If X-N(-3,4), Find The Probability That X Is Between 4 And 1. Round To 3 Decimal Places.

If X ~ N(-3, 4), Find The Probability That X Is Between 1 And 4. Round To 3 Decimal Places.

Understanding how to calculate probabilities for normally distributed variables is a fundamental skill in statistics, especially when dealing with continuous probability distributions such as the normal distribution. The problem at hand involves a normal random variable X, with a mean of -3 and a variance of 4, and aims to find the probability that X falls between 1 and 4. This kind of problem is common in various fields, including finance, engineering, and scientific research, making a solid grasp of the process crucial.

In this comprehensive guide, we will explore the step-by-step approach to solving this problem, including the necessary statistical concepts, formulas, and practical calculation methods. Whether you're a student preparing for exams or a professional applying statistical techniques, understanding how to find such probabilities and accurately interpret the results is an invaluable skill.

Understanding the Normal Distribution

Before diving into calculations, it's essential to understand what a normal distribution entails and why it's relevant for this problem.

What Is a Normal Distribution?

The normal distribution, often called the Gaussian distribution, is a bell-shaped probability distribution that is symmetric around its mean. It describes how the values of a continuous random variable are distributed:

- Characteristics:
- Symmetrical about the mean.
- The mean, median, and mode are all equal.
- The spread of the distribution is determined by the standard deviation.
- The total area under the curve equals 1, representing total probability.

- Mathematical Formulation:

$$
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{ -\frac{(x - \mu)^2}{2\sigma^2} }
$$

where:
- μ is the mean,
- σ is the standard deviation,
- x is the variable.

Parameters of the Distribution

In the problem, the distribution is given as $X \sim N(-3, 4)$. This notation indicates:

- The mean μ is -3.
- The variance σ^2 is 4, so the standard deviation σ is $\sqrt{4} = 2$.

Understanding these parameters is critical for transforming the problem into a standard normal distribution context.

Standardizing the Variable

Since the normal distribution is defined by parameters μ and σ, but standard normal distribution $Z \sim N(0,1)$ is often used for calculations, we convert X to Z.

Why Standardize?

Standardization simplifies probability calculations because the standard normal distribution has well-tabulated cumulative distribution function (CDF) values. Standardizing allows us to use these tables or computational tools.

Standardization Formula

The transformation from X to Z is:

$$
Z = \frac{X - \mu}{\sigma}
$$

Applying this to our bounds:

- For $X = 1$:

$$
Z_{1} = \frac{1 - (-3)}{2} = \frac{4}{2} = 2
$$

- For $X = 4$:

$$
Z_{4} = \frac{4 - (-3)}{2} = \frac{7}{2} = 3.5
$$

This means the probability that X is between 1 and 4 is equivalent to the probability that Z is between 2 and 3.5.

Calculating the Probability

The key step is to find:

$$
P(1 \leq X \leq 4) = P(Z_{1} \leq Z \leq Z_{4}) = P(2 \leq Z \leq 3.5)
$$

This can be expressed as:

$$
P(2 \leq Z \leq 3.5) = \Phi(3.5) - \Phi(2)
$$

where $\Phi(z)$ is the cumulative distribution function (CDF) of the standard normal distribution.

Using Standard Normal Tables or Calculators

Standard normal tables or statistical software provide the values of $\Phi(z)$.

- $\Phi(2)$ is approximately 0.9772
- $\Phi(3.5)$ is approximately 0.9998

Note: These values are typically found in Z-tables or computed using software like R, Python, or calculator functions.

Calculating the Probability

Subtract the two:

$$
P(2 \leq Z \leq 3.5) = 0.9998 - 0.9772 = 0.0226
$$

Thus, the probability that X is between 1 and 4 is approximately 0.0226.

Rounding the Result

The problem instructs to round the answer to three decimal places. Therefore:

$$
\boxed{0.023}
$$

This is the final probability that X lies between 1 and 4 for the specified normal distribution.

