculations, it's crucial to understand what the problem asks:

- You are given two points A and B with coordinates:
- A(2, -3)
- B(3, -2)
- Point C divides the segment AB in the ratio of 1:2.
- The task is to find the length OC, where O is the origin (0,0), and C is the point that divides AB as specified.

This problem combines concepts such as coordinate geometry, the section formula, and distance measurement.

Key Concepts Needed to Solve the Problem

1. Coordinates of Points

- Each point in a plane is represented by an ordered pair (x, y).

- For example, point A is at (2, -3).

2. Dividing a Line Segment in a Given Ratio

- When a point C divides the segment AB in the ratio m:n, the coordinates of C can be found using the section formula:

$$
C(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)
$$

where:
- (x_1, y_1) are the coordinates of point A,
- (x_2, y_2) are the coordinates of point B,
- m:n is the ratio of division.

3. Distance Formula

- To find the length of OC, the distance between points O(0,0) and C(x, y) is given by:

$$
OC = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}
$$

Step-by-Step Solution Process

Step 1: Identify the given points and ratio

- Point A = (2, -3)
- Point B = (3, -2)
- Ratio of division = 1:2

Step 2: Apply the section formula to find point C

- Since C divides AB in the ratio 1:2, we assign:
- m = 1
- n = 2

- Coordinates of C:

$$
x_c = \frac{m \times x_2 + n \times x_1}{m + n} = \frac{1 \times 3 + 2 \times

2}{1 + 2} = \frac{3 + 4}{3} = \frac{7}{3}
\]

$$
y_c = \frac{m \times y_2 + n \times y_1}{m + n} = \frac{1 \times (-2) + 2 \times (-3)}{3} = \frac{-2 - 6}{3} = \frac{-8}{3}
$$

- Therefore, point C has coordinates:

$$
C \left(\frac{7}{3}, -\frac{8}{3} \right)
$$

Step 3: Calculate the length OC

- The coordinates of O are (0, 0).
- Distance OC is:

$$
OC = \sqrt{\left(\frac{7}{3}\right)^2 + \left(-\frac{8}{3}\right)^2} = \sqrt{\frac{49}{9} + \frac{64}{9}} = \sqrt{\frac{113}{9}} = \frac{\sqrt{113}}{3}
$$

- Approximate value:

$$
\sqrt{113} \approx 10.63
$$

$$
OC \approx \frac{10.63}{3} \approx 3.54
$$

Thus, the length of OC is approximately 3.54 units.

Additional Insights and Related Concepts

Understanding the Significance of the Ratio

- The ratio 1:2 indicates that point C is closer to point A than point B.
- If the ratio were different, the coordinates of C would change accordingly, affecting the calculated length OC.

Applications in Real-World Scenarios

- Dividing a segment internally in a specific ratio is useful in:
- Engineering and design (e.g., dividing a beam into sections)
- Navigation (calculating intermediate points along a route)
- Computer graphics (finding midpoints or specific division points)

Visual Representation

- Plotting points A and B along with point C helps in understanding the division visually.
- Using graph paper or digital plotting tools can make the concept clearer.

Summary of the Solution

Step	Description	Result
1	Identify points and ratio	A(2, -3), B(3, -2), ratio 1:2
2	Apply section formula to find C	$C \left(\frac{7}{3}, -\frac{8}{3} \right)$
3	Calculate distance OC	$OC \approx 3.54$ units

Conclusion

Solving the problem of finding the length OC when a point C divides a line segment AB involves a clear understanding of coordinate geometry principles. By applying the section formula, we accurately determine the coordinates of C, and then use the distance formula to find OC. This approach is widely applicable for various problems involving line divisions, midpoints, and distances in the coordinate plane.

Understanding these foundational concepts enhances your ability to tackle more complex geometric problems, making coordinate geometry an essential skill in mathematics and related fields.

FAQs

1. **What is the section formula?** – It is a formula used to find the coordinates of a point dividing a line segment in a given ratio.
2. **How do I find the length of a segment in coordinate geometry?** – Use the distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
3. **Can this method be used for external division?** – Yes, but the formula adjusts for external division, involving ratios that add up differently.
4. **What are practical applications of dividing segments in ratios?** – Engineering, navigation, computer graphics, and design.

