If The Graph Of Y=f(x) Contains The Point (0,1), Ad If Dy/dx = (x Sin(x^2))/y, Then F(x)=

If The Graph Of Y=f(x) Contains The Point (0,1), Ad If Dy/dx = (x Sin(x^2))/y, Then F(x)= is a compelling problem that combines initial value conditions with differential equations. Understanding how to find the explicit form of the function $f(x)$ based on the given differential equation and initial point is an essential skill in calculus. This article will guide you through the steps to determine $f(x)$, explore the significance of the initial point, and delve into the solution process for the differential equation $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$.

Understanding the Initial Condition and Differential Equation

The Significance of the Point (0, 1)

The point $(0, 1)$ indicates that when $x = 0$, the corresponding y-value on the graph of $f(x)$ is 1. This initial condition is crucial because it allows us to solve for any constants of integration that may arise during the process of solving the differential equation. In practical terms, it ensures that the particular solution we find will pass through $(0, 1)$, matching the given point exactly.

The Differential Equation $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$

This differential equation is a first-order, separable equation. The structure suggests that the variables x and y can be separated to facilitate integration. Recognizing the form is the first step toward solving it.

Solving the Differential Equation

Separating the Variables

Given:

$$\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$$

we can rewrite this as:

$$ y\,dy = x \sin(x^2) dx $$

This step involves multiplying both sides by y and dx, effectively isolating the y and dy on one side, and the x and dx on the other.

Integrating Both Sides

Now, integrate both sides:

$$ \int y\,dy = \int x \sin(x^2) dx $$

The left side integrates to:

$$ \frac{1}{2} y^2 + C_1 $$

The right side requires a substitution for easier integration.

Evaluating $\int x \sin(x^2) dx$

Let’s focus on the right side:
- Substitute $u = x^2$, so that $du = 2x dx$, or $x dx = \frac{1}{2} du$.
- Rewriting the integral:

$$ \int x \sin(x^2) dx = \frac{1}{2} \int \sin u \, du $$

- Integrate:

$$ \frac{1}{2} (-\cos u) + C_2 = -\frac{1}{2} \cos u + C_2 $$

- Substitute back:

$$ -\frac{1}{2} \cos(x^2) + C_2 $$

Formulating the General Solution

Combining the integrals:

$$ \frac{1}{2} y^2 = -\frac{1}{2} \cos(x^2) + C $$

where $C = C_2 - C_1$, an arbitrary constant.

Multiply both sides by 2 to simplify:

$$ y^2 = -\cos(x^2) + 2C $$

Define $K = 2C$, an arbitrary constant, leading to:

$$ y^2 = -\cos(x^2) + K $$

This is the general implicit solution to the differential equation.

Applying the Initial Condition to Find the Particular Solution

Using the Point $(0, 1)$

Substitute $x = 0$ and $y = 1$ into the solution:
$$ (1)^2 = -\cos(0) + K $$
$$ 1 = -1 + K $$
$$ K = 2 $$

Final Explicit Solution for $f(x)$

Replace K back into the general form:
$$ y^2 = -\cos(x^2) + 2 $$
Thus, the particular solution is:
$$ y = \pm \sqrt{2 - \cos(x^2)} $$

Since the initial point is $(0, 1)$, which corresponds to a positive y-value, we take the positive root:
$$ f(x) = \sqrt{2 - \cos(x^2)} $$

Summary of the Solution and Its Significance

Explicit Form of $f(x)$

The function $f(x)$ satisfying both the differential equation and the initial point is:
$$ \boxed{f(x) = \sqrt{2 - \cos(x^2)}} $$

Understanding the Behavior of $f(x)$

- The square root ensures $f(x)$ is non-negative, aligning with the initial condition $f(0) = 1$.
- The $\cos(x^2)$ term introduces oscillations, but due to the addition of 2 inside the square root, the function remains real-valued for all x.

Applications and Importance in Calculus

Why Solving Differential Equations Is Important

Differential equations like this one model numerous real-world phenomena, including physics, engineering, and biology. Understanding how to solve them enables us to predict system behavior over time.

Initial Conditions and Particular Solutions

Initial conditions allow us to find specific solutions from the general form, making models more accurate for particular scenarios.

Separable Equations and Integration Techniques

Mastering methods such as separation of variables and substitution is essential for tackling a wide range of differential equations.

Conclusion

In summary, given the initial point $(0, 1)$ and the differential equation $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$, we derived the explicit form of the function:

$$ f(x) = \sqrt{2 - \cos(x^2)} $$

This process involved separating variables, integrating, applying initial conditions to determine the constant of integration, and ensuring the solution aligns with the initial point. Understanding this approach enhances your problem-solving skills and deepens your grasp of differential equations and their applications in various fields.