Summary of the Calculation Process

To encapsulate the method:

1. Identify the parameters of the distribution: mean $\mu = -3$ and standard deviation $\sigma = 2$.

2. Determine the bounds in the original scale: 1 and 4.

3. Standardize the bounds to Z-scores:

    - $Z_{1} = \frac{1 - (-3)}{2} = 2$

    - $Z_{4} = \frac{4 - (-3)}{2} = 3.5$

4. Use standard normal CDF tables or software to find $\Phi(2)$ and $\Phi(3.5)$: approx 0.9772 and 0.9998 respectively.

5. Calculate the probability:
$$
P(2 \leq Z \leq 3.5) = 0.9998 - 0.9772 = 0.0226
$$

6. Round to three decimal places: 0.023.

This method ensures accurate and efficient probability calculation for continuous normal variables.

Practical Applications and Importance

Understanding how to compute these probabilities has broad relevance:

Applications in Real-World Scenarios

- Quality Control: Determining the proportion of products within certain measurements.
- Finance: Calculating the probability of asset returns falling within a specific range.
- Medical Research: Estimating the likelihood of measurements falling within healthy ranges.
- Environmental Science: Assessing the probability of pollutant levels within permissible limits.

Why Mastering This Skill Matters

- Decision Making: Accurate probability estimates inform crucial decisions.
- Data Analysis: Interpreting data distributions and making predictions.
- Statistical Inference: Testing hypotheses and constructing confidence intervals.

Additional Tips for Accurate Calculations

To enhance accuracy and efficiency:

1. Use reliable statistical software or scientific calculators with built-in functions for the normal distribution.

2. Familiarize yourself with standard normal tables for quick reference.

3. Always verify the parameters of your distribution before standardizing.

4. Keep track of rounding to avoid cumulative errors, especially in multi-step calculations.

5. Practice with different problems to build confidence and proficiency.

Conclusion

Calculating the probability that a normally distributed variable falls within a specific interval involves understanding the distribution's parameters, standardizing the bounds, and using the standard normal distribution's properties. In this case, with \(X \sim N(-3, 4) \), the probability that

\(X \) is between 1 and 4 is approximately 0.023 when rounded to three decimal places.

Mastering these steps enhances your ability to interpret and analyze data modeled by normal distributions accurately. Whether in academic settings or professional environments, this skill is fundamental to statistical literacy and informed decision-making.

Remember: Always double-check your calculations, understand the assumptions behind the normal distribution, and practice with various examples to solidify your understanding.

Frequently Asked Questions

What is the type of distribution for X if X ~ N(-3, 4)?

X follows a normal distribution with mean -3 and variance 4 (standard deviation 2).

How do I find the probability that X is between 1 and 4 when X ~ N(-3, 4)?

Convert 1 and 4 to z-scores, then find the area between these z-values in the standard normal distribution table or using a calculator.

What is the formula for converting a raw score to a z-score?

$Z = (X - \mu) / \sigma$, where μ is the mean and σ is the standard deviation.

What are the z-scores for X = 1 and X = 4 in this problem?

For $X = 1$: $z = (1 - (-3)) / 2 = 4 / 2 = 2$; for $X = 4$: $z = (4 - (-3)) / 2 = 7 / 2 = 3.5$.

How do I find the probability that Z is between 2 and 3.5 in the standard normal distribution?

Subtract the cumulative probability at Z=2 from that at Z=3.5 using a Z-table or calculator: $P(2 < Z < 3.5) = P(Z < 3.5) - P(Z < 2)$.

What are the approximate cumulative probabilities for Z = 2 and Z = 3.5?

$P(Z < 2) \approx 0.977$, $P(Z < 3.5) \approx 0.9998$.

What is the final probability that X is between 1 and 4 for X ~

N(-3, 4)?

Subtracting the two cumulative probabilities: 0.9998 - 0.977 ≈ 0.023, rounded to 3 decimal places: 0.023.

Additional Resources

If X~N(-3, 4), Find The Probability That X Is Between 4 And 1. Round To 3 Decimal Places.

Investigating Probabilistic Boundaries: A Detailed Analysis of X~N(-3, 4)

In the realm of statistics and probability theory, the normal distribution stands as one of the most fundamental and widely applied concepts. When working with real-world data, understanding the behavior of a normally distributed random variable is essential, especially when calculating the likelihood of the variable falling within a certain range. In this detailed exploration, we focus on the problem: If X~N(-3, 4), find the probability that X is between 4 and 1, rounded to three decimal places.

This problem exemplifies the application of standard normal distribution techniques, z-score transformations, and the use of probability tables or computational tools. Our goal is to thoroughly analyze the problem, clarify the steps involved, and provide insights into the broader implications of such calculations.