By mastering these concepts, you can confidently handle a variety of problems involving line segments, ratios, and distances in coordinate geometry.

Frequently Asked Questions

What are the coordinates of point C that divides the line segment AB in the ratio 1:2?

Using the section formula, point C divides AB in the ratio 1:2. Coordinates of C are:
C = ((13 + 22)/(1+2), (1-2 + 2-3)/(1+2)) = ((3 + 4)/3, (-2 - 6)/3) = (7/3, -8/3).

How do you find the point C that divides the segment AB in ratio 1:2?

Apply the section formula: for points A(x_1, y_1) and B(x_2, y_2), and ratio m:n, the point dividing AB is (($mx_2 + nx_1$)/(m + n), ($my_2 + ny_1$)/(m + n)). In this case, m=1, n=2, so C = ((13 + 22)/3, (1-2 + 2-3)/3) = (7/3, -8/3).

What is the length of segment OC if O is the origin and C is at (7/3, -8/3)?

Distance OC = $\sqrt{[(7/3)^2 + (-8/3)^2]}$ = $\sqrt{[(49/9) + (64/9)]}$ = $\sqrt{(113/9)}$ = ($\sqrt{113}$)/3.

How do you calculate the length of OC based on point

C's coordinates?

Use the distance formula from the origin O(0,0) to C(7/3, -8/3):
$OC = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{(7/3)^2 + (-8/3)^2} = (\sqrt{113})/3$.

What is the significance of dividing the line segment AB in the ratio 1:2?

Dividing AB in the ratio 1:2 means that point C divides the segment into two parts where one part is twice as long as the other, with C located closer to A than to B, according to the ratio.

Can you explain the section formula used to find point C?

The section formula states that if a point C divides a segment AB internally in the ratio m:n, then C has coordinates $((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$. It essentially finds a weighted average of the endpoints based on the ratio.

What are the original coordinates of points A and B?

Point A has coordinates (2, -3) and point B has coordinates (3, -2).

What is the final value of OC in simplified form?

The length OC is $(\sqrt{113})/3$, which is approximately 3.377 units when calculated numerically.

Additional Resources

Understanding the Problem: Coordinates, Ratios, and Geometric Concepts

When dealing with coordinate geometry problems involving points and line segments, it's essential to understand the fundamental concepts such as coordinate points, division ratios, and how to derive unknown points or lengths from given data. The problem at hand involves two points, A and B, with known coordinates, and a third point C that divides the segment AB in a specific ratio. The ultimate goal is to find the length OC, where O is typically the origin (0,0).

This review provides a comprehensive exploration of the problem, starting from the basics of coordinate geometry, moving through the calculation of the dividing point C, and finally determining the distance OC. We will delve into relevant formulas, step-by-step calculations, and geometric interpretations to ensure a thorough understanding.

Fundamentals of Coordinate Geometry

Before tackling the problem directly, it’s vital to revisit some core concepts in coordinate geometry.

Coordinate Points

- A point in the coordinate plane is represented as (x, y), where:
- x is the horizontal coordinate (along the x-axis)
- y is the vertical coordinate (along the y-axis)
- For example, point A(2, -3) is located 2 units to the right of the origin and 3 units below it.

Line Segment and Coordinates

- A line segment AB connects points A(x_1, y_1) and B(x_2, y_2).
- The length of AB can be calculated using the distance formula:

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

- The coordinates of a point dividing AB in a ratio \(m:n \) can be found using the section formula.

Dividing a Line Segment in a Given Ratio

The key to solving the problem lies in understanding how a point C can divide a segment AB in a specific ratio.

The Section Formula

- If point C divides segment AB internally in the ratio \(m:n \), then the coordinates of C are given by:

\[
C_x = \frac{m x_2 + n x_1}{m + n}
\]

\[

$$
C_y = \frac{m y_2 + n y_1}{m + n}
$$

- Here, (x_1, y_1) and (x_2, y_2) are coordinates of points A and B respectively, and $m:n$ is the ratio in which C divides AB.

Important note:
- The ratio is often expressed as $\text{AC} : \text{CB}$.
- For an internal division, the point C lies between A and B.

Applying the Section Formula to Find Point C

Given data:

- $A(2, -3)$
- $B(3, -2)$
- C divides AB in the ratio $1:2$

Step 1: Understand the Ratio

- $m:n = 1:2$
- This means $\text{AC} = 1$ part, $\text{CB} = 2$ parts, with C closer to A than B.