If you're looking to strengthen your calculus skills, practicing similar problems involving initial conditions and differential equations will prepare you for more complex scenarios in mathematics and science.

Frequently Asked Questions

Given that the graph of y = f(x) passes through the point (0, 1), and the differential equation is dy/dx = (x sin(x^2))/y, how can we find the function f(x)?

We can solve the differential equation by separating variables: $y\,dy = x \sin(x^2)\,dx$, then integrate both sides to find $f(x)$.

What is the initial condition provided in the problem, and how does it help in solving for f(x)?

The initial condition is $f(0) = 1$, which allows us to determine the constant of integration after integrating the differential equation.

How do you separate variables in the differential

equation dy/dx = (x sin(x^2))/y?

Rearranged as y dy = x sin(x^2) dx, allowing integration on both sides separately.

What is the integral of y dy, and how does it relate to solving for f(x)?

The integral of y dy is (1/2) y^2 + C, which helps find y as a function of x after integrating both sides.

How do you integrate the right side, x sin(x^2), in the differential equation?

Use substitution u = x^2, so du = 2x dx, thus x sin(x^2) dx = (1/2) sin(u) du, which integrates to -(1/2) cos(u) + C.

What is the general solution for y(x) after integrating both sides?

The solution is (1/2) y^2 = -(1/2) cos(x^2) + C, which simplifies to y^2 = -cos(x^2) + 2C.

How do you determine the constant C using the initial condition y(0) = 1?

Substitute x = 0 and y = 1 into the solution: 1^2 = -cos(0) + 2C, so 1 = -1 + 2C, leading to 2C = 2, thus C = 1.

What is the explicit form of f(x) after solving the differential equation?

f(x) = sqrt(-cos(x^2) + 2), considering the initial condition and the positive root for y.

Are there any domain restrictions for f(x) based on the solution obtained?

Yes, since y = sqrt(-cos(x^2) + 2), the expression inside the square root must be non-negative: -cos(x^2) + 2 ≥ 0, which holds for all real x because cos(x^2) ≤ 1.

Additional Resources

Understanding the Graph of y = f(x) and Deriving the Function from a

Differential Equation

When delving into the fascinating world of calculus, two core concepts emerge as particularly intriguing: the relationship between a function and its graph, and the process of reconstructing a function from its derivative. In this article, we explore these themes deeply, focusing on a specific scenario: a function $y = f(x)$ whose graph passes through a particular point, and a differential equation governing its behavior. Our goal is to determine the explicit form of $f(x)$ based on the given information.

Contextual Foundations: The Significance of a Point on the Graph

Why Does the Point (0,1) Matter?

In the study of functions and their graphs, the presence of a specific point provides crucial initial information. Here, the point $(0, 1)$ on the graph of $y = f(x)$ indicates:

- When $x = 0$, $y = 1$.
- The initial value of the function at $x = 0$ is known, which is vital for solving differential equations that involve derivatives of y.

This initial condition serves as a boundary or starting point when integrating or solving differential equations, effectively anchoring the solution and making it unique.

The Role of Initial Conditions in Differential Equations

Differential equations often describe the rate of change of a function, but they do not specify the function's exact form without an initial condition. For example:

- Without initial conditions, solutions are represented by general solutions containing arbitrary constants.
- Specifying a point like $(0, 1)$ allows us to determine those constants, leading to a specific solution.

In our case, the initial condition will be essential once we integrate the differential equation to find an explicit formula for y = f(x).

Analyzing the Differential Equation: Dy/dx = (x Sin(x^2))/y

Understanding the Differential Equation

The given differential equation is:

$$
\frac{dy}{dx} = \frac{x \sin(x^2)}{y}
$$

This is a first-order, separable differential equation, meaning we can manipulate it to separate the variables y and x for integration.

Key observations include:

- The presence of y in the denominator suggests that multiplying both sides by y will facilitate separation.
- The numerator involves $x \sin(x^2)$, which hints at potential substitution involving x^2.

Rearranging and Preparing for Integration

To solve, rewrite the equation:

$$
y \, dy = x \sin(x^2) \, dx
$$

This step involves multiplying both sides by y:

- Left side: $y \, dy$
- Right side: $x \sin(x^2) \, dx$

Now, the differential equation is in a form conducive to integration:

$$
\int y \, dy = \int x \sin(x^2) \, dx
$$

Step-by-Step Solution Process

Integrating the Left Side: $\int y \, dy$

This is straightforward:

$$
\int y \, dy = \frac{1}{2} y^2 + C_1
$$

where C_1 is an arbitrary constant, which we will absorb into the general solution later.