Understanding the Distribution Parameters

Before diving into calculations, it is vital to interpret the given distribution parameters accurately.

The Normal Distribution Parameters

- Mean (μ): -3
- Variance (σ^2): 4

The variance is 4, which implies the standard deviation (σ) is:

$$
\sigma = \sqrt{4} = 2
$$

Distribution Summary

Thus, the random variable X follows a normal distribution with mean -3 and standard deviation 2, denoted as:

$$
X \sim N(\mu = -3, \sigma^2 = 4)
$$

Clarifying the Range: Between 4 and 1

The problem asks for the probability that X is between 4 and 1, i.e.,

$$
P(1 \leq X \leq 4)
$$

Note that the order of bounds is from 1 to 4, which is standard in probability calculations when the lower bound is less than the upper bound.

Visual Representation

Plotting the distribution with the specified bounds:

- The mean at -3 is well below both bounds.
- The interval from 1 to 4 is located to the right of the mean.

This positional understanding helps anticipate the probability's magnitude.

Step-by-Step Solution Approach

To find $P(1 \leq X \leq 4)$, we follow standard steps:

1. Convert bounds to z-scores

The z-score transformation standardizes the variable:

$$
z = \frac{X - \mu}{\sigma}
$$

Calculate for both bounds:

- For $X=1$:

$$
z_1 = \frac{1 - (-3)}{2} = \frac{4}{2} = 2
$$

- For $X=4$:

$$
z_2 = \frac{4 - (-3)}{2} = \frac{7}{2} = 3.5
$$

2. Use Standard Normal Distribution (Z-table or computational tools)

Now, the probability becomes:

$$
P(1 \leq X \leq 4) = P(z_1 \leq Z \leq z_2) = P(2 \leq Z \leq 3.5)
$$

3. Find the cumulative probabilities

Using standard normal distribution tables or computational tools:

- $P(Z \leq 3.5)$
- $P(Z \leq 2)$

From Z-tables or calculator:

- $P(Z \leq 2) \approx 0.9772$
- $P(Z \leq 3.5) \approx 0.9998$

4. Calculate the probability between the two z-scores

$$
P(2 \leq Z \leq 3.5) = P(Z \leq 3.5) - P(Z \leq 2) = 0.9998 - 0.9772 = 0.0226
$$

5. Final answer rounded to three decimal places

$$
\boxed{0.023}
$$

Thus, the probability that X is between 1 and 4 is approximately 0.023.

Broader Context and Implications

Why is this calculation important?

Understanding probabilities within a normal distribution helps in various fields:

- Quality control: Estimating the likelihood of measurements falling within specified ranges.
- Finance: Assessing the probability of asset returns within certain thresholds.
- Medicine: Determining the probability that a biological measurement lies within healthy bounds.
- Engineering: Evaluating the chance of system parameters remaining within safe operational limits.

Sensitivity to Parameters

Small changes in the mean or standard deviation can significantly impact the calculated probability. For example:

- If the mean were shifted closer to the bounds, the probability would increase.

- A larger standard deviation would spread the distribution, affecting the probability of being within the same range.

Limitations and Assumptions

The calculation assumes:

- The data indeed follows a normal distribution.
- The bounds are accurately specified and correspond to the actual measurement units.

Any deviation from these assumptions might necessitate alternative models or non-parametric approaches.

Additional Considerations

Effect of Changing the Range

If the bounds were reversed (from 4 to 1), the probability remains the same, as the interval length is the same, and probability is symmetric:

$$
P(4 \geq X \geq 1) = P(1 \leq X \leq 4) \approx 0.023
$$

Extension to Other Distributions

While the normal distribution is the focus here, similar techniques apply to other distributions, with adjustments based on their specific properties and tables.

Summary and Final Remarks

In conclusion, the probability that $X \sim N(-3, 4)$ falls between 1 and 4 is approximately 0.023, when rounded to three decimal places. This calculation involves:

- Recognizing the distribution parameters.
- Transforming bounds to z-scores.
- Consulting the standard normal distribution.
- Computing the difference in cumulative probabilities.

This example underscores the importance of understanding distribution parameters and z-score transformations in probability calculations, which are fundamental skills in statistical analysis and decision-making.

References

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This comprehensive review aims to clarify the process of solving probability problems involving normal distributions, emphasizing clarity, step-by-step reasoning, and real-world relevance.

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