Step 2: Use the Section Formula

Plugging into the formula:

$$
C_x = \frac{(1)(x_2) + (2)(x_1)}{1 + 2} = \frac{1 \times 3 + 2 \times 2}{3} = \frac{3 + 4}{3} = \frac{7}{3}
$$

$$
C_y = \frac{(1)(y_2) + (2)(y_1)}{1 + 2} = \frac{1 \times (-2) + 2 \times (-3)}{3} = \frac{-2 - 6}{3} = \frac{-8}{3}
$$

So, the coordinates of C are:

$$
C\left(\frac{7}{3}, -\frac{8}{3}\right)
$$

Summary:

- $C \left(\frac{7}{3}, -\frac{8}{3} \right)$

Calculating the Length OC

Now that we have the coordinates of point C, the next step is to find the length of OC, where O is the origin (0,0).

Step 1: Coordinates of O

- $O(0, 0)$

Step 2: Use the Distance Formula

The distance between O(0,0) and C$\left(\frac{7}{3}, -\frac{8}{3} \right)$:

$$
OC = \sqrt{\left(\frac{7}{3} - 0 \right)^2 + \left(-\frac{8}{3} - 0 \right)^2}
$$

$$
OC = \sqrt{\left(\frac{7}{3} \right)^2 + \left(-\frac{8}{3} \right)^2}
$$

$$
OC = \sqrt{\frac{49}{9} + \frac{64}{9}} = \sqrt{\frac{113}{9}}
$$

$$
OC = \frac{\sqrt{113}}{3}
$$

Final answer:

$$
\boxed{
OC = \frac{\sqrt{113}}{3}
}
$$

This represents the length from the origin to point C in simplest radical form.

Geometric Interpretation and Additional Insights

Understanding the geometric implications of this problem enriches comprehension.

Location of Point C

- Since C divides AB in a ratio of 1:2, it is closer to A than B.
- The coordinates reflect this, lying proportionally nearer to A.
- The division ratio indicates the weighted average of A and B's coordinates.

Significance of the Length OC

- The length OC (from origin O to point C) provides insight into the position of C relative to the origin.
- The value $\frac{\sqrt{113}}{3}$ is approximately 3.55 units, indicating how far C is from O.

Additional Calculations and Variations

- One could also calculate the length of AB for completeness.
- If the ratio were different, the section formula would adapt accordingly.
- For external division or other ratios, the formulas change slightly.

Extensions and Related Problems

The problem's framework opens avenues for exploring related questions:

- Finding the length of AB:

$$
AB = \sqrt{(3 - 2)^2 + (-2 + 3)^2} = \sqrt{1^2 + 1^2} = \sqrt{2}
$$

\]

- Changing the ratio: How does the position of C shift if the ratio is altered?
- External division: If C divides AB externally, the section formula involves subtraction.
- Midpoint calculation: When ratio is 1:1, C is the midpoint, simplifying calculations.

Practical Applications of Coordinate Geometry

Understanding such problems is not purely academic; they have real-world applications:

- Navigation and GPS: Calculating intermediate points along a route.
- Engineering design: Determining points of division on structural elements.
- Computer graphics: Positioning objects based on coordinate ratios.
- Robotics: Path planning and point positioning.

Summary and Key Takeaways

- The problem involves fundamental concepts of coordinate geometry, including the section formula, distance formula, and ratios.
- The coordinates of point C dividing AB in the ratio 1:2 are $\left(\frac{7}{3}, -\frac{8}{3} \right)$.
- The length OC from the origin to point C is $\frac{\sqrt{113}}{3}$, approximately 3.55 units.
- Mastery of these concepts facilitates solving a wide array of geometric problems involving points, lines, and ratios.

Final Remarks

This detailed exploration underscores the importance of understanding the foundational formulas and their applications in coordinate geometry. Problems like these serve as excellent practice for developing spatial reasoning and algebraic manipulation skills. They also exemplify how abstract mathematical principles translate into precise calculations that describe real-world situations.

Whether you are a student preparing for exams or a professional applying geometry in technical fields, grasping these concepts enhances problem-solving efficiency and deepens geometric intuition.

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