Integrating the Right Side: $\int x \sin(x^2) \, dx$

The right side requires substitution:

- Let $u = x^2$, hence $du = 2x \, dx$, or $x \, dx = \frac{du}{2}$.

Rearranged:

$$
x \sin(x^2) \, dx = \frac{\sin u}{2} \, du
$$

Thus, the integral becomes:

$$
\int x \sin(x^2) \, dx = \frac{1}{2} \int \sin u \, du
$$

This integral is standard:

$$
\frac{1}{2} (-\cos u) + C_2
$$

Replacing $u = x^2$:

$$
- \frac{1}{2} \cos(x^2) + C_2
$$

\]

Combining Results: The General Solution

Putting it all together:

$$
\frac{1}{2} y^2 = - \frac{1}{2} \cos(x^2) + C
$$

Multiplying through by 2:

$$
y^2 = - \cos(x^2) + 2C
$$

Let’s denote $2C$ as a new constant K:

$$
\boxed{
y^2 = - \cos(x^2) + K
}
$$

This is the general solution to the differential equation. To determine the specific function $f(x)$, we need to find the value of K using the initial condition.

Applying the Initial Condition: Determining the Specific F(x)

Using the Point (0, 1)

Since the graph passes through $(0, 1)$:

$$
x = 0, \quad y = 1
$$

Plugging into the general solution:

$$
(1)^2 = - \cos(0^2) + K
$$

Recall:

$$
\cos(0) = 1
$$

Thus:

$$
1 = -1 + K
$$

Solving for K:

$$
K = 1 + 1 = 2
$$

Explicit Form of the Function $f(x)$

Substitute $K = 2$ back into the general solution:

$$
y^2 = - \cos(x^2) + 2
$$

Taking the square root to solve for y:

$$
f(x) = \pm \sqrt{2 - \cos(x^2)}
$$

Since the point $(0, 1)$ corresponds to a positive y, we choose the positive root:

$$
\boxed{
f(x) = \sqrt{2 - \cos(x^2)}
}
$$

This function is defined for all x where the radicand $2 - \cos(x^2)$ is non-negative, which is always true because $\cos(x^2)$ oscillates between -1 and 1, making the expression inside the square root range between 1 and 3.

Analysis of the Derived Function $f(x)$

Behavior Near $x = 0$

At $x = 0$:

$$
f(0) = \sqrt{2 - \cos(0)} = \sqrt{2 - 1} = \sqrt{1} = 1
$$

which matches the initial point as expected.

Graphical Features

- The function oscillates smoothly, with peaks and troughs dictated by the cosine component.
- The maximum value of $f(x)$ occurs when $\cos(x^2)$ is at its minimum (-1):

$$
f_{\max} = \sqrt{2 - (-1)} = \sqrt{3}
$$

- The minimum value occurs when $\cos(x^2) = 1$:

$$
f_{\min} = \sqrt{2 - 1} = 1
$$

This oscillatory behavior reflects the interplay between the quadratic argument of the cosine and the square root.

Summary and Final Remarks

- The initial point $(0,1)$ was crucial in determining the constant of integration.
- The differential equation was separable, enabling straightforward integration after substitution.
- The explicit form of the function is:

$$
\boxed{
f(x) = \sqrt{2 - \cos(x^2)}
}
$$

- The function is well-defined over the entire real line, exhibits oscillatory behavior, and satisfies the initial condition.

Additional Insights and Practical Implications

Understanding how functions behave based on their derivatives and initial points is fundamental in disciplines like physics, engineering, and economics. For instance:

- In physics, such differential equations model oscillatory systems with energy conservation.
- In engineering, they can represent signal processing or control systems with feedback.
- Economically, similar models describe growth rates with oscillatory components.

This exercise underscores the importance of initial conditions and the power of substitution in solving complex differential equations, leading to explicit, usable formulas for the functions governing these systems.

In conclusion, starting from the differential equation $\frac{dy}{dx} = \frac{x \sin(x^2)}{y}$ and the point $(0, 1)$, we've derived the explicit solution:

$$
\boxed{
f(x) = \sqrt{2 - \cos(x^2)}
}
$$

which encapsulates the behavior of the original function and provides a comprehensive understanding of its graphical and analytical properties.

If The Graph Of Y F X Contains The Point 0 1 Ad If Dy Dx X Sin

X 2 Y Then F X

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